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When am I Richer than You?
Toward a Meaningful Comparison of Income of Nations

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&

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August 15, 2025

Abstract

Per-capita national income remains a primary indicator of wellbeing in an international comparison. Accordingly, the World Bank has popularised the concept of constant-dollar income of a given base year (say 2015), which converts income denominated in constant local currency units (LCU) to a common base year and uses the market exchange rate of that year to derive the ‘2015 constant-USD’ income which is directly comparable across nations. The issue we address here centres on the evolving base year; with each change, a new constant USD income series is born embodying the base-year market exchange rate, which tends to be volatile. Predictably, the prior ranking of income of any two countries may well get reversed. To be concrete, as per 2010-base, Japan’s per capita GDP appeared to have peaked in 1991 at 10.4 percentage points above the US level, but by the constant 2015 metric, the 1991 Japanese income was only about 3/4th. Was Japan actually richer than the US in 1991? In an attempt to answer this, we propose replacing the market rate by a smoothed exchange rate. We then compare the smoothed exchange rates as well as the resulting constant-dollar income series with the PPP exchange rate and PPP income. We show that some of the smoothed rates yield exchange rates that are at least as stable as the PPP, measure the exchange rate movement at par with PPP, and the predicted income levels appear mutually consistent with the PPP income.

[232 words]

Key Words: Constant USD Income, GDP Deflator, Market Exchange Rate, PPP Exchange Rates, Smoothed Exchange rates, Per capita Real Income

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1. Introduction

In spite of the progress made in measuring the concepts of human capability and wellbeing in their quintessential multi-dimensionality, per capita national income and its growth continue to be the top line figures that attract the most attention. The focus of the current paper is on the concept of per capita income and its measurement in the international context over a long horizon. We start off with the widely used per capita 'constant-USD' income series (GDP or GNI) that the World Bank (WB) group highlights. We find the description of their construction on the WB data portal to be sketchy. Moreover, we take issues with the cardinality of the concept especially between points of base change.

The 'constant dollar income' concepts are based on *constant* local currency unit (LCU) data, ensuring that one is dealing with *real* rather than *nominal* values. However, for comparability across nations the constant LCU data must be converted to a 'standard' currency, typically the USD. It relies on the base-year market exchange rate of the host-country's currency, and hence whenever the WB alters the base as it does every few years, the 'constant USD' values are directly altered by a multiplicative shift factor, namely the change in the host country's market exchange rate in the interim, which can be volatile in nature.[1] The latter feature injects an element of randomness in the resulting income concept to the extent the observed exchange rates may deviate, at least over the period in question, from what one may consider their long run equilibrium levels. With each base change, the historical ranking of income of any two countries often get reversed, which is awkward and sometimes implausible, but in view of the construction methodology, entirely predictable.

Since all WB constant-dollar income measures are anchored on the exchange rate of a given date, currently 2015 (i.e., the base year in the latest World Development Indicators, WDI, December 2024), potentially these offer a misleading comparison of the relative income of nations. Below we illustrate the above point by contrasting the WB constant income series of 2010 (in use till Sept 2021) with that for 2015 (launched in Oct 2021). In the historic context of Japan-US incomes of the late 1980s and 1990s, the 2010 constant series show per-capita income of Japan having matched or exceeded the US level between 1988 and 1998, while peaking in 1991 at about 10.4 percentage points ahead. Awkwardly, utilizing the 2015 constant data leads to a reversal of the above finding dramatically putting Japanese income of 1991 at

[1] The exchange rate used in constructing the 'constant exchange rate deflator' happens to be an average of market exchange rates over a period of one year (i.e., the base year). The averaging notwithstanding, this exchange rate is often highly volatile as we demonstrate below.

mere three-fourths of the US level! Were the Japanese richer or poorer than the Americans in 1991?

While the purchasing power parity (PPP) income of a country may well be a superior measure of living standard than the constant-USD measures, it is often neglected in historical time-series analysis since it originated in 1990, and initially only for a limited set of countries.[2] Hence, the constant-USD incomes become the default real income benchmarks, which in the WDI portal go as far back as 1960 for most countries. A closer look into the WDI's constant dollar methodology reveals that all constant-dollar income series (including PPP) are scalar transforms of the constant LCU series, and indeed all constant-dollar measures are linear translates of each other.

In spite of innovations in the PPP methodology, the constant-PPP exchange rate (most recently calibrated to 2021) yields just another estimate of 'true income'. We naturally ask if indeed there were other valid exchange rates that may capture the underlying economic fundamentals better or just as good as the constant PPP exchange rates.

The analysis of the paper thus delves into the construction of alternative but robust constant-dollar income concepts (per-capita GDP/GNI) that eliminates the anomalies implicit in the WB constant dollar methodology thereby permitting a more durable comparison of incomes across countries over time. In light of our findings, we propose that the market exchange rate of a currency be replaced by its forecast (e.g., following moving average or exponential methods) yielding a constant 'smoothed' series of the exchange rate. Depending on the precise smoothing technique utilised, the smoothed exchange rates and the resulting income magnitudes appear more stable between points of base year change and are thus presumed more reliable than the extant WB measures. We then contrast the exchange rates so derived with model-based estimates such as the PPPs as well as the 'fundamental equilibrium exchange rate' (FEER) concept arising out of the 'equilibrium exchange rate' (EER) literature.[3] EERs, by construction, average out transitory and random errors in observed market data. We also explore a methodology that may allow a comparison of alternative exchange rates, both their levels as well as the direction of change over an episode (e.g., between adjacent points of base change), to gauge their plausibility and hence that of the underlying income series. Some of the smoothing techniques yield exchange rates that appear as stable as the PPP, and we thus identify a class of 'locally' stable exchange rates.[4]

[2] For most East European countries, this series started around 2000, and for many small developing countries, even later.

[3] Indeed, IMF adopted the FEER concept in 2012 to monitor 'exchange rate imbalances' between currencies and to design policies to ensure global macroeconomic stability. Though often viewed as a model for the medium term, Saadaoui (2015) provides evidence that FEER performs very well as a predictor of long-run equilibrium exchange rates.

[4] The 'local' qualifier in the present context refers to the period between adjacent points of base change.

With the foregoing serving as an introduction, the rest of the of the paper proceeds as follows. In section 2, we offer a historical digression of 'who was richer' debate of the late 1980s to early 1990s in the Japan-US context that had been couched in the concept of current USD incomes. A switch to the WB constant-USD income concept shed additional light on the story, though the former's volatility between successive base years renders it a poor cardinal anchor. This leads us, in section 3, to examine the methodology behind the construction of the concept of the constant-dollar income in some detail where we see that the fault lies in its reliance on the base-year market exchange rate, which happens to be stochastic in a temporal perspective. Hence, in section 4, we probe the derivation of more resilient measures of constant dollar income by invoking the concept of a 'smoothed' exchange rate and its measurement. We further argue that the constant-PPP income series can also be extrapolated back to dates prior to 1990 so long as consistent constant-LCU data is available for that epoch. Section 5 provides an illustration of how the latter concept can be made operational in the historical Japan-US context. We then contrast the constant smoothed exchange rates vis-à-vis the FEER and the constant-PPP measures as to their mutual stability between adjacent base years as well their trajectory between points of base change (namely a currency appreciation or depreciation). We then reflect on the implications of the analysis on an evaluation of whether the Japanese were indeed as rich as the Americans in the late 20th century! Section 6 offers some concluding remarks.

2. The Japan-US per Capita Income, A Prologue

2.1 *Current Dollar Income*: The Economist magazine, on 25th October 1986 published a cartoon on its cover page featuring a Japanese character holding a diminutive Uncle Sam figure on its palm uttering the slogan: 'Now I'm richer than you'. The lead article inside, referring to current USD figures, observed that the Japanese per capita income was 17,000 vs 16,000 for the US (late October of 1986).[5] Pursuant to the Plaza Accord of 1985, G5 treasuries actively intervened in the currency markets bringing about a realignment to ease the prevailing current account imbalances, and in the process, propelling the Japanese Yen (JPY) dramatically upward.[6] In the week of 20th October, 1986, to coincide with *the Economist* issue in question,

[5] The historical national income data, especially valued at a given date, does not appear accessible; standard data sources yield average values on an annual basis. It is unclear how *the Economist* estimated the figures it cited; however, given its reputation, we take these values as being approximately correct by conventional accounting norms. The per capita values are typically arrived at by dividing the total annual GDP of a country by its estimated mid-year population. Given continuous innovation in national income accounts, published data for a given year may very well be different between points in time. The latest version of current dollar WB data accessed in December 2024 reports 1986 Japanese per capita income to be significantly lower than that of the US, 17,452 vs 19,071. Only in 1987, did Japan overtake US ($21,142 vis-à-vis 20,039), but then Oct 25, 1986 (the *Economist* publication date) was not far behind the advent of 1987!

[6] Recall that the JPY became very volatile following high inflation during the second world war. To stabilize the currency, the occupying US government fixed the exchange rate at 360 yen to the dollar in 1949 as part of the nascent Bretton Woods system. This rate prevailed till 1971, when the US abandoned the Gold Standard, effectively in favour of a flexible currency regime, which emerged in 1973. The JPY was believed highly undervalued by then resulting in high current account surplus, which annoyed the US and other major trading

the yen reached 154 to the USD, close to the high of 152 for calendar 1986. Acknowledging the role of the rising Japanese currency, the editorial argued, " … most estimates of what exchange rates 'should be', if they fairly reflected differences in productivity and costs, suggest that today's actual yen-dollar rate is not far from that hypothetical norm. So, the conclusion stands, that the Japanese are richer than the Americans" (*The Economist*, 25 October 1986, p15).

Indeed, as cited already, WDI data however shows the Japanese per capita GDP (current USD) to have exceeded that of the US first ever in 1987 at 21,142 vs 20,039 for the US. This essentially reinforces the point made by the *Economist* editors cited above. The mighty Yen held supreme till 1995, even touching 80 JPY to the USD in April of that year and moderating somewhat to become more or less stable for a while. In the process, the Japanese per capita GDP peaked in 1995 at 154% of the US value (current USD)! Oblivious to the fact that the real per capita output growth in Japan had appreciably slowed by then, some financial media sources even speculated that further increases in the Japanese currency might take its total GDP, again measured in current USD, close to that of the US, about twice as large in population.[7]

Figure 2.1 reveals that Japanese per capita GDP (in current USD), on the back of a surging currency, rose above that of the US in 1987 and remained at or above the parity till about 2000. Was this phenomenon primarily driven by the fast-rising relative value of the Japanese currency? To a curious Martian, the above fairy-tale rise of the Japanese income would fail to impart any meaningful knowledge of how well the *real* economy was performing vis-à-vis the US in those times. It may be instructive to educate the Martian a little.

While the October 1986 Economist leader was not the first to rightly acknowledge the ascent of the Japanese economy, which continued well into the 1990s, the hypothesis that perhaps uniquely in a class by itself, Japan was well poised to lead the industrial world into the 21st century did not quite materialize. The latter view had support even among serious scholars. Moses Abbramovitz, for example, wondered aloud "today there is a widely held opinion that America is about to fall behind a new candidate for leadership, Japan" (1986, p396).[8] The

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nations. This led the US Treasury along with then other G5 Finance Ministers to enact the Plaza (Hotel) Accord on September 22, 1985. "Public statements from the officials were backed up by foreign exchange intervention, selling dollars in exchange for other currencies in the foreign exchange market" (Frankel, 2015, p.5). Pursuant to the accord, all major currencies underwent realignment thereby setting in motion the spiralling rise in the value of yen against all major currencies.

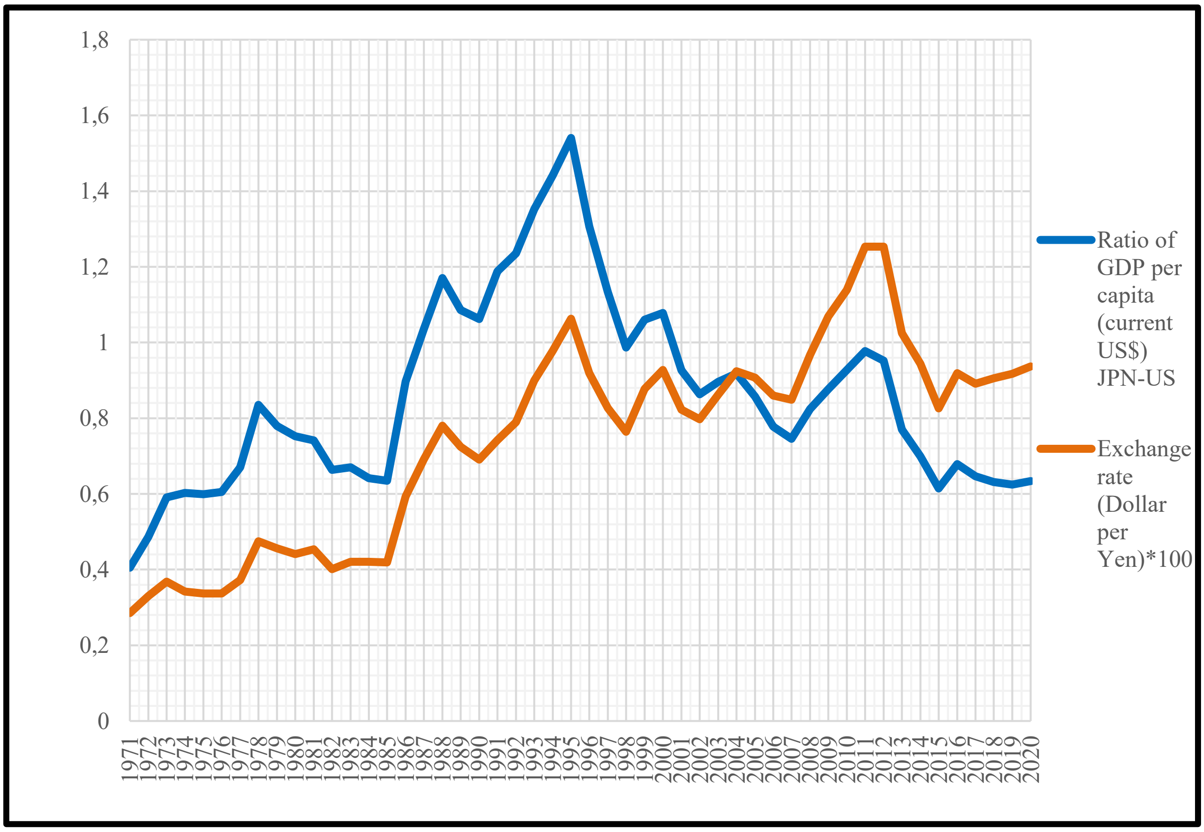
[7] Sam Jameson of the *Los Angeles Times* remarked, "… should the yen gain in value to reach an exchange rate of 69 to the dollar-- a level that some predict is a matter of months away -- Japan's GDP would match that of the United States at $6.74 trillion" (08 May 1995). He however hastened to ask if such a claim would be meaningful.

[8] Krugman (1994) cited the much earlier claims to the same effect by the futurist Herman Kahn (1970) and Ezra Vogel (1979), the Harvard sociologist.

forecast had gone awry! At the dawn of the new millennium, Japan stood a distant observer in the race to the top waiting to slip further down over the next two decades.[9]

2.2 *Real Growth in Japan*: The truth is that the apparent catch-up to the US level of income in the late 1980s was no flash in the pan. In terms of the real growth of per capita GDP, Japan to its credit had fared robustly not just in the immediate post-war reconstruction phase (say the 50s and the 60s), growth had retained its momentum well into the 1970s and 80s. Below, we contrast the ongoing low-growth phase, say 1992-2020, with the preceding era of high growth, which had gone on far longer. For the sake of comparability, we focus on the exact time length, 29 years, namely 1963-91, as representing the high-growth phase. During this period, Japan was the undisputed growth leader in the industrial world with its real per capita GDP (in constant LCU) growing at 4.93 per cent per annum (simple average) vis-à-vis 2.31 percent for the US, less than half the Japanese rate (Figure 2.2). Krugman claimed that during the 1970s and 80s, Japan grew both by adding resources as well as by generating efficiency in the production process. He went on to claim that as of the early 1990s it was attempting to catch up to the US level of efficiency (1994, pp74-75).

Fig 2.1 The Japan-US per capita GDP ratio (current USD) vis-à-vis the Exchange rate, USD/JPY *100, 1971-2020

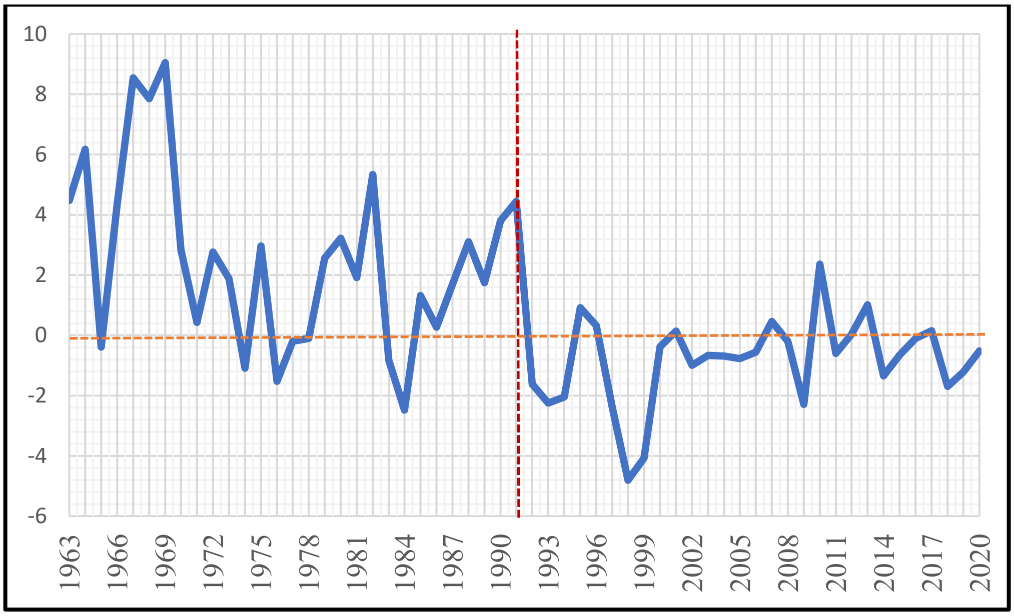


[9] Japan's rank in per capita income differs markedly between current USD and PPP. In current USD, Japan held the 6th position in 2000 (39.2K), but steeply fell to the 25th in 2020 (40.2K). However, in 2017 PPP, it was totally stable though far behind, 32nd in 2000 (36.3K) and 31st in 2020 (40.2K).

Source: Authors' construction based on WDI data

Subsequently, Kohli's (2003) analysis brought out Krugman's remarks into sharper focus. He found that technology's contribution to growth in Japan, namely the total factor productivity, was almost one percent per annum during 1967-1996 (precisely, 0.99 percent, *geometric* average over the 30 years).[10] Though well behind that of many European countries, this rate was more than double that of US' 0.4 percent over the same period (p.426).[11] Recall that Japanese real growth (4.7 percent) had been significantly higher than that of US' (3.0 percent) during the period analysed by Kohli, all figures in the latter paper being geometric averages.[12]

Figure 2.2 Difference (percentage point) in per capita GDP growth rate, JPN-US, 1963-2020



Source: Authors' construction based on WDI data

Turning to the recent low-growth phase, 1992-2020, we see that the Japanese per capita output has grown by a meagre 0.61 percent per annum (simple average). This rate was less than half that of the 1.44 percent pace set by the US over the same period, the mature economy standard.[13] Though a full-throttle analysis of the Japanese financial crisis and its consequences

[10] Kohli's growth accounting analysis was one of the first to adopt an open economy framework. He also employed a trans-log production function instead of the standard Cobb-Douglas.

[11] He also found that capital accumulation had indeed been the first factor of real growth in Japan at about 3.5 percent, well ahead of 1.3 percent for the US. On the downside, both US and Japanese growth had been reduced due to adverse terms-of-trade changes, though a little more harshly for Japan, 0.2 percent per annum vs 0.1 percent for the US.

[12] Recall that during the period of high growth in Japan, 1963-1991, it grew by 4.9 percent per in *per capita* terms and stood at more than double the US rate of 2.3 percent (simple averages). The period Kohli analysed, namely 1967-96, matches well with the period cited above, and he found the *geometric* average of real GDP growth (*total*, not per-capita) over the 30 years at 4.7 percent for Japan vis-à-vis 3.0 percent per annum for the US. In view of the differential rate of population growth between the two countries, these figures are mutually consistent.

[13] During 1992-2020, the per capita income growth turned negative 7 times in Japan as against just thrice for the US; in the latter's case that occurred during the Great Recession (2008 and 2009) and, of course, during the pandemic-hit 2020.

on economic growth and more particularly on productivity growth would take us far afield, a few remarks would be in order. Baily et al (2020) have analysed the growth slowdown in mature economies (namely, US, Germany, and Japan) using OECD Stats data. The dataset for Japan spans from 1985 to 2017; though, for US and Germany, the series extends to 2018. They find that the slower growth in Japan can be attributed to a mix of (i) a declining rate of hours worked (since 1995),[14] (ii) a slower TFP growth, and (iii) a lower contribution of capital deepening than seen earlier.[15]

2.3 *WB's Constant Dollar Income*: Admittedly, much of the euphoria surrounding the rise in Japanese income in the late 1980s was fuelled by exchange rate changes. What about constant price income? Figure 2.3 tracks constant price data between 1971-2020. It is seen that as per the 'constant-2010 USD' series, Japan's per capita GDP appeared to have just eclipsed that of the US (though just barely) in 1988, while peaking in 1991 at 10.4 percentage points above the US level. However, WB abandoned the 2010-base in favour of 2015-base in October 2021, presumably in keeping with the base change in the US national income accounts. By the new metric ('2015-constant USD'), the 1988 Japanese income was mere 69 percent of the US level, gradually moving up to about 3/4th (76 percent) the US level in 1991.[16] Note the difference between the dates when the Japanese income appears to have peaked, 1995 in nominal data versus 1991 in real data; more on this below.

What explains the large discrepancy? Though the WB constant price series, both GDP and GNI, are based on constant-LCU data, as stated already, the base year of comparison changes over time, which has proved far from innocuous. The shift from 2010-constant USD to the constant 2015-base dramatically altered the income level of countries adversely affected by the change in market exchange rates between the two dates. Herein arises the arbitrariness of the methodology. In the present context, the Yen depreciated against the Dollar by exactly 27.5 percent between 2010 and 2015.[17] It is hard to think that this shift was following any exchange rate fundamentals (e.g., productivity differences, differences in preferences or in government

[14] While the hours worked in Japan had been growing slowly even during 1970-85 (at 0.1 percent per annum), it declined rather vigorously at the pace of 1.0 percent yearly during 1995-2004, and then by 0.1 percent annually from 2004-2018.

[15] TFP growth fell precipitously to 0.3 precent (all figures being annual averages) in the most recent sub-period they analysed, 2004-17, vis-à-vis earlier figures of 1.3 (1995-2004) and 1.7 (1985-1995). Capital's contribution similarly fell from 1.6 percent (1985-95) to 0.7 (1995-2004) and then to 0.5 (2004-17), respectively. The Baily et al calculations of 1985-95 need not be inconsistent with the figures derived by Kohli for the 1964-1996 discussed above due to different estimation methodologies and the definition of productivity. Recall that Kohli used the GDP level as opposed to GDP per hour as the output indicator and all the averages were geometric instead of arithmetic.

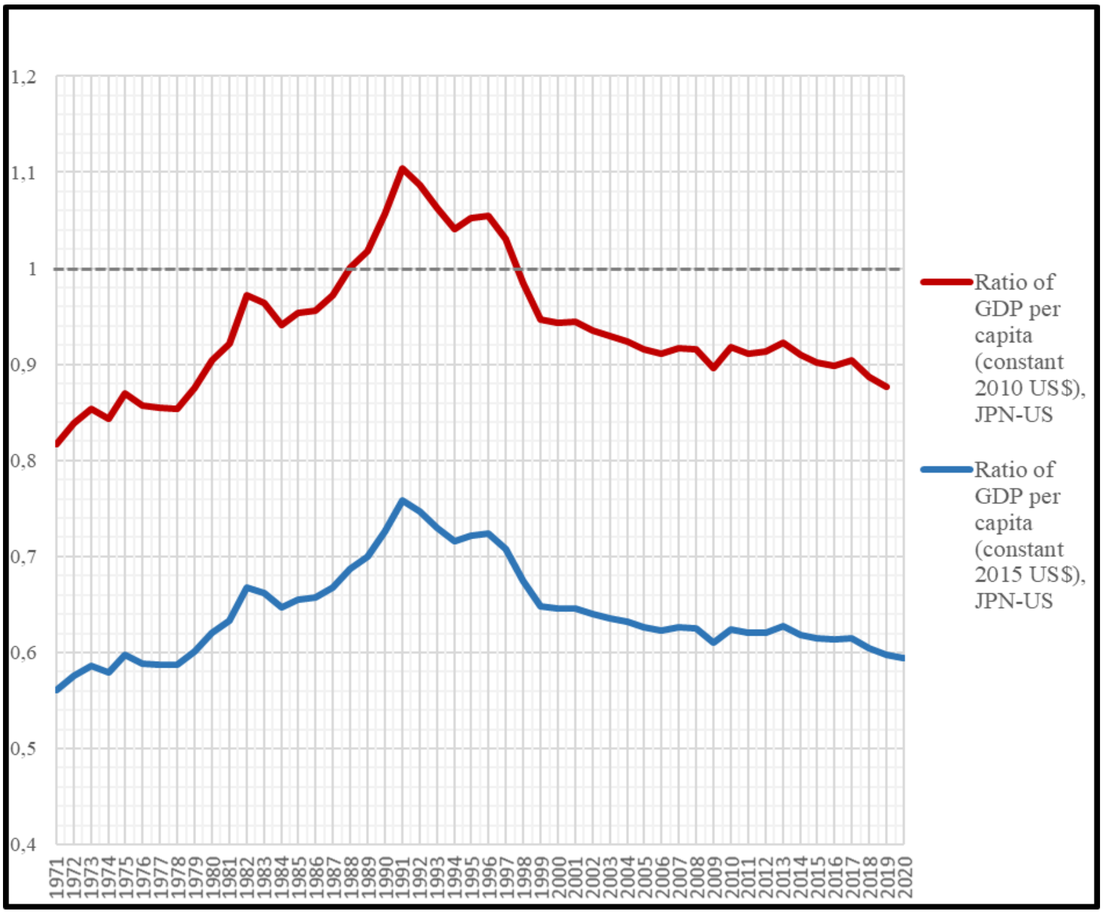
[16] Between 2010 and 2015, the relative (JPN/US) income decline was therefore 31.3 percent vis-à-vis the currency decline of 27.5 precent; and thus, the balance of the difference was due to inflation between the previous Japanese base year and 2010. See Box 2.1 below for a more precise analysis of the constant dollar ratios.

[17] The official 2010 value of Yen per dollar was 87.78, which was the highest value ever recorded till then. Similarly, the 2015 value of Yen, (121.04/USD), was the weakest against USD between 2003-15. Thus, both these values may be considered outliers, each in the opposite end of the spectrum!

spending) as subsequent corrections have demonstrated. Equally plausibly, there are contexts, where the market exchange rates may well track economic fundamentals and the observed shift may end up being persistent.

This just about brings us back to square one, namely the unreliability of the path of the market exchange rate and hence its role in measuring a country's standard of living. Was the Japanese currency overvalued in 2010 while undervalued in 2015? If so, by what margin? How will we ever know if the *Japanese were truly richer than the Americans during 1988-98?* How wrong was the *Economist*?

Fig 2.3 The GDP Per Capita, 1971-2020: Constant 2010 USD vs constant 2015 USD



Source: Authors' construction based on WDI data

2.4 *Some Elementary Regressions*: We undertake an econometric analysis to decipher if the association observed in Fig 2.1 was mere spurious. We also examine how well the constant price data actually measure real income. The results are summarised in Box 2.1.

The econometric tests reported below focus on the relative per capita income of Japan and US over the period 1971-2020. The model specification is based on the following reasoning. Given that we are trying to discover the properties of the per capita income series, not the determinants of income, the basic hypothesis is that the underlying macro variables explaining the systematic variations in income would be captured by the time trend. An income that is 'real' ought to be independent of the nominal variables, such as the market exchange rate, which in turn is determined by a variety of both real (e.g., volume of trade flows), capital, and speculative financial flows.

A visual inspection of the data reveals an abrupt change in the time path of the JPN-US ratio of current dollar per capita income that occurs in 1995, matching a similar shift in the nominal exchange rate, indicating a possible structural break. We thus incorporate this fact in our estimating model (via a set of dummy variables) in order to obtain more stable, i.e., time invariant, parameter estimates. In other words, we do not explore the causes of a probable structural shift.[18]

As described in Box 2.1, the analysis endorses the hypothesis of a primary role of the relative nominal exchange rates of the two currencies in driving the behaviour of the Japan/US relative income when measured in current USD. It also supports the idea of a structural break in the data in 1995, and the exchange rate variable retains its full significance in both time segments (1971-1995 and 1996-2020). A similar examination of the constant 2015 USD per capita income also admits a structural break; however, one that occurred back in 1991 thus pointing to a discrepancy in the patterns between nominal and real data. The latter also does not support any role of the market exchange rate variable, except maybe of a spurious nature. The apparent peaking of Japan's current dollar relative income in 1995 therefore does not reflect the underlying strength of the economy, instead the power of a rising Yen. These outcomes accord well with our *a priori* hypothesis.

Interestingly the constant USD income confirming the structural break as of 1991 coincides with what was apparent from an examination of the per capita real GDP growth in Japan as reviewed above. In the constant dollar regression, the exchange rate is found to be insignificant on both sides of 1991, while the 'time trend' is significant on both sides. Further, the latter *coefficients on the two sides differ significantly from one another*. This implies that there has been a change in the way time trend was affecting the GDP ratio (before and after the break). In terms of economic fundamentals, earlier we reviewed the findings by Kohli (2003) that during the faster growth phase, Japanese TFP growth (abut 1.0 percent annually) was more than double that of US. Baily et al (2020), on the other hand, found that the growth slowdown in Japan was caused among other by different factors such as declining reduced work hours, a muted capital deepening, as well as by much reduced TFP growth. The regression results are therefore consistent with the earlier research.

[18] Alternatively, one could require the regression model to detect structural breaks if any in the time series.

Box 2.1: Relative Japan-US Per Capita Income and the Relative Exchange rate, 1971-2020

Current USD Data, 1971-2020: The plots in Fig 2.1 of (i) the ratio of GDP per capita, Japan-US, in current USD and (ii) the exchange rate (USD per JPY) against time, 1971-2020, appear to exhibit a pattern of strong co-movement. Given the interpretation of the 'current dollar GDP', this is not surprising. The plot in addition shows a very pronounced change of trend in the ratio of GDP per capita – a switch from a strongly rising to a declining trend around 1995. We wish to statistically examine: (a) how strong the co-variation or correlation is, (b) wonder if this correlation can be 'spurious', and (c) ask if the observed change in the trend, about 1995, is statistically significant.

Accordingly, we propose the following naïve model for the underlying process that generates the per-capita GDP ratio between Japan and US:

$$(GDP\ ratio\ per\ capita)_t = \alpha + \gamma(Time\ trend)_t + error$$

which implies that all systematic variation in the per capita GDP ratio is captured by the time trend variable. However, given that GDP is in current dollar terms, it would be natural to add the exchange rate as an explanatory variable in the model:

$$(GDP\ ratio\ per\ capita)_t = \alpha + \beta(Exchange\ rate)_t + \gamma(Time\ trend)_t + error$$

We propose a further innovation by way of introducing a dummy variable. This is to capture a probable 'structural break' in the process generating the dependent variable, which is visually evident in Fig 2.1.

Thus, we restate the model allowing for the structural break:

(1) $(GDP\ ratio\ per\ capita)_t = \alpha_i + \beta_i(Exchange\ rate)_t + \gamma_i(Time\ trend)_t + error_t$,

$where\ \ i = 1\ for\ 1971 < t \leq 1995\ and\ i = 2\ for\ t > 1995.$

This however implies two separate regressions. Hence the dummy variable representation becomes:

(2) $(GDP\ ratio\ per\ capita)_t = \alpha_1 + \delta_1 D_t + \beta_1(Exchange\ rate)_t + \delta_2(Exchange\ rate)_t \times D_t + \gamma_1(Time\ trend)_t + \delta_3(Time\ trend)_t \times D_t + error_t$,

where $\alpha_1 + \delta_1 = \alpha_2$, $\beta_1 + \delta_2 = \beta_2$, $\gamma_1 + \delta_3 = \gamma_2$ and $D_t = 0$ for $1971 < t \leq 1995$ and $D_t = 1$ for $t > 1995$. Therefore a *test for the presence of a structural break* would amount to testing if $\delta_1 = \delta_2 = \delta_3 = 0$.

The estimated version of the final equation with 1971-2020 data yields:

(2a) $GDP\ ratio\ per\ capita = 0.096 + 1.37\ D_t + 1.36\ (Exchange\ rate)_t$

(2.5**) (14.0***) (11.6***)

$- 0.74\ (Exchange\ rate)_t \times D_t + 0.0008\ (Time\ trend)_t - 0.03\ (Time\ trend)_t \times D_t$

(- 5.02***) (0.2) (- 6.5***)

$R^2 = 0.964$, R^2 (adj) = 0.959; t-ratios in parentheses.

'**' and '***'denote statistical significance at 95% and 99% levels respectively.

The coefficient of exchange rate for the two time-segments, i.e., before 1996 and after are $\beta_1 = 1.36$ and $\beta_2 = 0.62$, (i.e., $1.36 - 0.74$), respectively. Both are highly significant.

…… 'Box' continues overleaf

Estimating the above without the dummy gives:

$$(1a) \qquad GDP\ ratio\ per\ capita = 0.52 + 1.6\ (Exchange\ rate)_t - 0.2\ (Time\ trend)_t$$

(8.7***) (10.6***) (- 8.5***)

$R^2 = 0.71$, R^2 (adj) = 0.695.

We thus reach two conclusions from the above regressions. (a) First, the hypothesis that the behaviour of the relative GDP per capita is strongly driven by the relative nominal exchange rate is clearly supported by the data. (b) Second, the results also support the hypothesis of a structural break, i.e., the hypothesis that $\delta_1 = \delta_2 = \delta_3 = 0$ is strongly rejected (as verified by a simple F-test using the 'residual sum of squares' from regressions (1a) and (2a).

Constant 2015 Dollar Incomes, 1971-2020: Given that the two graphs in Figure 2.3 are vertical translates of each other, one could work with either of the two data series, 2010 or 2015, and obtain similar results. We choose the latter series. We again wish to check for possible 'structural breaks' in the constant dollar measure of the GDP ratio since here too we observe an episode of persistent rise followed by steady decline, but now the turning point occurs in 1991, much earlier than in the current price data. Thus, the dummy would now take on the value 0 for 1971-1990 and 1 for all subsequent years, 1991-2020.

Estimation results:

$$(3) \qquad GDP\ ratio\ per\ capita = 0.45 + 0.4\ D_t + 0.004\ (Exchange\ rate)_t$$

(28.0***) (14.0***) (0.07)

$$- 0.007\ (Exchange\ rate)_t \times D_t + 0.008\ (Time\ trend)_t - 0.01\ (Time\ trend)_t \times D_t$$

(- 0.1) (5.4***) (- 8.1***)

$R^2 = 0.85$, R^2 (adj) = 0.83.

The dummy variable model above actually collapses into (4) if we set $\delta_1 = \delta_2 = \delta_3 = 0$:

$$(4) \qquad GDP\ ratio\ per\ capita = 0.6 + 0.2\ (Exchange\ rate)_t - 0.003\ (Time\ trend)_t$$

(33.9***) (3.86***) (- 3.36***)

$R^2 = 0.27$, R^2 (adj) = 0.24.

From eq (3), once again, note that both time trends are highly significant, though one propelled by sustained accelerated growth, while the other embodying steady deceleration, with both episodes measured relative to US. It is also apparent that eq (4), which fits the data poorly, is ill specified. Here the coefficient of the exchange rate turns out to be significant, but not so in (3). Thus, the correlation between GDP ratio and the exchange rate in (4) may well be spurious.

International Dollar (PPP) Data, 1990-2020: The WB GDP series in constant international dollar also happens to be a scalar multiple of the constant price GDP data (either LCU or constant USD). In a regression context, these would all prove to be observationally equivalent; the size of coefficient(s) would naturally depend on the unit of measurement of both the dependent and independent variables. Moreover, since the international dollar data is only available for a shorter period, the estimation is likely to give different results vis-à-vis those based on the longer constant USD measures reported above.

…… 'Box' continues overleaf

Box 2.1: Japanese-US per Capita Income [continued]

In the PPP data (either current or 2017 constant) we do not observe any role for the exchange rate variable in determining the ratio of per capita real income between Japan and US. Using *current* PPP data, we obtain the following:

(5a) $GDP\ ratio\ per\ capita = 1.007 + 0.03\ (Exchange\ rate)_t - 0.0062\ (Time\ trend)_t,$

(33.9***) (1.0) (-13.0***)

where $R^2 = 0.87$, R^2 (adj) = 0.86. As one would expect, the coefficient of only the time trend variable is statistically significant in equation (5a). The *constant* 2017 PPP series also yields very similar results:

(5b) $GDP\ ratio\ per\ capita = 0.99 - 0.002\ (Exchange\ rate)_t - 0.0054\ (Time\ trend)_t$

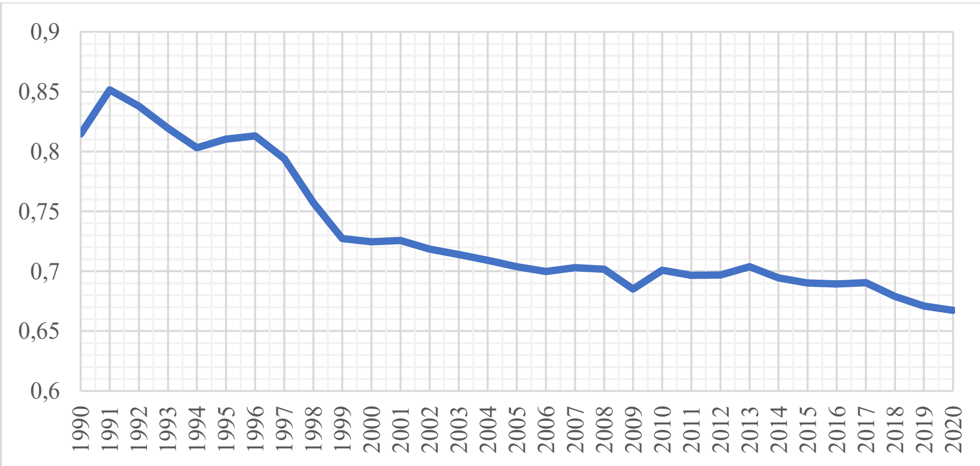
(29.4***) (-0.0061) (-10.2***)

$R^2 = 0.82$, R^2 (adj) = 0.80.

The exchange rate variable in the constant-2017 international dollar estimation turns out to be even more insignificant than in the current PPP data.

The above exercise reinforces the primary conjecture posed in this section, namely that the per-capita income ratio in current USD mainly reflected the behaviour of the market exchange rate between the two currencies. The idea of a structural break in the data is supported by the regressions reported above. The PPP data, especially the constant-2017 series, does not admit any material role of the exchange rate and thus can be presumed to reflect economic fundamentals governing the real growth of income.

Figure 2.4: Ratio of GDP per capita, constant 2017 PPP, JPN-US



Source: Authors' construction based on WDI data

There being hardly any observation prior to 1991, testing the hypothesis of a structural break in PPP data becomes a moot point, though see Figure 2.4 and Fig 5.5.[19] We however argue below that there is no conceptual reason why one cannot utilise the constant LCU data of period earlier than 1990 to configure the missing PPP income of earlier years. We turn to the PPP methodology in section 4 below.

To conclude this section, we note that the magical surge in the Japanese per capita national income (current USD) especially during the 1985-95 episode was more an artefact of the dramatic currency gain than real growth. The constant dollar incomes measure real growth as well as does the PPP income; these series support the hypothesis that the Japanese relative income had peaked in 1991, much earlier than predicted by nominal data. However, the choice of the base-year required in the constant dollar methodology fails to offer a credible cardinal measurement of relative income between the two countries due to the apparent fallibility of the market exchange rate to consistently track evolving economic fundamentals. The issue of an ideal benchmark for an international comparison of per capita income thus remains an open question.

3. The WB Constant-Dollar Income Methodology

Let us briefly examine how the WDI data portal describes the computation of the constant-dollar per capita income values, or any other constant series for that matter. For the ease of notation, we discuss the national income of a reference host country (without indexing it). The numéraire country is of course the US. Let $\{x_t\}$ denote the host country income in *current* LCU at time-t, while $\{y_t\}$ be its *constant* local currency counterpart. At a given point in time, therefore, the value of the constant series would eliminate all host country inflation that had occurred since its base change, whenever that may have happened. The object now is to construct the *constant USD value* of the host country income calibrated to a common base year as in the latest WDI dataset, namely 2015. We label the latter value by $\{z_t\}_{2015}$.

3.1 *The Constant Exchange Rate Idea*: Published data appears to show that the series $\{z_t\}$ is effectively obtained as:

$$(6) \qquad \{z_t\}_{2015} = [\{y_t\}/ ER^{constant}{}_{2015}],$$

where $\{ER^{constant}{}_{2015}\}$ denotes what we call, the 'constant-2015 exchange rate deflator', or in other words, the 'constant-2015 dollar' exchange rate. The latter in turn appears to be constructed as follows:

$$(7) \qquad \{ER^{constant}{}_{2015}\} = [(ER^{Market}{}_{2015}) / (x_{2015}/y_{2015})],$$

[19] The constant 2017 PPP data thus finds the Japanese per capita income to have indeed reached its highest level in 1991 at about 85% of the US value of the year, a far cry from the 110.4 % as yielded by the 2010 constant USD incomes, but well above the 76% as per the 2015 constant USD series.

where ER^{Market}_{2015} denotes the official exchange rate for 2015, i.e., the annual average. First observe that the constant-2015 exchange rate, (7), is independent of time-t, and hence, a scalar. Next, we note that right-hand-side of (7) merely divides the nominal exchange rate of 2015, by the domestic GDP deflator relevant for the period in question, (x_{2015}/y_{2015}), namely inflation between the point of base change in the host country national income accounts and 2015. Thus, the constant-2015 USD income, $\{z_t\}$ reflects true growth in income as measured by the constant LCU series, $\{y_t\}$. For Bangladesh data, for example, the values in equation (7) are:

(7a) $\{ER^{constant}_{2015}\} = [(77.7021)/(97{,}007.44/52{,}789.06)] = [77.7021/1.8376]$

$= 42.2836,$

where the 2015 GDP deflator factor in question happens to be 1.8376.[20] Thus, while the nominal Bangladesh exchange rate in 2015 was 77.7021 BDT per dollar, its 'constant 2015 USD' value came to 42.2836. Though none of this appears to be described in any WB document, the constant exchange rate has an intuitive interpretation; it is merely the value of 2015 market exchange rate (77.7) in BD's base-year prices (here 2006).[21]

The advantages of the resulting constant USD are plain. Not only does it eliminate all inflation from the host country GDP data, *at the reference point*, namely 2015 here, the 'constant dollar' income becomes identical to the current dollar income.[22] In other words, the procedure effectively converts the host-country national accounts data from its historic base to a new common base of 2015, *in USD terms*, for all countries in the WDI dataset rendering them directly comparable. Hence the coinage, 'constant-2015 USD' income.

3.2 *A Stochastic Shift Factor*: The construction described above however comes with baggage. Restating eq (6), we obtain:

(6a) $\{z_t\}_{2015} = [(y_t / ER^{Market}_{2015})\ (x_{2015}/y_{2015})],$

which illustrates that the 'constant-USD' income is inversely related to the market exchange rate of the base year.[23] Thus, any base change by WDI, e.g., the recent switch from 2010 to 2015 in Sept/Oct 2021, would directly impact the value of the resulting 'constant-USD income' of all countries by a multiplicative shift factor that is stochastic in nature.[24] Consequently, any two countries whose exchange rates are differentially affected by transient shocks, would be

[20] Here we are using the 2006 base-year national accounts data for Bangladesh, which was in effect till the Fall of 2021. Thus, BD producer price level had increased by about 84% between 2006 and 2015.

[21] After having worked out the above analysis, it occurred to us that there exists an alternative interpretation of the WB constant dollar income concept that does not invoke the idea of a 'constant dollar exchange rate'. See the Appendix.

[22] Letting t =2015, it follows from (6a) that $\{z_{2015}\}_{2015} = [(x_{2015}/ER^{Market}_{2015})]$ = current USD income.

[23] It seems that WDI changes its base whenever that occurs in the US national accounts.

[24] Letting θ denote the ratio (1/Exchange rate), the multiplicative shift parameter cited above can be written as $\Delta\theta T$ ($\equiv \theta_T - \theta_{T-1}$), a scaler, where T and (T-1) are the two adjacent base years. Even though both θ_T and θ_{T-1} are observed data, the behaviour of $\Delta\theta T$ remains unpredictable *ex ante*.

affected very differently by this construction even though their long-run paths may turn out to be similar. Exchange rates, unlike most macro aggregates (e.g., GDP), are known to be highly sensitive to speculative shocks, geopolitics, and the like (Benigno et al, 2011). We have already seen the havoc that the recent base change from 2010 to 2015 in WDI data played to the comparison of real income between Japan and USA (Fig 2.3). What is concerning is that such fluctuations in the shift factor between adjacent base years are not merely restricted to JPY/USD rates or even a handful of cases. Even a cursory look at the data suggests that the phenomenon is widespread, both for advanced (e.g., GBP/USD or CAD/USD) as well as emerging nation currencies (BDT/INR).

The short discussion above (including that in the appendix) underscores two points. First is the paucity of adequate explanation and interpretation of the procedures that the world bank staff follows in processing national income data, and secondly the awkward reliance of the constant dollar income concepts on the vagaries of the market exchange rate. There has been some prior reference in the literature to concerns over the WB data practices.[25]

4. The 'Smoothed' Constant USD Income: A Definition

It appears that there have been several distinct approaches in the literature to the conceptual measurement of the relative *purchasing power of income* between countries, i.e., an exchange rate that equalizes prices between the respective countries. Most well known is the constant PPP exchange rate, based on actual consumption and price data of different countries, that has been in continuous innovation over the past five decades or so. The second is the equilibrium exchange rate (EER), a concept entirely reliant on modeling of time series data deciphering how the former ought to be determined. While both PPP and EER concepts are premised on the 'the law of one price', as we see below, the resulting exchange rate figures often come out as dramatically different from each other.

Given this tension, we explore the notion of 'constant smoothed exchange rates', which are both conceptually and empirically far easier to estimate than either the EER or the PPP price relatives. The smoothing process, effected by statistical techniques, removes random variation from the observed exchange rate data by replacing actual values of a given date by its forecasted value based on past observations.[26] Ideally, smoothing could have generated the underlying true data if we knew the data generating process or had a theory that discovers real underlying data from noise (e.g., as claimed by EER enthusiasts). However, lacking a rigorous methodology, our approach here is an eclectic one. We examine several alternative smoothing

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[25] In an evaluation of WB research, the analysts, while underscoring the centrality of data to the Bank's own research and for development research in general, found that " … data activities are organized haphazardly, whether in collection, archiving, or dissemination. The Development Economics data group is not as centrally involved with researchers in the collection and dissemination of Bank data as is desirable" (Banerjee et al, 2006, p6).

[26] There is a long tradition of using smoothed series for variables such as the exchange rate in international monetary economics and related topics. See for example, the paper by Garcia et al (2011).

techniques, and then compare the resulting values with the model-based forecasts, namely the FEER and PPP. This latter exercise follows a normative framework, that entails a comparison of the spread between the estimates of exchange rates at two points in time as well as the direction of the change in the interim (i.e., appreciation vs depreciation).

4.1 *Smoothed Constant Dollar Income*: In our quest for a robust measure of per-capita income, we propose that the market rate be replaced by a constant 'smoothed' series. Let us denote the latter by $\{ER^{Smoothed}{}_{2015}\}$, which can be obtained from $(ER^{Market}{}_{2015})$, calibrated to the WDI base year, 2015, by adopting standard forecasting means such as the moving average and exponential techniques. Therefore, following (6a), the 'smoothed' series, $\{z_t(s)\}_{2015}$, can be written as

$$(8) \qquad \{z_t(s)\}_{2015} = [(y_t / ER^{Smoothed}{}_{2015})\,(x_{2015}/y_{2015})].$$

Below we illustrate how such smoothing may improve the reliability of the dataset so modified and thus yield a more informed concept of income relevant for a meaningful international comparison.[27] Since most data to be analysed in the present context would be in the format of 'ratios' for two countries (say, A and B), from (8) we get:

$$(9) \qquad [\{z_t(s)\}^A/\{z_t(s)\}^B]_T = \{(y_t)^A/(y_t)^B\}[\{ER^B(s)\}/\{ER^A(s)\}]_T\,[(x_T/y_T)^A/[(x_T/y_T)^B]],$$

where we denote the relevant base year by subscript-T and let (s)-denote smoothed values. Therefore, we see that the relative values of constant USD income between a pair of countries depend on three factors, (a) their relative real incomes (in constant local currency, $\{(y_t)^A/(y_t)^B\}$), (b) the relative value of one's currency, albeit smoothed, vis-à-vis the other $\{ER^B(s)\}/\{ER^A(s)\}$, and finally on (c) the relative GDP deflators, $[(x_T/y_T)^A/[(x_T/y_T)^B]$. The last element underscores the fact that each country has a different base year in the national income accounts and hence the value of the deflator as of the date of the WB base year (e.g., 2015) can vary substantially even if the underlying *annual* inflationary pressures were comparable in magnitude.

4.2 *PPP Income*:[28] As stated by Deaton and Heston, the PPPs 'aim is to measure how much local currency is needed to buy as much as does the currency in the numeraire country, usually the US dollar' (2010, p1). In spite of continuous innovation in the PPP methods, primary limitations arise out of the differences between countries at different levels of economic development, and with that the variations in the quality of national accounts data, comparability of similar goods, the range and quality of non-traded goods, and the extent of goods produced for household consumption. Moreover, prior of 1996, most direct price surveys by ICP had

[27] Note that the shift parameter in question presently, $(1/ ER^{Smoothed}{}_{2015})$, unlike that in (6a), is a stable scalar, a sort of 'expected value'.

[28] The PPP methods were pioneered by the International Comparison Program (ICP) led by Irving Kravis, Alan Heston, and Robert Summers in the late 1960s - early 1970s; see Kravis et al (1978). What began as a joint project between United Nation's Statistical Office (UNSO) and UPenn, become a WB-UNSO managed initiative beginning 1993. The results of the ICP 2017 have been summed up by Deaton and Schreyer (2022).

focussed only on advanced countries, while for others, the informational gap was filled by extrapolations and guesses.

Questions have also arisen on the evident volatility of PPP values between successive rounds of the ICP exercise. The standard practice has been, as Pinkovskiy & Sala-i-Martin (2020) observes, "… to revise both current and all past estimates of GDP once a new ICP price survey is conducted, essentially throwing out the price data from the previous rounds" (p73). To this Deaton and Schryer (2022) counter that "… in future rounds, the ICP plans to move to a rolling data program, where new data are collected and incorporated on a continuous basis, and this should further guarantee the consistency over time that users require and expect" (p4).

All said and done, it is clear that 'true comparable incomes' remain unobserved. Though both Japan and US are advanced economies, cultural differences might call into question the comparability of consumption baskets and their prices. The latter issues would be more challenging when comparing a more disparate set of countries.

How is constant PPP measured? Let $\{\varphi_t\}$ denote current PPP ('international dollar') values, while the 'constant-2017 PPP' series is, in turn, denoted by $\{\varphi(k)_t\}$. Predictably from WB PPP methodology, we see that:

(10) $\{\varphi(k)_t\} = [\{y_t/y_{2017}\} \times \{\varphi_{2017}\}]$, $t = 1990, …., 2020, …$, or, equivalently as

(10a) $\{\varphi(k)_t\} = [\{y_t\}/\{y_{2017}/\varphi_{2017}\}]$.

Expression (10a) is more easily interpretable since the constant LCU income of year-t is being divided by a price index, namely the 'constant 2017 PPP exchange rate'. In the present context the latter magnitude happens to be JPY 104.74/USD.[29]

Given the ready comparability of (8) and (10a), we argue that conceptually the constant PPP exchange rate may not be entirely alien to the notion of a 'smoothed exchange rate' to the extent the latter exercise succeeds in eliminating noise in the observed data. In any event, it may be instructive to examine how good a 'smoothed exchange rate constant-dollar income' would fare vis-à-vis the 'constant-PPP' or EER magnitudes.

4.3 *Derivation of Constant Smoothed PPP Income for Different Base Years*: Though the constant PPP base year has just been updated to 2021 (as of December 2024), we retain the 2017 benchmark as it is closer to the WDI constant dollar income base of 2015, which is of immediate focus in the present analysis. Nevertheless, in view of different base years used in constant USD income and PPP, a base conversion is in order. We could derive the 'constant smoothed USD income' for 2017 (the same as for PPP). This would have been straight forward. All we needed do is to set T = 2017 in the expression (9) above.

[29] Note the uniformity in the construction of each of the constant price income concepts (constant dollar, smoothed, and PPP) as described by expressions (6a), (8), and (10a), respectively.

Alternatively, one can construct the constant-PPP series with 2015 or 2010 as the base year, which may be more relevant if the presumption were that 2015 exchange rates were less reliable in view of values yielded by forecasting methods as detailed below. Here all is needed is first to convert the constant LCU series to an index by dividing by the 2015/2010 value, as in (10), and then multiply the coefficient by the 2015/2010 current PPP income figure as appropriate. Since we use the World Bank GDP series (for Japan) in constant-LCU that covers the period 1960-2021, the *derived* constant PPP Dollar income series can also run from 1960 to 2021.

In other words, we do not see any conceptual problem in using the constant PPP Dollar series (for Japan) for periods before 1990. The reason is obvious! Consider the per capita income of Japan in constant 2010 LCU. To convert this to constant 2010 PPP dollar, we use the PPP exchange rate for 2010.[30] This exchange rate considers relative purchasing power between Japan and US for 2010 alone. Information about relative purchasing power prior to or after 2010 (between these two countries) has no role here. The fact that, the relative price information across countries that is used to construct the PPP conversion factor does not exist for periods before 1990, should not matter here.[31]

The constant-PPP exchange rate (JPY/USD) happens to be 111.71 for 2010 and 103.47 for 2015, as against the market rates of 87.78 and 121.04, respectively, for these two points in time. Not only the PPP rates are vastly different, and they lie in a tighter range than the market rates, the direction of the change also has been reversed. As per PPP, JPY firmed up against the USD in 2015 vis-à-vis 2010, quite a reversal from the dramatic weakening as the market data implied. Use of these rates would logically render the per capita incomes (constant 2010 vs constant 2015) much closer as we examine further below.

This section therefore accomplished two tasks, the first of which has been to lay down the idea of a smoothed exchange rate for a given date, which can be used to deduce per-capita income series of a country in smoothed constant-dollar terms and hopefully avoid the volatility exhibited by WB measures of constant-dollar incomes. Secondly, we demonstrate that the current PPP exchange rate data can be leveraged to derive constant-PPP per-capita income of a country for any base year, which can be extended as far as back in time (say 1960) for which constant-LCU data existed.

[30] WB publishes the *current* annual PPP exchange rate of a currency for all data points. The only issue in choosing an arbitrary base year is that the former may not coincide with the ICP survey years (e.g., 2011 or 2017) and thus the 2010 PPP rate must have been in the nature of an imputation or interpolation and would not fully enjoy the informational basis attributable to the periodic surveys, especially for countries for which direct price data has been utilised.

[31] An Excel worksheet that details the arithmetic with actual data, 1960-2021, can be obtained from the authors upon request.

5. Smoothing Methodology: An Illustration

Smoothing methodologies are a part of a larger literature on statistical forecasting. Statistical methods of smoothing typically include 'simple moving average' and 'exponential smoothing'. While in simple moving average all past observations are weighted equally, exponential methods assign exponentially decaying weights on past data. Moreover, general models of exponential smoothing typically allow for multiple smoothing parameters to capture elements such as the trend, seasonality, and cycles/periodicity (Nau, 2014).

Exponential smoothing models are used to generate optimal time series forecasts by minimizing a given function of the forecast errors (e.g., the root mean squared error, RMSE) with respect to the smoothing parameters. Optimal forecasts are in effect designed to track the observed data closely. However, models of exponential smoothing can also generate forecasts without resorting to optimization. This involves choosing a set of starting values for the parameters which are held constant as one runs the forecasting routine. However, depending on the number of parameters in the model, the choice of alternative sets of starting values can lead to a large number of forecasts. Hence one might require some stopping rule based on some notion of a 'desired level of smoothing'.

From the perspective of exchange rates, we merely seek methods that by and large even out the peaks and troughs from its path. Thus, the success of smoothing may be gauged not by its capacity to mimic the actual data, but visibly smoothen the series vis-à-vis competing forecasts. Below we offer a methodology for choosing among the alternatives.

5.1 *Moving Average*: First let us examine the 'simple moving average' (SMA) exchange rates. By definition, 'moving average of a certain order-p, SMA(p)' of a series $\{\chi_t\}$ will generate a new series S_t, such that

$$(11) \qquad S_t = \frac{\sum_{k=0}^{p-1} \chi_{t-k}}{p} .$$

In other words, each observation in the new series is an arithmetic mean of the corresponding observation in the original series as well as past observations going back p-1 periods.[32] Here each observation receives an equal weight. Given χ_t is a volatile series, (e.g., yearly series of JPY per USD), the degree of smoothness in the derived series, S_t, will increase as the number of intervals (p) also increases (Figure 5.1). While Fig 5.1 displays various lengths of the moving average, SMA (9) appears to offer the smoothest series, though at the loss of some initial observations.[33] In what follows, we have chosen to focus on the 5-period moving average, SMA(5) for our illustration, which appears to smooth out the fluctuations/spikes about as well

[32] The moving average definition (11) includes the current value to derive an estimate of the same, while some authors exclude the current observation.

[33] From eq (11) it follows that choosing an order-p implies that the effective length of the dataset-S would lose (p-1) values at the start of the sample.

as those of a higher order. Especially in the period after 1990 or so, there appear no kinks in the SMA(5) smoothed series vis-à-vis the market rates.

5.2 *Exponential Smoothing*: While the SMA-idea is intuitive, as Nau states, focussing only on a limited number of past observations and allowing "… no weight on any previous observations is usually not the best way to average values that are arriving consecutively in time" (2014, Ch 3, p7). While there are many versions of exponential smoothing, the first is the simple exponential smoothing (SES). Though based on the moving average idea, a new feature is that past data are weighted in an 'exponentially decreasing manner'.[34] SES can be described by the following recursive process:

(12) $\mu(\chi, 1) = \chi_0$, and

(13a) $\mu(\chi, k) = \alpha\chi_{k-1} + (1-\alpha)\,\mu(\chi, k-1)$, or,

(13b) $$\mu(\chi, k) = \alpha\chi_{k-1} + \alpha(1-\alpha)\chi_{k-2} + \alpha(1-\alpha)^2\chi_{k-3} + \cdots$$

where μ-denotes the predicted/forecast value, $\alpha \in (0,1)$ the smoothing factor, and k-the length of the series, i.e., the number of data points (Nau, Chapter 3). It assumes no trend in data. Written out fully (as in 13b), this structure implies placing decreasing weights to past observations. Notably, only past observations (i.e., excluding the current one) figure in forecasting the current value. For $\alpha = 0.5$, the most recent observation attracts the highest weight, 0.5, and declining thereafter. Thus, for k = 4 (i.e., k = 0, 1, 2, and 3),[35]

(14) $\mu(\chi, 3) = [0.5\ \chi_2 + 0.25\ \chi_1 + 0.25\ \chi_0]$.

Note however that neither the SMA-model nor the SES allowed for a trend in the time series in question, which would appear to contradict, for example, the pattern in the JPY/USD exchange rate as can be gleaned from Figure 5.1 or 5.2 below. There happen to be several smoothing techniques, generally known as 'double exponential smoothing' (DES) that attempt to deal with the trend issue. 'Brown's linear exponential smoothing' (BLES) is one such method; however here the same parameter (denoted by α above) is utilized to adjust both the level and the trend in data (Brown, 1963). Consequently, the level and the trend terms cannot vary independently of each other. Later Holt (1957) generalized the BLES model by adding a separate smoothing constant for the trend, say $\beta \in (0,1)$. To avoid a proliferation of acronyms, below we refer to this model simply as DES.

Finally, for our discussion, we cite two additional methods: the Holt-Winters 'triple exponential smoothing' (TES) and the 'generalized exponential smoothing' (GES). The Holt-

[34] SES method is also known as 'exponentially weighted moving average model' or even, 'simple exponential smoothing'.

[35] Thus, given our convention, the past three observations are used in (14) to predict the fourth, χ_3, i.e., $\mu(\chi, 3) = \alpha\chi_2 + \alpha(1-\alpha)\chi_1 + (1-\alpha)^2\chi_0$.

Winters method assumes that the data series has both a *trend* as well as a recurring/repetitive pattern or seasonality (Winters, 1960), an element that was absent from the DES method. GES allows for even more complex time series patterns, such as both linear and non-linear trends, etc. In order to generate the smoothed series, both methods minimize RMSE with respect to the smoothing/damping parameters.

What do the exponentially smoothed exchange rates look like vis-à-vis the market? Visually examining the plots in Fig 5.2, it is seen that SES (with $\alpha = 0.4$) is the only one that appears to even out the troughs and peaks in the market data. All other methods seem to move these spikes around, perhaps a little less so for the BSES. TES in particular tends to heighten the spikes and widen the duration especially on the upside, i.e., when JPY seems to rise in value vis-a-vis the USD.

5.3 *The 'Fundamental Equilibrium Exchange Rate' (FEER) Idea*: The advantages of using some notion of an equilibrium exchange rate as a substitute for the market exchange rate in the computation of constant exchange rate deflator is two-fold. First, EERs are, by definition, less volatile than market exchange rates. Second, unlike the standard smoothing techniques reviewed above, there are strong theoretical arguments in support of EER especially in the long run, since it is solely determined by the market fundamentals. Transitory factors however are also relevant in the determination of a short-run equilibrium. The above argument has been summed up, for example, by Driver and Westaway (2005):

$$\begin{matrix} Market\ exchange \\ rate\ at\ t \end{matrix} = \begin{matrix} Economic\ fundamentals \\ that\ determine\ exchange \\ rate\ at\ t \end{matrix} + \begin{matrix} transitory\ factors \\ at\ t \end{matrix} + \begin{matrix} random\ factors \\ at\ t \end{matrix}$$

A time-honoured theory of the determination of EER is the notion of PPP or the 'law of one price'. Following this, prices across countries are equalized in the long run by EER. To the extent PPP holds, (and that the constant dollar income series are constructed using EER), comparing constant dollar income between countries will then truly amount to comparing the real standard of living between countries, the ultimate goal.[36]

A sojourn into the alternative strategies of estimating the PPP model would take us far afield. For the purposes of illustration, we utilize JPY/USD values for 2010 and 2015 estimated by the 'fundamental equilibrium exchange rate' (FEER) model (Cline, 2015).[37] At JPY 84 and 107 vis-à-vis market rates of 87.78 and 121.04, respectively for these two dates, the FEER estimates reported by Cline do not appear to accomplish a dramatic narrowing of the spread,

[36] In principle, the conditions under which the exchange rate between countries should follow PPP are rather stringent (Driver and Westaway, 2005).

[37] See Rossi (2013) for a comprehensive survey of the EER literature and Itskhoki and Muhkin (2021) for a more recent update.

possibly reflecting the frustration of scholars in the field for the lack of correlation between exchange rates and other macroeconomic variables.[38]

5.4 *Outcome of the Smoothing Exercise*: Prior to examining the smoothed exchange rates, we ask how to gauge the quality of smoothing. We propose two normative criteria: (a) the extent smoothed estimates narrow the spread between the 2010 and 2015 values; and (b) the direction of the movement of the exchange rate estimates between the two dates. The idea of 'narrowing of the spread' makes good intuitive sense, as the market exchange rates for both 2010 and 2015 appear to be complete outliers with each lying on the opposite side of the spread. The point is that if one believed that JPY at 87.8/USD was overvalued in 2010 but undervalued in 2015 at 121/USD, the 'true' values should have been somewhere in between at these two points in time. Thus, while entirely normative in character, we propose that one primary indicator of the success of the smoothing process is the narrowing of the spread (ideally in percentage terms) between the estimated values for 2010 and 2015 vis-à-vis the observed market rates for the same dates.

Table 5.1 Smoothed vs Actual, FEER and PPP Exchange Rates

Smoothing Regime ↓	2010	2015	Difference: Basis Points [%][39]	Ratio of 2010/2015 Exchange Rates
(i) SMA-5	103.75	96.84	7 [6.9]	1.07
(ii) SES (alpha = 0.4)	103.85	96.81	7 [7.0]	1.07
(iii) BSES	93.57	105.95	(-)12 [-12.4]	0.88
(v) DES	87.21	99.59	(-)12 [-13.3]	0.88
(vi) TES	79.01	102.41	(-) 23 [-25.8]	0.77
(vii) Constant PPP	111.71	103.47	8 [7.7]	1.08
(viii) FEER	84	107	(-) 23 [-24.1]	0.79
(ix) Market	**87.78**	**121.04**	(-) **33 [31.9]**	**0.73**

Source: Figures in rows (i) - (vii) have been derived by the authors based on WDI data

As for the second criteria, we flag the direction of the exchange rate movement between 2010 and 2015. The market rates reveal a near 38-perent *depreciation* of the Japanese currency (vis-à-vis USD) over this period. By contrast, the derived constant PPP rates instead point to an *appreciation*, albeit in the smaller range of about 8% (Fig 5.3B and Table 5.1). Moreover, the

[38] The figures of JPY 84 and 107 are reported without decimals and were dubbed 'FEER consistent' dollar rates for 2010 and 2015. Since we do not have access to Cline's full dataset, we cannot display a plot of these rates for a possible comparison with those derived in this paper (i.e., as in Figures 5.1, 5.2, and 5.3).

[39] All percentage figures shown here within the square brackets are arc measures, namely the absolute difference divided by the average of the 2010 and 2015 values.

PPP rates (Fig 5.3B) appear very stable over the last decade in the dataset (i.e., 2012-2021), and, if anything, registering a feeble appreciation of the Japanese currency. Why the stress on PPP-exchange rates as the benchmark? Well, though not without flaws of their own as noted above, these may be the 'closest' thing to the notion of 'equilibrium exchange rates? The smoothed rates, if in effect exhibit properties similar to constant PPP values, may therefore form what we can call 'the class of locally stable exchange rates' deserving of attention in future research.

To recap the smoothing analysis, we display the exchange rate estimates for SMA(5), SES, and BSES for a shorter period for expositional clarity (Figure 5.3A, 1986-2021) and in Fig 5.3B which also adds PPP line (but dropping BSES) over the full dataset, 1990-2021. The PPP line shows no spikes at all and mingles well with several of the smoothed series starting about 2006 or so.[40] We explore the implications of these exchange rates on the measurement of the relative per-capita income of JPN-USA in section 5.5 below.

A summary of the smoothed exchange rates is presented in Table 5.1 along with Cline's FEER estimates and the derived constant-PPP values for 2010 and 2015. Focussing on the direction of the change, we find that remarkably SMA-5 and SES both report a currency appreciation of an identical magnitude, 7 percent, which compares well with an eight percent gain as per constant-PPP. These three methods qualify as members of the class of locally stable rates, which behave very differently from the exchange rates that minimise the forecast errors and thereby closely track the market rates, namely BSES, DES and TES. The latter set report a deprecation of the JPY between 2010 and 2015, which we conclude as improbable and accordingly, in the discussion to follow, we drop these forecasts. FEER values also fall in the improbable range, though falling short of the extent of currency deprecation exhibited by the market data. Given the latter's novelty as a method and for completeness, we retain the FEER estimates for further illustration below, as sort of reflecting the contrarian view.

We conclude this sub-section by observing that judged by the two criteria cited above, the market rates stood far and away from the likely 'true' rates. This conclusion appears to confirm our priors based solely on the visuals (Figures 5.1 and 5.2).

5.5 *Relative Per-Capita Income of Japan and USA*: How do these exchange rate forecasts as summed up in Table 5.1 affect the time-path of relative per-capita income? Figures 5.4 to 5.7 illustrate the various cases. One striking observation is that PPP, SMA(5), and SES exchange rates suggest that the relative Japan/US per capita income ratio had historically remained very similar at all data points for both the 2010 or the 2015 smoothed exchange rates (Figures 5.5, 5.6A and 5.6B, respectively). Indeed SMA(5) and SES rates are so close that we had to put the two in separate figures (e.g., Fig 5.7A and 5.7B) otherwise the per capita income lines would

[40] To the extent PPPs serve as a credible and independent check on the relative value of a currency, it would appear that up until 2006 or so, the market rates were way out of line from what the real value of JPY may have been, an issue worth a closer examination.

overlay each other. Some key statistics for each method are presented in Table 5.2. The table indicates that three models, namely SMA-5, SES, and constant PPP predict a very stable pattern of incomes between points of base change, and hence are more reliable.

Does this analysis offer any guidance as to the burning question of the late 1980s-early 1990s about who was richer, US or Japan? One immediate observation is that the relative income of Japan had indeed peaked in 1991 (Figures 2.3 and 5.4-5.7), and thus the question can be further narrowed to one of determining its value in 1991. The relevant data is summarised in Table 5.2, column 3. Two of the entries in column 3 suggest a relative income in excess of unity in favour of Japan, namely the WB 2010 constant-dollar measure as well as the 2010 FEER. Figure 5.4 contrasts the Japan/US relative per-capita real income series obtained by FEER vis-à-vis the WB constant dollar methods. The FEER income series retains similar distance between the 2010 and 2015 base as does the WB constant dollar.

Table 5.2: Ratio of JPN-USA per-capita GDP, 1988-98
(Under Alternate ER Regimes)

Year → / **↓ ER Regimes and base-year**	**1988**	**1991**	**1998**	**Growth in relative per-capita GDP, 1991 and 1988 (%)**	**Mean (1988, 1991, & 1998)**
2010 FEER	1.046	1.154	1.028	10.30 10.296 ≈ 10.3	1.08
2010-Const USD (WB)	1.001	1.104	0.984	10.3	1.03
2010-Const Smoothed Dollar, SMA-5	0.847	0.934	0.833	10.3	0.87
2010-Const Smoothed Dollar, SES	0.846	0.933	0.832	10.3	0.87
2010-Const PPP	0.787	0.868	0.773	10.3	0.81
2015 FEER	0.775	0.856	0.762	10.426≈10.4	0.80
2015-Const USD (WB)	0.685	0.757	0.674	10.4	0.71
2015-Const Smoothed Dollar, SMA-5	0.856	0.946	0.842	10.4	0.88
2015-Const Smoothed Dollar, SES	0.857	0.946	0.842	10.4	0.88
2015-Const PPP	0.801	0.885	0.788	10.4	0.82

Note: Growth rates in relative GDP per capita' is defined to be identical under all ER-regimes. This growth rate is however not identical for the two base years, (10.3 in constant 2010 and 10.4 in constant 2015). This is because many of the 2010 estimates of GDP were re-estimated/ revised in 2015.

Source: Authors' calculations based on estimates utilizing WDI data

Moreover, the 2010-base year FEER value renders Japanese per-capita income to have equalled or exceeded that of the US throughout the period 1986 to 1998, sort of validating the Oct 1986 Economist magazine's assertion, though the latter was based on the market data. However, we have already remarked on the inherent volatility of these estimates between points of base change, i.e., between 2010 and 2015, thus questioning their reliability.[41]

Though we have stopped referring to the RMSE minimising forecasts (e.g., BSES, DES, and TES), they too display large variations in the predicted 1991 income ratios between two base years. In contrast to these large 'errors', SMA(5), SES, and PPP, yield the smallest of deviations between the points of base change, at 1.3%, 1.4%, and 1.9%, respectively.[42] Thus, going by the latter methodology, Japan had come close to catching up with the US, reaching somewhere between 88 and 94 percent of US real income in 1991.[43]

Figure 2.2 above records the relative growth rates in per-capita income in each country, which tells us that the Japanese per-capita income grew in excess of the growth rates in the US in each of 1989, 1990 and 1991 by 1.74, 3.8 and 4.4 percentage points, respectively, thus cumulatively by about 10.3% between 1988 and 1991. Column 5 (Table 5.2) illustrate the same point, namely that relative growth in per-capita income in JPN exceeded that of the USA by about 10.3 percent. As remarked already, all constant dollar measures of per-capita income are directly based on per-capita constant LCU income, and hence all these methods, though differing in the *level* of per-capita income, measure the growth rate in income accurately.[44]

We sum up the discussion of this section as follows. First, we have shown that a smoothening of market exchange rates by statistical methods may be useful in deriving exchange rates for different base years (2010 and 2015 here) that do not suffer from the volatility evident in market rates. Use of the smoothed rates are then contrasted vis-à-vis PPP rates for the same base years. In the JPY-USD context, we see that both SMA-5 and SES yield exchange rates between these two base years that lie in an even tighter band than the PPP.[45] The resulting smoothed exchange rates, along with constant PPP have been termed 'locally stable rates', which can be utilised to determine more reliable estimates of the relative national income of countries. In the comparison of JPN-USA *real income*, we analyse the historical claim if Japan had outclassed

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[41] The basis point difference in the relative income of Japan between 2010 and 2015 values for WB constant dollar measure was 35 (or, 37%, by the arc measure) while for FEER it came to 29 (also 29% by the arc measure).

[42] These percentage figures are all arc measures.

[43] We had decided to drop SMA(9) on grounds of data economy, which yielded the smoothest graph in Fig 5.1. In retrospect, this method performs almost as well as SMA(5), SES or constant PPP in terms of the criteria cited above, namely the tightness of the band between 2010 and 2015 (10 % vis-à-vis 7-8% for the former) and also points to an appreciation of JPY over the interval. Adoption of this smoothed exchange rate would lead to the 1991 value of the relative JPN-USD per-capita income to be a little higher between 0.971 and 0.985 instead of between 0.934 and 0.946 for SMA(5).

[44] Any discrepancy away form 10.3 clearly indicates rounding errors in higher order decimal points.

[45] Conceivably the eventual choice of the relevant smoothing technique(s) may depend on the data at hand, both as to its temporal as well as its geographic dimensions. However, the criteria prosed above, namely an evaluation of the volatility of the forecast exchange rates between adjacent dates of base change and its direction (namely appreciation/deprecation), may prove helpful in most cases.

the US in the late 1980s or early 1990s. Our best guess is that Japan came close to catching up with the US in per-capita real income in 1991, though did not overtake it. SMA(9) predicts the Japanese income at about 98% of the US level in 1991.

6. Conclusion

A comparison of national income (either per-capita or aggregate) between and among countries is a fundamental issue in economic analysis. The WB constant-dollar income methodology, in practice for a long time, has significant practical advantages, most notably, a device to render the world income database directly comparable for the given base year. Moreover, the series can go as far back as national accounts data is available and is therefore widely used. However, as we discover here, these constant-dollar measures yield income estimates that entail dramatic fluctuations whenever the base-year changes. The source of the volatility relates to the high variability of the market exchange rate over time. Base changes in WDI data, supposedly innocuous by design, effectively introduce a volatile multiplicative shift parameter in constant LCU data, which may imply large and implausible values of constant-dollar income of a country relative to another. This aspect of the WB data methodology, while entirely predictable, appear not to have come under the scanner until now. The latter fact calls the cardinality of such income measures seriously into question.

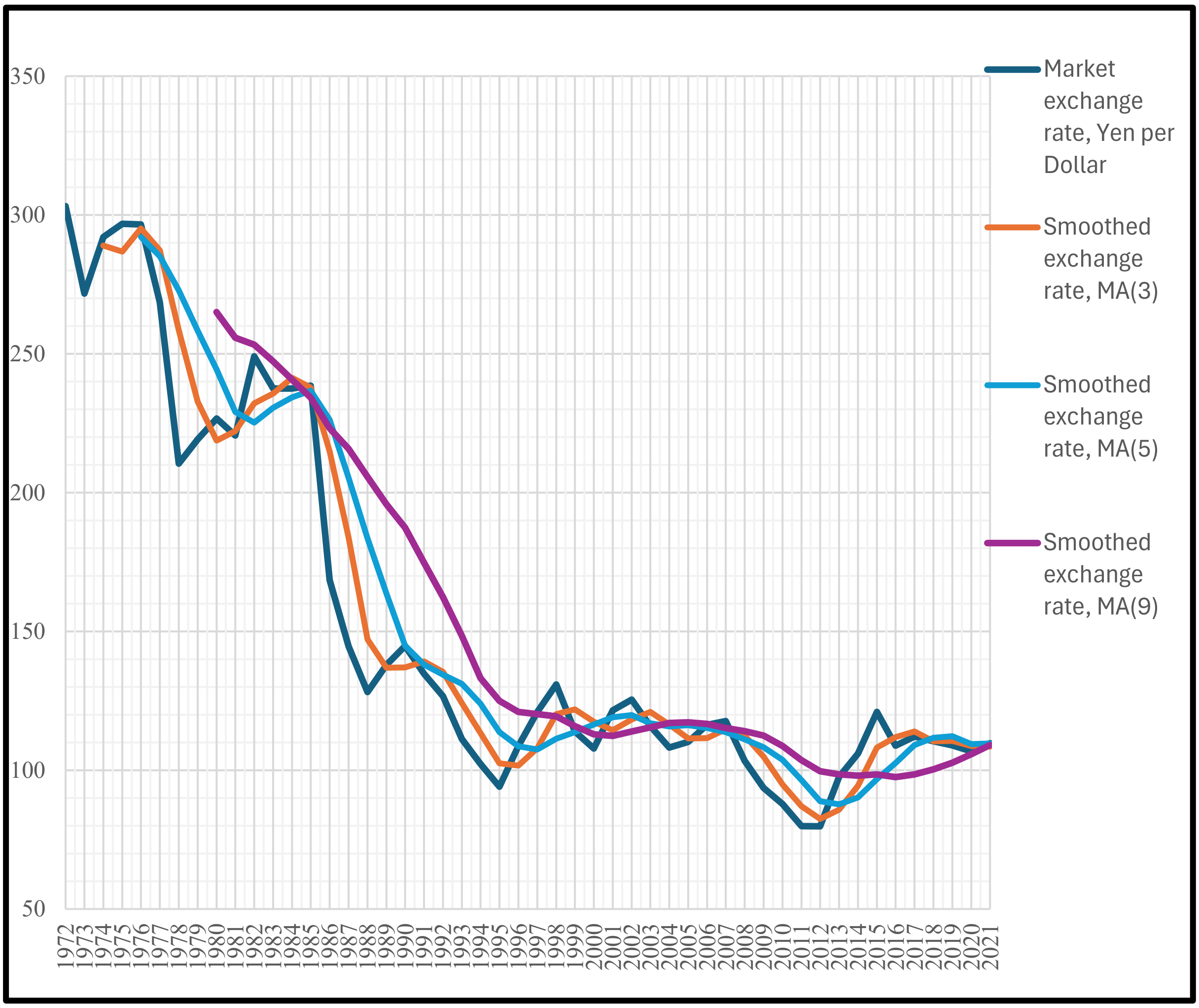
By contrast the PPP exchange rates, which too are periodically calibrated to different base years as per new price surveys conducted by ICP, appear to be much more stable. While PPP data originated in 1990, and accordingly, the WDI database reports these series dating back to 1990 for all subsequent editions. The latter element poses a hindrance for researchers working with time series data, where either for historical or the econometric reasons, one may require a longer series.

In view of the presumed failure of market exchange rates to reflect the underlying economic fundamentals, we propose that smoothing the market exchange rates may turn out to be a viable strategy to rescue the integrity of constant-dollar income measures. These measures of course can retain the length of the data series intact, except for the loss of a few initial observations depending on the specific smoothing techniques utilised (e.g., as in moving average of a certain order). These smoothed rates can then be utilised to compare relative income of nations.

As a by-product of our analysis, by following WB methods, we find that the constant LCU data of a country (for any base year, say 2010) can be used to create a constant 2010 PPP series for that country, by simply using the PPP exchange rate for 2010. The 'derived' series can be taken as far back for which one has reliable constant LCU data for base 2010. This allows us to derive and compare Japan-USA relative per-capita income of a year prior to 1990. We therefore argue that the 'derived PPP incomes' can be another alternative to the smoothed exchange-rate incomes of a country. A comparison between smoothed and PPP exchange rates suggest that that there exist smoothing techniques that are capable of yielding exchange rates that are, much like PPP, more stable than the market rates and which can lead to measures of per capita income that appear more plausible between base year changes than the standard World Bank concept of constant-dollar income that has been the mainstay of real income yardstick for long.

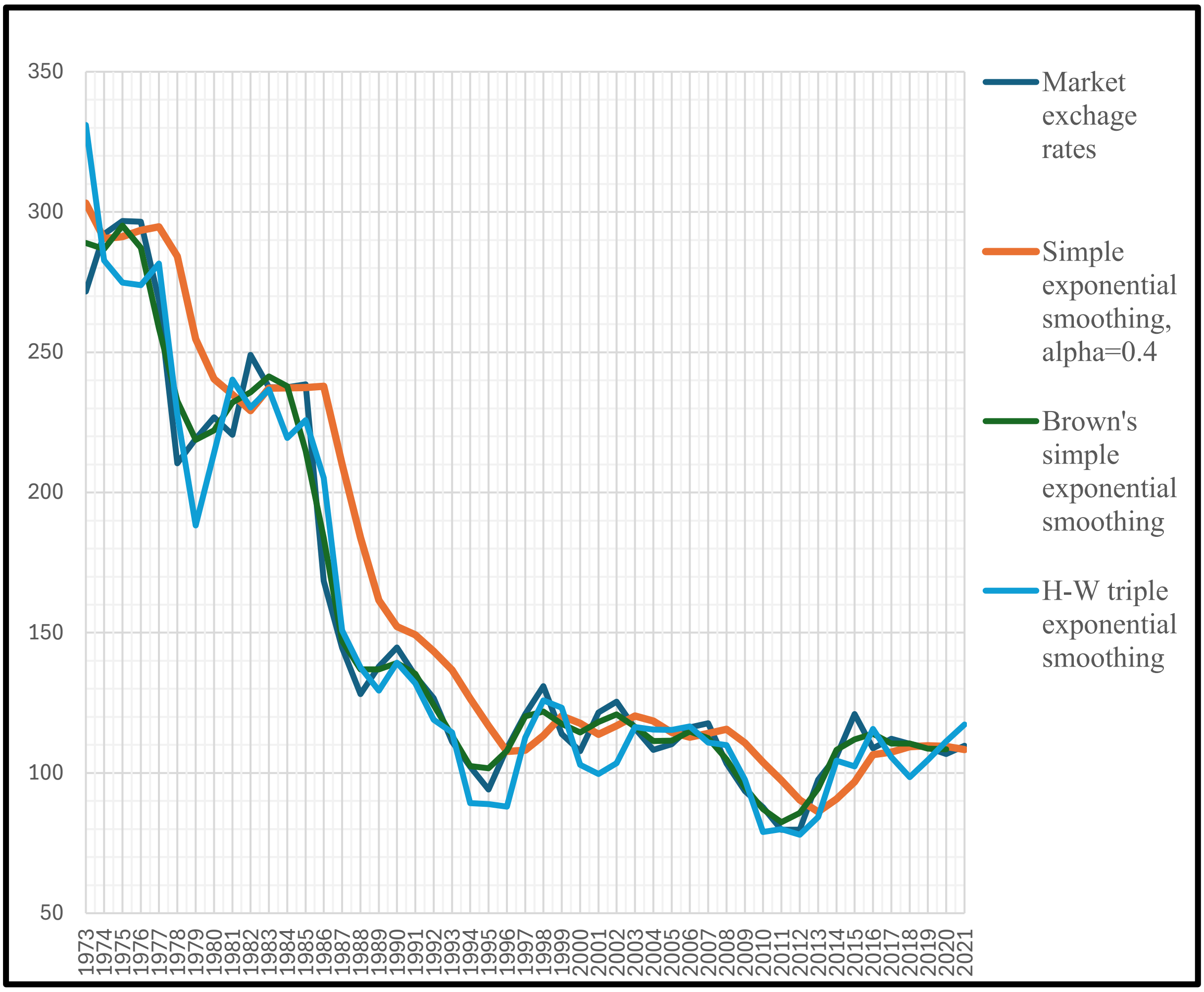
Finally, in the specific JPY-USD context, we see that both SMA-5 and SES yield exchange rates for these base years that lie in an even tighter band than the constant-PPP, while each being mutually consistent in determining the relative appreciation/depreciation of the currency over time. The relative per-capita income predicted by each of these exchange rates appear comparable to each other. On the historical curiosity if Japan had ever been richer than the US in the late 20th century, the present analysis suggests that the per-capita real income in faster-growing Japan peaked in 1991 but remained just below the US level.

Fig 5.1 Moving Average Smoothing of Exchange Rates, JPY/USD, 1972-2021



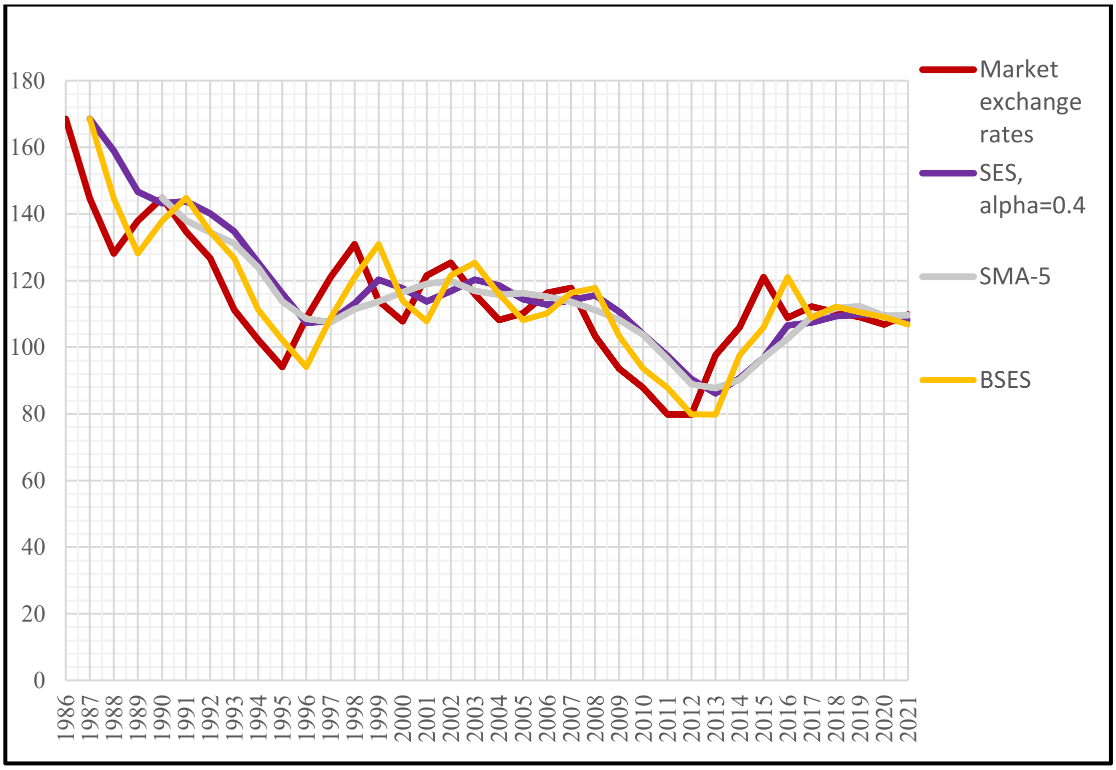
Source: Authors' construction based on WDI data

Fig 5.2 Exponential Smoothing of Exchange Rates, JPY per USD, 1972-2021



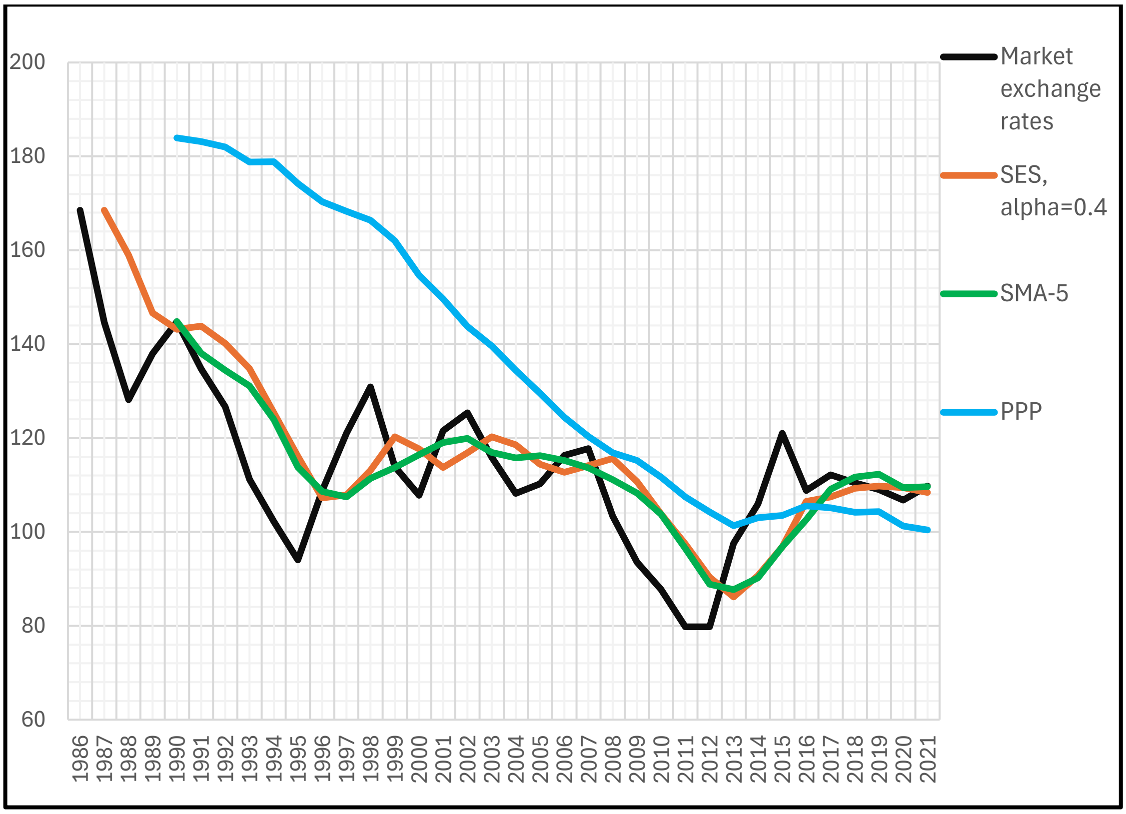
Source: Authors' construction based on WDI data

Fig 5.3A Selected Smoothed Exchange Rates, JPY per USD, 1986-2021



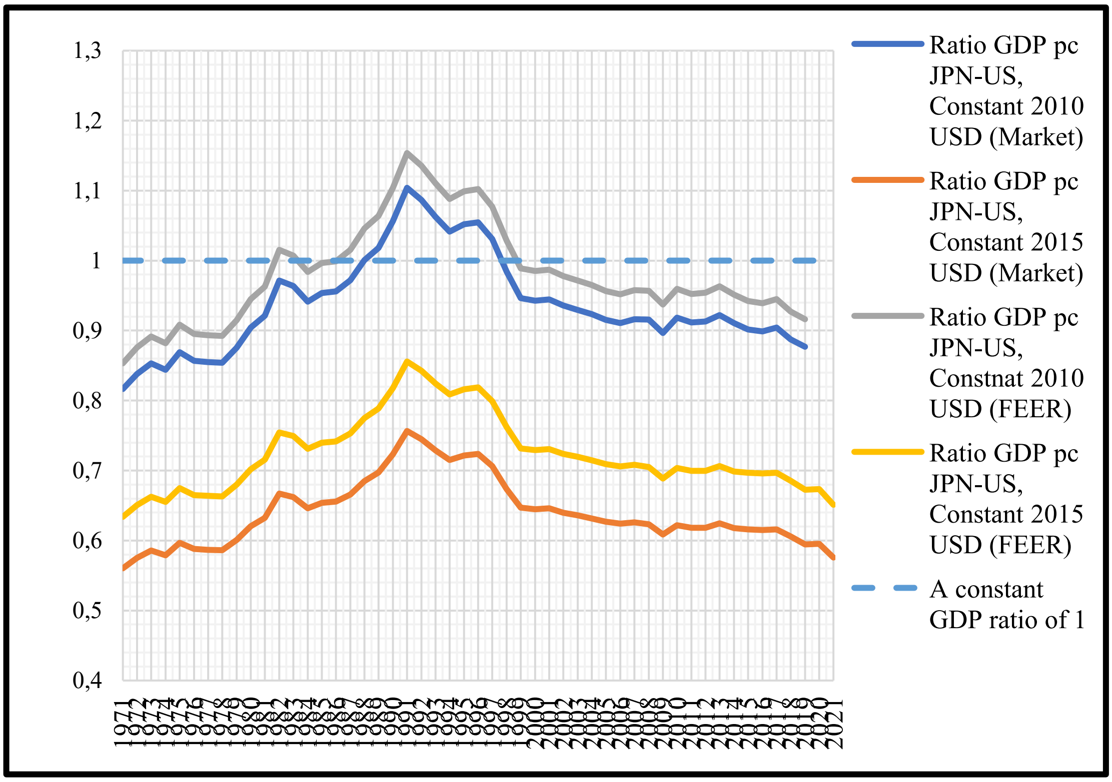
Source: Authors' construction based on WDI data

Fig 5.3B Selected Smoothed Exchange Rates, JPY per USD, 1986-2021



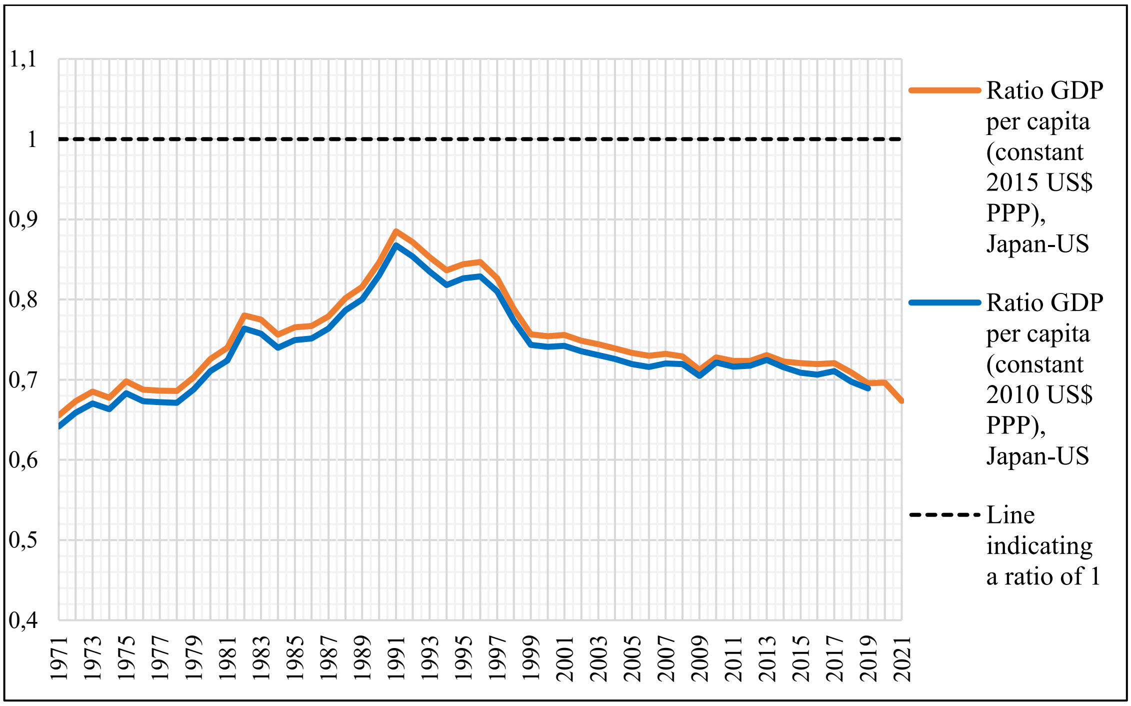
Source: Authors' construction based on WDI data

Fig 5.4 Per capita GDP Ratio, JPN-US, 1971-2021
(WB Constant Dollar vs FEER)



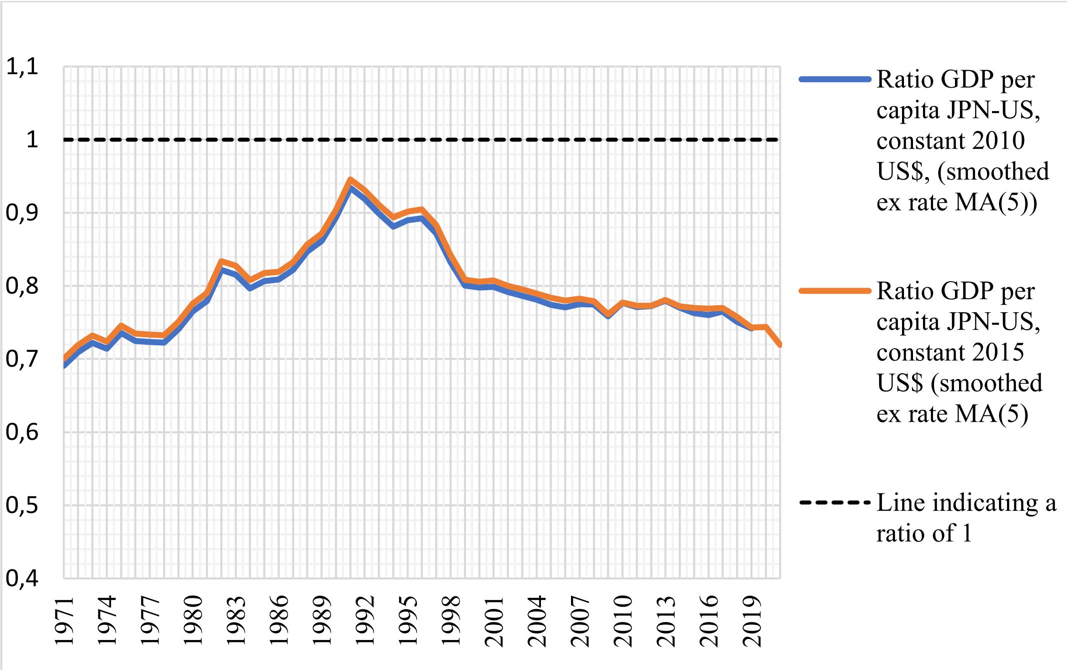
Source: Authors' construction based on WDI and related data

Fig 5.5: Per capita GDP Ratio, JPN-US, 1971-2021
(Constructed Constant PPP, 2010 and 2015)



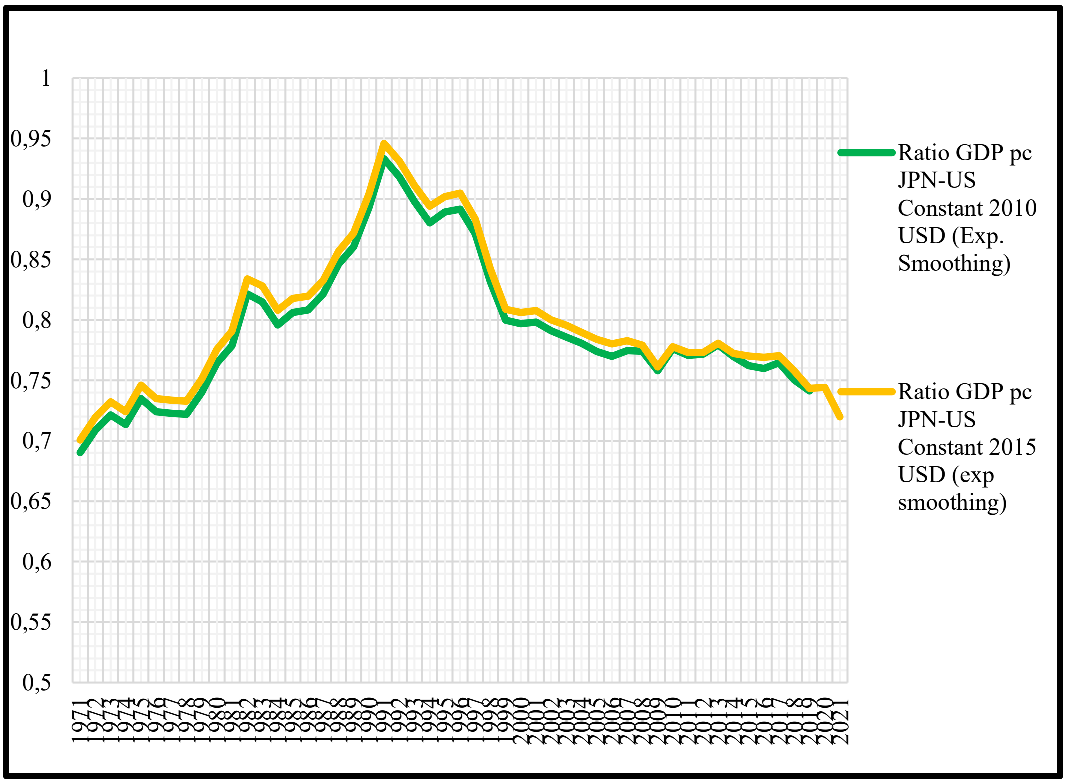
Source: Authors' construction based on WDI data

Fig 5.6A Per capita GDP Ratio, JPN-US, 1971-2021
(Smoothed 2010 and 2015 Exchange Rates, SMA-5)



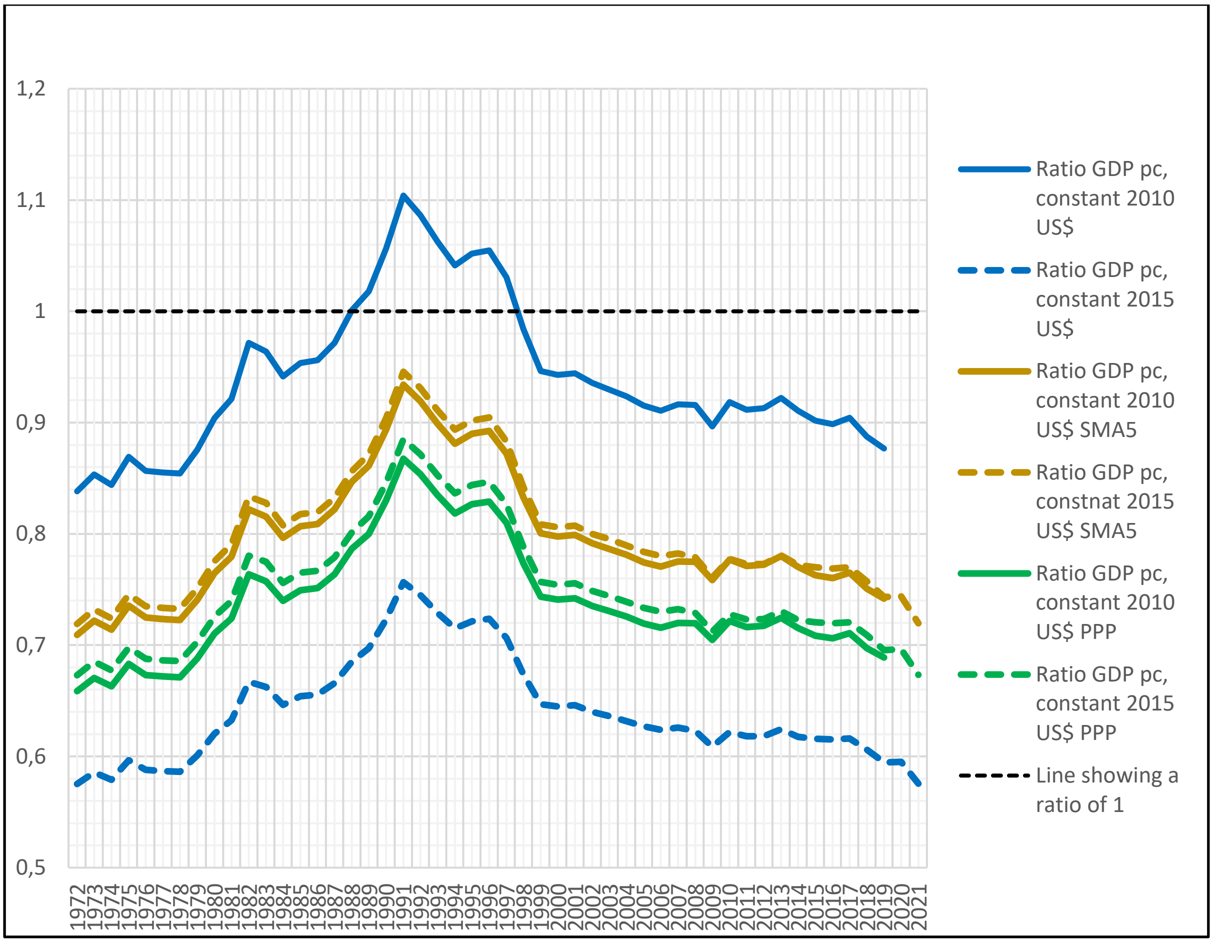
Source: Authors' construction based on WDI data

Fig 5.6B Per capita GDP Ratio, Japan-US, 1971-2021
(Smoothed Exchange Rate, SES)



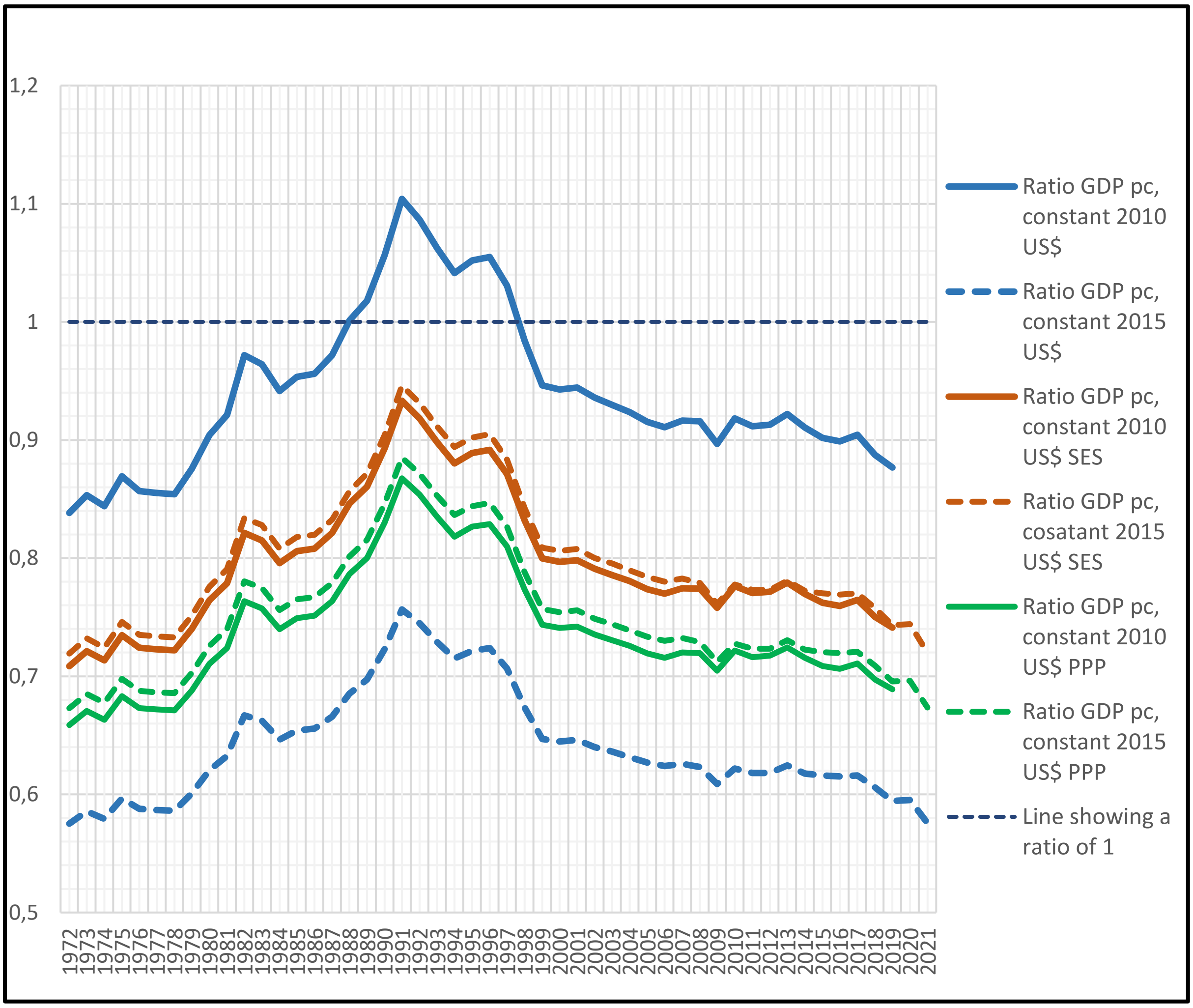
Source: Authors' construction based on WDI data

Fig 5.7A Ratios of per capita GDP, JPN/USA, 1971-2021
(Alternate Exchange Rate Regimes including SMA-5)



Source: Authors' construction based on WDI data

Fig 5.7B Ratios of per capita GDP, JPN/USA, 1971-2021
[Alternate Exchange Rate Regimes including SES (α =0.4)]



Source: Authors' construction based on WDI data

Appendix: An Alternative Interpretation of the WB Constant Dollar Income Concept and its Derivation

Below we present an alternative interpretation of how to obtain the 2015 constant USD series for a host country per capita GDP or similar concepts. This requires conversion of the host country *constant* LCU data from its historical base (T) to 2015, which we label 'derived' constant 2015 LCU per capita GDP.

Using the above notation, we can proceed in two simple steps.

Step-1: Convert the host country constant price data, $\{y_t\}_T$, from its historical base (T) to 2015, i.e., $\{y_t\}_{2015}$. First restate

(A.1) $$\{y_t\}_T = (\textstyle\sum_i p_{iT} * x_{it}),$$

where p_i denotes the price of the i-th good (i = 1, …, M) and x_i, the quantity. Definitionally constant 2015 LCU host country per capita income can be written as

(A.2) $$\{y_t\}_{2015} = (\{y_t\}_T) \times (x_{2015}/y_{2015}) = (\textstyle\sum_i p_{iT} * x_{it}) \times \left(\frac{\sum_i p_{i2015} * x_{i2015}}{\sum_i p_{iT} * x_{i2015}}\right).$$

We refer to the $\{y_t\}_{2015}$ series as the '*derived* constant 2015 LCU per capita GDP in year-t'.

Step-2: It is now evident that the constant 2015 USD per capita GDP in year-t, denoted above by $\{z_t\}_{2015}$ can be obtained by dividing the series in (A.2) by the host country market exchange rate as of 2015, i.e.,

(A.3) $$\{z_t\}_{2015} = [(\{y_t\}_{2015}) / ER^{Market}_{2015}].$$

Given (A.2), it is immediate that equation (A.3) is mathematically equivalent to equation (6a) in the text.

One can once again illustrate the calculation above by using the Bangladesh data as reviewed above. Recalling that the per capita constant LCU, $(y_{2015}\}_T$ = 52,789.06 (from eq. 2a), the expression $\{y_t\}_{2015}$ is merely = (52,789.06 ×1.8376) = 97,007.44; (see eq, 2a). Hence from (A.3), $\{z_{2015}\}_{2015}$ = USD 1248.45. Once we have converted the constant LCU data to base of 2015, the 2015 value of current LCU and constant 2015 LCU become identical. This also implies that the per capita income in current USD and '2015 constant USD' would also be identical for that year.

While the exposition in the text (section 3) followed the WB guidance on the matter, the above interpretation proceeds from first principles and hence may be claimed to be more intuitive. However, as we illustrate below, the WB statement as to its derivation often seems clumsy at best.

What do WB documents claim? While the above construction is simple enough, it is frustrating that WB documents offer a vague description of this concept and how it is actually obtained. The only reference we find states, " … when we convert the constant price data to U.S. dollars, we preserve the growth rates observed in the local price series. That is, we convert the constant LCU series to an index by dividing each value by its 2015 value (i.e., yielding 2015 = 1), and then multiplying this index by the 2015 current USD value of the series using the 2015 average official exchange rate [(https://datahelpdesk.worldbank.org/ knowledgebase/ articles/114968), accessed 01 Nov 2021].

Is this explanation helpful? Again, rewriting (6a), we have

(A.4) $$\{z_t\}_{2015} = [(x_{2015}/ ER^{Market}_{2015}) (y_t/y_{2015})].$$

… cntd/overleaf

Now it is transparent that the expression in the second set of brackets, namely (y_t/y_{2015}), is indeed the resulting index cited in the quote. However, in what sense is the factor multiplying the index, ($x_{2015}/ER^{Market}_{2015}$), a scalar, "the 2015 current USD value of the series". The 'series' in the quote refers to the 'the constant-LCU series', but as we see from (A.4), mathematically it ought to be the *current* LCU data, x_{2015}. Taken literally, using Bangladesh data (as in eq. 7a above), the constant LCU per capita income 2015 was 52,789.06, while the exchange rate was 77.7021. As per the WB quote, this would have yielded a value of USD 679.38, a figure that is not interpretable. But the current USD value (x_{2015}/77.7021) gives us the correct figure, USD 1248.45. The correct statement would have read "… multiplying this index by the 2015 current USD value of the *current* LCU series." One may however choose to ignore the anomaly as an unintended flaw in the semantics.

References

Abbramovitz, Moses, (1986), "Catching Up, Forging Ahead, and Falling Behind," *Journal of Economic History*, June, 46(2), pp. 385-406.

Almås, Ingvild (2012), "International Income Inequality: Measuring PPP Bias by Estimating Engel Curves for Food." *American Economic Review* 102 (1): 1093–1117.

Baily, Martin N., Barry P. Bosworth and Siddhi Doshi,(2020), "Lessons from Productivity Comparisons of Japan, the United States, and Germany", *International Productivity Monitor*, 38 (Spring).

Banerjee, Abhijit V., Angus S. Deaton, Nora Claudia Lustig, Kenneth S. Rogoff, and Edward Hsu, (2006) *An Evaluation of World Bank Research, 1998 - 2005* (September 24, 2006). Available at SSRN: https://ssrn.com/abstract=2950327 or http://dx.doi.org/10.2139/ssrn.2950327.

Benigno, Ginaluca, Pierpaolo Benigno, and Salvatore Nisticò, (2012), "Risk, Monetary Policy, and the Exchange Rate," *NBER Macroeconomics Annual* 2011, vol 26, 247-309.

Brown, R. G. (1963), *Smoothing, Forecasting and Prediction of Discrete Time Series*, Englewood Cliffs, NJ: Prentice-Hall.

Cline, William R (2015), "Estimates of Fundamental Equilibrium Exchange Rates," PB15-20, Washington, DC: Peterson Institute for International Economics, November.

Deaton, Angus and Alan Heston (2010), "Understanding PPPs and PPP-based National Accounts", *American Economic Journal: Macroeconomics*, 2(4), 1–35.

Deaton, Angus and Paul Schreyer (2022), "GDP, wellbeing, and health: thoughts on the 2017 round of the International Comparison Program," *Review of Income and Wealth,* 68, Number 1, March.

Driver, Rebecca L and Peter F Westaway (2005), "Equilibrium Exchange Rate Determination", in *Exchange Rate, Capital Flows and Policy*, in Driver, Rebeca L, Peter Sinclair, and Cristoph Thoenissen, Eds, Routledge, 2005.

Garcia, Carlos, J., Jorge E. Restrepo, and Scott Roger (2011), "How Much Should Inflation Targeters Care About the Exchange Rate?", *Journal of International Money and Finance*, 30 (7), 1590-1617.

Holt, C. C. (1957), *Forecasting Seasonals and Trends by Exponentially Weighted Moving Averages*. ONR Memorandum, Vol. 52, Carnegie Institute of Technology, Pittsburgh.

Itskhoki, Oleg and Dmitry Mukhin (2021), "Exchange Rate Disconnect in General Equilibrium," *Journal of Political Economy*, 129(8): 2183-2232.

Kohli, Ulrich (2003), "Growth Accounting in The Open Economy: International Comparisons", *International Review of Economics and Finance*, 12, 417–435.

Kravis, Irving B., Alan Heston and Robert Summers (1978) [Kravis et al., 1978), "Real GDP per Capita for More Than One Hundred Countries", *Economic Journal*, Vol 88, Issue 350, 215-42.

Krugman, Paul (1994), 'The Myth of Asian Miracle', *Foreign Affairs*, 73(6), 62-78.

Nau, Robert (2014), *Forecasting* (www.people.duke.edu/~rnau/forecasting.htm).

Pinkovskiy, Maxim and Xavier Sala-i-Martin (2020), "Shining a Light on Purchasing Power Parities," *American Economic Journal: Macroeconomics*, October 2020, Vol. 12, No. 4 (October 2020), pp. 71-108.

Rossi, Barbara (2013) "Exchange Rate Predictability". *Journal of Economic Literature*, 51(4), 1063-1119.

Saadaoui, Jamel (2015), "Global Imbalances: Should We Use Fundamental Equilibrium Exchange Rates?" *Economic Modelling* 47, 383-398.

Winters, P. R. (1960), "Forecasting Sales by Exponentially Weighted Moving Averages," *Management Science*, *6*(3), 324–342.