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Overlapping ownership and input prices ☆

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ABSTRACT

This paper studies the effect of overlapping ownership in a setting where firms must contract with an input supplier before competing in the product market. Horizontal ownership among the competing firms can here affect the input prices set by the supplier. I derive conditions for when overlapping ownership raises, reduces, or has no effect on input prices. The key factor is how demand curvature varies with total output. When overlapping ownership reduces input prices, the cost reduction is in turn passed on to consumers. This indirect effect offsets – and can even outweigh – the direct negative effect of overlapping ownership on product market competition.

1. Introduction

In many markets, we observe that firms have partial ownership stakes in each other (cross-ownership) or that institutional investors hold shares in several competitors (common ownership) (Backus et al., 2021; Heim et al., 2022). These practices can be collectively referred to as *overlapping ownership arrangements* (OOAs) (López and Vives, 2019). The ability of such OOAs to reduce product market competition is well established theoretically (e.g., Reynolds and Snapp, 1986) and has recently been documented empirically in the airline industry (Azar et al., 2018). Intuitively, OOAs can limit competition by inducing firms to internalize externalities on each others' profits. This theory of harm has received much interest from antitrust agencies and policy makers in recent years (e.g., OECD, 2017; US Federal Trade Commission, 2018; European Commission, 2020).

In the literature, the effects of OOAs have traditionally been studied using the classic oligopoly models (e.g., the Cournot and Bertrand models). When analyzing OOAs in such frameworks, an implicit assumption is that the level of overlapping ownership does not affect the firms' input costs. This assumption can be reasonable if firms produce inputs themselves or source them in the world market. In practice, however, inputs are often bought from powerful upstream firms. For example, in the airline industry, airlines use planes from suppliers such as Airbus and Boeing. Also, there is empirical evidence of OOAs among supermarkets (e.g., Schmalz, 2018; Leigh and Triggs, 2021), who buy products from large brand manufacturers. Moreover, another type of upstream firm is a trade union that negotiates wages on behalf of its workers (see Lommerud et al., 2005).

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The assumption that OOAs do not affect input costs is less compelling in the presence of such *vertical relationships* between the jointly held firms and their suppliers. First, input costs will then be a function of the input prices charged by upstream firms. Second, there is empirical evidence suggesting that input prices are responsive to changes in downstream ownership (e.g., McGuckin and Nguyen, 2001; Fee and Thomas, 2004; Bhattacharyya and Nain, 2011). Given this, new questions about OOAs emerge.

For example, will an increase in overlapping ownership elicit a strategic response from suppliers in the form of a change in input prices? If so, will input prices increase or decrease? How would such an input price effect interfere with the standard view of OOAs? Can the effect on input prices mitigate the direct, anti-competitive effect of OOAs in the product market?

This paper adds to our understanding of OOAs by addressing these questions. I study a canonical model of a vertical market structure in which an upstream monopolist supplies multiple downstream firms who in turn produce final goods and compete à la Cournot in the downstream market. In this setting, I analyze how horizontal OOAs among the downstream firms affect the input prices set by the supplier, as well as output and welfare. A key feature of the model is that I consider a general consumer demand function and derive results in terms of primitives of this function. That approach distinguishes my paper from a nascent literature that has looked at similar questions while restricting attention to specific demand forms (see Section 2 for a detailed discussion).

The key result of the paper is that the effect of overlapping ownership on input prices depends on the curvature (i.e., the elasticity of the slope) of the consumer demand function – and, more specifically, on *how the curvature varies with total output*. For example, I find that an increase in overlapping ownership among the downstream firms leads to a higher input price if demand curvature is increasing in output. Conversely, an increase in overlapping ownership leads to lower input prices if demand curvature is decreasing in output. Finally, OOAs have zero effect on input prices if and only if demand curvature is constant with respect to output. In short, the output-derivative of demand curvature is a sufficient statistic for the effect of overlapping ownership on input prices.

The above result means that when demand curvature is non-constant, OOAs work through two channels in my model. On the one hand, OOAs have a direct effect on competition in the product market, as is known from previous models. On the other hand, OOAs also have an indirect effect on the input prices set by the supplier. The net impact of OOAs on total output and welfare is jointly determined by these two effects. Of particular interest is the case of decreasing demand curvature in which OOAs reduce input prices. Here, the reduction in input prices is passed on to consumers and thus offsets the standard anti-competitive effect. If pass-through is very strong (which happens if demand is extremely convex), the indirect effect can even dominate the direct effect such that OOAs raise output and welfare.

The paper proceeds as follows. I start in Section 2 by placing the paper more accurately within the related literature. Section 3 then lays out the model. The equilibrium of the model is derived in Section 4. In Section 5, I present my main comparative statics results on how the equilibrium input prices and output levels depend on the level of overlapping ownership. Section 6 concludes and discusses the potential policy implications. Some formal material is relegated to the Appendix.

2. Related literature

This paper is related to a nascent theoretical literature that examines the effects of downstream OOAs in vertically related markets.[1] This literature shares some similarities with the more established literature on retail mergers (e.g., Lommerud et al., 2005; Inderst and Shaffer, 2007; Gaudin, 2018; Ghosh et al., 2022). However, a fundamental difference between the modeling approaches in the two strands of literature is the following: After a horizontal merger, the acquiring firm obtains full control over the merging partner's decisions. By contrast, after a small increase in the level of overlapping ownership, downstream firms continue to make their decisions independently and non-cooperatively.

Hu et al. (2022a) study a model with two downstream firms and one supplier who sets input prices and can invest in cost-reducing R&D. They show that when the level of overlapping ownership increases, downstream firms reduce their outputs, to which the supplier responds by reducing its investment. In their model, OOAs therefore lead to higher input prices.

Li and Shuai (2022) consider a model with asymmetric ownership, e.g., one downstream firm holding a share in its rival while the rival does not hold such a share (see also Hu et al., 2022b). They show that the upstream supplier offers a lower input price to the downstream firm who holds the ownership share. This effect occurs because this buyer produces a lower quantity of the final good and therefore has a more elastic demand for the input.

Shuai et al. (2022) also study a model with asymmetric ownership, but where there also are "outside" downstream firms (that are not part of the OOA), and potentially multiple suppliers. They find, similar to Li and Shuai (2022), that suppliers respond to downstream ownership by reducing the input price of the acquiring firm. However, their analysis also shows that the input price effects for the acquired firm and the outsider depend delicately on the upstream market structure.

Chen et al. (forthcoming) examine a model with multiple downstream firms and one supplier. In their model, the OOAs are not purely horizontal but also involve vertical ownership between the downstream firms and the upstream firm (more on this below). They find that a higher ownership level reduces the elasticity of market-wide derived demand, thus allowing the supplier to raise its input prices.

[1] There are some differences within this literature in terms of whether authors focus on partial cross-ownership (also called cross-holding or minority ownership), common ownership (also called horizontal shareholding), or both. While cross-ownership and common ownership are not identical in practice, they typically give rise to similar incentive effects in theoretical models. In this paper, I follow López and Vives (2019) in considering both practices under the umbrella term OOAs (see Section 3 for more details).

The key difference between my paper and the above papers is the specification of consumer demand. In particular, whereas I consider a general demand function, the above papers restrict attention to linear and constant elasticity demands.[2] This restriction is critical because these demand forms have constant curvatures. Thus, the above models do not pick up the relationship between overlapping ownership and input prices that works through *variations* in demand curvature. My paper provides a clean and general characterization of this relationship and shows that it can be positive or negative depending on the demand function. By contrast, the above-cited papers identify relationships that are specific to the demand systems they use, and whose directions (i.e., whether OOAs raise or reduce input prices) depend on other features of the models (R&D, asymmetries, vertical ownership).

The closest paper in the traditional literature on OOAs is probably López and Vives (2019), who study a model in which oligopolistic firms set quantities and also invest in cost-reducing R&D. They show that a higher level of ownership can increase investment and reduce costs when certain conditions on demand curvature and R&D spillovers are met. My paper complements López and Vives (2019) by illustrating that even in the absence of R&D investment, OOAs can lead to lower costs through the pricing decision of an upstream supplier.

On a methodological level, the paper is related to Weyl and Fabinger (2013) and Mrázová and Neary (2017) who emphasized the role of pass-through rates and demand curvature in analysis of imperfect competition. These concepts have also previously been applied to vertical settings. For example, Adachi and Ebina (2014b) use pass-through rates and (inverse) demand curvature to study mark-ups and vertical integration in bilateral monopoly. A related paper by Adachi and Ebina (2014a) extends the mark-up analysis to upstream and downstream Cournot competition. To my knowledge, my paper is the first to connect demand curvature – and particularly how it varies with output – to the effects of downstream OOAs. (Further discussion of related results is deferred to Section 5.1.1.)

It is also important to distinguish my focus on *horizontal* ownership between firms at *the same* level of the supply chain from questions about *vertical* ownership between firms at *different* levels of the supply chain. There is a larger literature on such vertical ownership which has shown that also this practice can have both pro- and anti-competitive effects. For example, vertical ownership can alleviate double marginalization (Flath, 1989) but may also induce foreclosure (Levy et al., 2018) or deter entry (Hunold, 2020). In a recent paper, Levy (2024) examines how partial vertical ownership affects relationship-specific investments. He shows that such ownership promotes investment and raises joint surplus provided that a small increase in ownership does not give a too large increase in the acquirer's corporate control of the target.[3] A special case where this holds is a "silent financial interest," i.e., a passive, non-controlling ownership stake, which is also what I consider. My paper can be seen as providing a complementary theory to Levy (2024) for how such non-controlling, partial ownership can affect the efficiency of vertical supply contracting (see also Section 6 for further discussion).

Finally, I am aware of two empirical papers that have considered the effect of overlapping ownership on input prices. First, the findings of He and Huang (2017) are consistent with common ownership giving rise to purchasing efficiencies that reduce input costs.[4] On the other hand, Nain and Wang (2018) find no evidence of partial ownership affecting supplier pricing. My paper develops a theory that can rationalize variations in such results and that gives insight on the mechanism through which OOAs may affect input costs.

3. Model

Consider an industry with one upstream supplier, U, and $n \geq 2$ downstream firms. The supplier produces an input at constant marginal cost $c \geq 0$. The downstream firms transform inputs into a homogeneous final good using a one-to-one technology. For simplicity, I normalize the production costs of the downstream firms to zero.[5]

The level of overlapping ownership among the downstream firms is $\lambda \in [0, 1)$.[6] In general, overlapping ownership induces (the manager of) each firm to internalize with weight λ the rival's profit in its own objective function (see (2) below). Moreover, as shown by López and Vives (2019, Section III), λ can represent both cross-ownership and common ownership, depending on the micro-foundation. Under cross-ownership, λ is an increasing function of the direct (symmetric) stakes that the firms have in each other. Under common ownership, external investors hold shares in the competing firms and λ increases with the number of investors and the size of their ownership positions.

The inverse demand function in the final-goods market is given by $P(Q)$ where $Q \triangleq \sum_i^n q_i$ is total output and q_i is the output of firm $i \in \{1, ..., n\}$. I assume that $P(Q)$ is thrice continuously differentiable and that $P'(Q) < 0$ for all $Q \geq 0$ such that $P(Q) > 0$. Furthermore, the *curvature* of inverse demand is defined as

[2] Hu et al. (2022a) analyze both linear demand and a parametric constant elasticity form (see Section 3 for formal details). Li and Shuai (2022) and Chen et al. (forthcoming) focus on linear demand in their main analysis and consider general demand in appendices. However, there they do not solve for the equilibrium input prices or output levels. Shuai et al. (2022) use general demand to analyze the downstream firms' incentives to engage in partial ownership with exogenous input costs. However, when analyzing endogenous input prices, they too focus on linear demand.

[3] Conversely, Levy (2024) shows that vertical ownership may instead discourage investment if the degree of corporate control rises sufficiently fast with the ownership stake.

[4] However, their results are also consistent with a theory in which OOAs promote cost-reducing investments (e.g., López and Vives, 2019).

[5] The model can easily be extended to constant marginal downstream costs with no changes in the main results.

[6] It is perhaps most realistic to think of λ as belonging to the lower end of this spectrum. For one, relatively small ownership stakes are what we typically observe in practice. Moreover, very high values of λ can be seen as logically inconsistent with the assumption that downstream firms choose quantities independently. Note that while I derive my main results by looking at small changes in λ, the insights do not hinge on the initial level of ownership.

$$\sigma(Q) \triangleq -\frac{P''(Q)Q}{P'(Q)}. \tag{1}$$

Note that $\sigma(Q)$ has the same sign as $P''(Q)$. Thus, $\sigma(Q)$ can also be interpreted as a measure of demand convexity. Demand is concave if $\sigma(Q) < 0$ and convex if $\sigma(Q) > 0$. For linear demand, $\sigma(Q) = 0$.

Importantly, I allow $\sigma(Q)$ to vary with Q. This is in contrast to the literature discussed in Section 2 that focuses on demand forms with constant curvatures. It is immediate that linear demand has constant curvature. Moreover, Hu et al. (2022a) consider the parametric form $P(Q) = a - bQ^{(1-z)}/(1-z)$ with $a, b \geq 0$ and $z < 1$, which has a curvature of $\sigma = -\left(bQ^{-(1+z)}z\right)Q/(-bQ^{-z}) = z$ for any Q.

The profit of U is $\pi_U = \sum_i^n (w - c)\, q_i$, where w is the per-unit input price (more on this below). The operating profit of downstream firm i is $\pi_i = [P(Q) - w]\, q_i$. For a given λ, firm i's objective function can therefore be written as

$$\phi_i \triangleq [P(Q) - w]\, q_i + \lambda \sum_{k \neq i}^{n-1} [P(Q) - w]\, q_k. \tag{2}$$

I assume that both the upstream and downstream second-order conditions are satisfied everywhere over the relevant interval (I return to the formal conditions under which this holds in Section 4).

The timing of events is as follows.

- $t = 1$: The supplier sets the per-unit input price, w.
- $t = 2$: Downstream firms purchase inputs and produce and sell final goods. Downstream competition takes place in a Cournot fashion, with each firm i choosing q_i simultaneously and non-cooperatively. Finally, consumers make their purchases and profits are realized.

The solution concept will be subgame perfect Nash equilibrium.

I conclude this section by discussing three important aspects of the $t = 1$ stage. First, I assume that the supplier can set the input price on a take-it-or-leave-it basis. An alternative approach would be to have the terms of trade determined via a bargaining process such as the Nash bargaining solution (see, e.g., O'Brien and Shaffer (2005) for an application of this method to vertical contracting). In that case, the impact of OOAs would also depend on the distribution of bargaining power between the upstream and downstream levels, and whether a breakdown of negotiations between the supplier and one downstream firm could be observed by other downstream firms.[7] My model eschews these issues in order to accentuate and focus on the strategic response of the upstream firm (the indirect effect), which is the key additional feature compared to the traditional literature on OOAs.[8]

Second, I assume that there is *one* input price that applies for *all* downstream firms. Since the downstream firms in my model are symmetric in terms of both their ownership positions, demands, and costs, it is not obvious why U would want asymmetric prices. Assuming outright that U sets a uniform input price for all downstream firms simplifies the equilibrium analysis. In practice, the ability of upstream firms to offer different input prices to similar downstream firms can be restricted by anti-discrimination laws such as the U.S. Robinson-Patman Act and Article 102(c) of the TFEU.

Third, I assume linear wholesale contracts (i.e., only a per-unit input price). This assumption, which is also made in the literature on downstream OOAs, can be defended on both theoretical and empirical grounds. In theory, more complex pricing schemes can be costly to implement (e.g., because of moral hazard problems) and may also be less efficient if trade occurs infrequently and volumes are subject to uncertainty (Dobson and Waterson, 2007). Moreover, we often observe the use of simple linear contracts in real-life input markets, e.g., between TV channels and distributors (Crawford and Yurukoglu, 2012) and between hospitals and suppliers of medical equipment (Grennan, 2013).

4. Equilibrium

In this section, I derive the subgame perfect Nash equilibrium of the above model. This analysis proceeds by backward induction.

4.1. Downstream stage ($t = 2$)

At $t = 2$, the first-order condition of downstream firm i is $\partial \phi_i / \partial q_i = 0$, which can be written as

$$P(Q) + P'(Q)\, q_i + \lambda \sum_{k \neq i}^{n-1} P'(Q)\, q_k = w. \tag{3}$$

[7] On this, see Iozzi and Valletti (2014) who study Nash bargaining between a supplier and competing retailers using a linear demand system. They show that the equilibrium input prices, and their comparative statics properties, depend critically on whether negotiation breakdowns are observable or unobservable. Moreover, they show that these relationships may also depend on whether downstream firms set quantities or prices and whether input prices can be renegotiated in case of disagreement. OOAs may further complicate the picture by introducing disagreement payoffs (outside options) also for the downstream firms, since a buyer who does not reach an agreement with the supplier can still earn a profit from its shares in rival firms. Carefully analyzing these issues with a general demand function lies beyond the scope of the present paper but could definitely represent an interesting avenue for future work (see Section 6).

[8] The related papers on downstream OOAs discussed in Section 2 also assume that upstream firms make take-it-or-leave-it offers.

The third term at the left-hand side of (3) reflects the standard anti-competitive effect of OOAs: Firm i takes into account with weight λ that marginally raising q_i lowers the price on the units of firms $k \neq i$. In the following, I focus on a different question: How do OOAs affect the input price at the right-hand side of (3) when this is chosen by an upstream supplier at a preceding stage of the game?

When all n downstream firms buy at the common input price w, we can write $q_i = q$ for all i and $q = \frac{1}{n}Q$. Lemma 1 below restates (3) in a more compact way and introduces some key variables that will feature throughout the analysis.

Lemma 1. *The equilibrium in the downstream market at $t = 2$ can be expressed as*

$$\mu(Q) = w \tag{4}$$

where

$$\mu(Q) \triangleq P(Q) + \alpha P'(Q) Q \tag{5}$$

is a firm's marginal revenue and

$$\alpha \triangleq \frac{1 + \lambda(n-1)}{n} \tag{6}$$

denotes the "Modified Herfindahl-Hirschman Index" (MHHI).[9]

Condition (4) simply states that each downstream firm equates its marginal revenue and marginal cost in equilibrium. Moreover, as can be seen from (5) and (6), an increase in overlapping ownership raises the MHHI (specifically, $\partial\alpha/\partial\lambda = 1 - 1/n > 0$) and thereby reduces a firm's marginal revenue for a given Q, in line with the standard intuition.[10]

Before proceeding with the analysis, I now pin down the downstream second-order condition. Following López and Vives (2019), we have that existence of a unique and symmetric downstream equilibrium (for a given w) and concavity of ϕ_i are ensured if the condition $\partial^2\phi_i/\partial q_i^2 + (n-1)\partial^2\phi_i/\partial q_i \partial q_k < 0$ holds globally. I will assume that this condition indeed holds. In Appendix A.1, I show that this can equivalently be expressed as follows:

Assumption 1. $\sigma(Q) < 1 + \frac{1}{\alpha}$ for all $Q > 0$.

This condition limits the permissible degree of convexity of inverse demand. Sufficient for Assumption 1 to hold, and thus for a unique and symmetric equilibrium to exist, is that $P(Q)$ is log-concave which corresponds to $\sigma(Q) < 1$. This is the same sufficient condition as found by Amir and Lambson (2000, Theorem 2.3) for the Cournot game without overlapping ownership.[11] I discuss demand curvature and Assumption 1 further in Section 5.2, where this becomes key for determining the effect of overlapping ownership on output and welfare.

4.2. Upstream stage ($t = 1$)

Let $q(w)$ be the derived demand function of one downstream firm for a given w, and let $Q(w) = nq(w)$ be the market-wide derived demand function. We can then simply write the supplier's profit at $t = 1$ as $\pi_U = (w - c)Q(w)$. Taking the first-order condition $\partial\pi_U/\partial w = 0$ gives

$$Q(w) + (w - c)\frac{\partial Q}{\partial w} = 0. \tag{7}$$

To solve for the equilibrium input price, we need the pass-through rate $\partial Q/\partial w$. This can be derived by evaluating the downstream equilibrium condition (4) at $Q(w)$, thus obtaining $\mu(Q(w)) = w$, and then differentiating this equation with respect to w. This yields

$$\frac{\partial Q}{\partial w} = \frac{1}{\frac{\partial\mu}{\partial Q}}. \tag{8}$$

Importantly, we have $\partial Q/\partial w < 0$ because $\partial\mu/\partial Q < 0$. To see that $\partial\mu/\partial Q < 0$, note from (5) that

$$\begin{aligned} \frac{\partial\mu}{\partial Q} &= (1+\alpha)P'(Q) + \alpha P''(Q)Q \\ &= P'(Q)\left[1 + \alpha - \alpha\sigma(Q)\right], \end{aligned} \tag{9}$$

[9] The MHHI was introduced by Bresnahan and Salop (1986) and has featured heavily both in the empirical literature and the recent policy debate on OOAs. In essence, for a given n, the MHHI captures the "additional" reduction in competitive incentives caused by overlapping ownership.

[10] Note also that in the limit where $n \to \infty$, we have $\lim_{n\to\infty} \alpha = \lambda$ and downstream quantity can thus be expressed purely in terms of λ and the demand function.

[11] Amir and Lambson (2000) use the approach of supermodular games and lattice theory, which is more general than my approach in the sense that it requires less strict assumptions about differentiability of the demand function etc. In recent work, Vives and Vravosinos (2022) extend Amir and Lambson's (2000) analysis to a symmetric degree of overlapping ownership. Interestingly, Vives and Vravosinos (2022, p. 9) derive a condition for existence and uniqueness of equilibrium which (for linear costs) is equivalent to Assumption 1, see their Proposition 1 (i) (b).

where the expression on the second line is clearly negative since $P'(Q) < 0$ and $1 + \alpha - \alpha\sigma(Q) > 0$ (the latter inequality is equivalent to Assumption 1).[12]

Moreover, note that $\partial Q/\partial w$ is the main determinant of the elasticity of the market-wide derived demand, which can be written as $-\left[w/Q(w)\right](\partial Q/\partial w)$. For example, we see from (8) that a steeper downstream marginal revenue curve gives a lower pass-through rate (i.e., lower in *absolute* terms, meaning that cost changes are passed on to a *lesser* degree) and thus a less elastic derived demand curve, *ceteris paribus*. Conversely, if the marginal revenue curve is flatter, the pass-through rate is higher and derived demand is more sensitive to input price changes.

By combining (7) and (8), we obtain the following result.

Lemma 2. *The equilibrium input price is given by*

$$w = c - Q\frac{\partial \mu}{\partial Q} \tag{10}$$

where the last term is evaluated at the equilibrium point.

The pricing rule in (10) is intuitive. First, the supplier always charges a mark-up above its marginal cost ($\partial\mu/\partial Q < 0$ implies $w > c$). Second, the size of the optimal mark-up reflects the elasticity of derived demand as explained above. For example, a steeper downstream marginal revenue curve gives a less elastic derived demand curve, which allows the supplier to set a higher input price. Conversely, if $\partial\mu/\partial Q$ is closer to zero, derived demand is more elastic and the supplier's market power is more limited.

Finally in this section, consider the supplier's second-order condition. Let

$$\tau(Q) \triangleq -\frac{\frac{\partial^2 \mu(Q)}{\partial Q^2} Q}{\frac{\partial \mu(Q)}{\partial Q}} \tag{11}$$

denote the curvature of the derived demand function, with $\mu(Q)$ given by (5). It can then be shown (see Appendix A.1) that the upstream second-order condition holds under the following assumption.

Assumption 2. $\tau(Q) < 2$ for all Q.

To sum up so far, (4) from Lemma 1 and (10) from Lemma 2 are the equilibrium conditions of the model. In the next section, I use these equations to address the main research questions about the effects of OOAs.

5. Main results

I now take a comparative statics approach in examining the effect of a marginal increase in the ownership level on the equilibrium values. Specifically, I study the effect on input prices in Section 5.1 and the effects on total output and welfare in Section 5.2. Note that total output in equilibrium is given by $Q = nq(w)$, where w is set according to (10).

The first step is to totally differentiate the equilibrium conditions with respect to λ. First, by differentiating (4) and using (5) and (6) we obtain

$$\frac{\partial \mu}{\partial Q}\frac{\mathrm{d}Q}{\mathrm{d}\lambda} + \frac{\partial \mu}{\partial \lambda} = \frac{\mathrm{d}w}{\mathrm{d}\lambda} \iff \frac{\mathrm{d}Q}{\mathrm{d}\lambda} = \frac{1}{\frac{\partial \mu}{\partial Q}}\left[\frac{\mathrm{d}w}{\mathrm{d}\lambda} - \frac{(n-1)P'(Q)Q}{n}\right]. \tag{12}$$

For future reference, note from the last equation in (12) that $\mathrm{d}w/\mathrm{d}\lambda < 0$ is a necessary condition for $\mathrm{d}Q/\mathrm{d}\lambda > 0$ to be possible. Second, differentiating (10) with respect to λ yields

$$\frac{\mathrm{d}w}{\mathrm{d}\lambda} = -\left[\frac{\mathrm{d}Q}{\mathrm{d}\lambda}\frac{\partial \mu}{\partial Q} + Q\left(\frac{\partial^2 \mu}{\partial Q^2}\frac{\mathrm{d}Q}{\mathrm{d}\lambda} + \frac{\partial^2 \mu}{\partial Q \partial \lambda}\right)\right],$$

or equivalently

$$\frac{\mathrm{d}w}{\mathrm{d}\lambda} = -\left(\frac{\partial \mu}{\partial Q} + Q\frac{\partial^2 \mu}{\partial Q^2}\right)\frac{\mathrm{d}Q}{\mathrm{d}\lambda} - \frac{(n-1)Q\left[P'(Q) + P''(Q)Q\right]}{n}. \tag{13}$$

The system given by (12) and (13) can then be solved simultaneously in order to determine $\mathrm{d}w/\mathrm{d}\lambda$ and $\mathrm{d}Q/\mathrm{d}\lambda$.

[12] Note also that $\partial P/\partial w = P'(Q)(\partial Q/\partial w) = 1/[1 + \alpha - \alpha\sigma(Q)]$. In the limit where $\lambda \to 1$ and thus $\alpha \to 1$, the latter expression approaches the pass-through rate under downstream monopoly, i.e., $\partial P/\partial w = 1/[2 - \sigma(Q)]$, see, e.g., Adachi and Ebina (2014a, p. 171).

5.1. Input prices

Starting with the effect on the input price, we find that

$$\frac{\mathrm{d}w}{\mathrm{d}\lambda} = \frac{-(n-1)\,Q^2}{n\left(2\frac{\partial\mu}{\partial Q} + Q\frac{\partial^2\mu}{\partial Q^2}\right)}\left[P''(Q)\frac{\partial\mu}{\partial Q} - P'(Q)\frac{\partial^2\mu}{\partial Q^2}\right]. \tag{14}$$

By using (9), (11), and the fact that

$$\frac{\partial^2\mu}{\partial Q^2} = (1+2\alpha)\,P''(Q) + \alpha P'''(Q)\,Q, \tag{15}$$

we can rewrite (14) as

$$\frac{\mathrm{d}w}{\mathrm{d}\lambda} = \frac{-\alpha\,(n-1)\,Q^2}{n\frac{\partial\mu}{\partial Q}\,[2-\tau(Q)]}\left\{Q\left[P''(Q)\right]^2 - P'(Q)\left[P''(Q) + P'''(Q)\,Q\right]\right\}. \tag{16}$$

In addition, we have from (1) that

$$\frac{\partial\sigma(Q)}{\partial Q} = -\left\{\frac{\left[P'''(Q)\,Q + P''(Q)\right]P'(Q) - Q\left[P''(Q)\right]^2}{[P'(Q)]^2}\right\}. \tag{17}$$

Thus, by combining (9), (16), and (17), we obtain

$$\frac{\mathrm{d}w}{\mathrm{d}\lambda} = \underbrace{\left[\frac{-\alpha\,(n-1)\,Q^2 P'(Q)}{n\,[1+\alpha-\alpha\sigma(Q)]\,[2-\tau(Q)]}\right]}_{>0}\frac{\partial\sigma(Q)}{\partial Q}. \tag{18}$$

Because $\alpha > 0$ and $P'(Q) < 0$, the term in brackets in (18) is positive due to the second-order conditions. Specifically, Assumption 1 implies $1+\alpha-\alpha\sigma(Q) > 0$ and Assumption 2 implies $2-\tau(Q) > 0$. Thus, $\mathrm{d}w/\mathrm{d}\lambda$ and $\partial\sigma(Q)/\partial Q$ have the same sign. This yields the following result.

Proposition 1. *An increase in the degree of overlapping ownership leads to ...*

- *a higher input price if demand curvature is increasing in output, i.e., $\partial\sigma(Q)/\partial Q > 0$,*
- *a lower input price if demand curvature is decreasing in output, i.e., $\partial\sigma(Q)/\partial Q < 0$,*
- *no change in the input price if demand curvature is constant with respect to output, i.e., $\partial\sigma(Q)/\partial Q = 0$.*

Proposition 1 is the key result of the paper. It shows that the derivative of the curvature of consumer demand with respect to total output is a sufficient statistic for the effect of overlapping ownership on input prices. Moreover, it shows that the effect may be positive, negative, or zero, depending on whether demand curvature is increasing, decreasing, or constant.[13]

To see the intuition behind Proposition 1, it is useful to first recall the basic relationship between demand curvature and pass-through: A demand function with a higher curvature – i.e., one that is more convex – yields a higher (in absolute terms) pass-through rate of input prices to final-goods output.[14] This means that if the curvature (convexity) is increasing, a higher total output leads to a higher curvature and higher total pass-through rate, *ceteris paribus*. Conversely, if the curvature is decreasing, a higher total output leads to a lower pass-through rate.

With this relationship in mind, consider now the impact of a small increase in overlapping ownership, inducing a downward pressure on downstream output (the standard, direct effect). If demand curvature is increasing, this output contraction gives a reduction in the curvature and pass-through rate. The lower pass-through rate in turn gives a less elastic derived demand curve, which leads the supplier to raise the input price. Thus, $\mathrm{d}w/\mathrm{d}\lambda > 0$. The converse argument applies with decreasing curvature. Here, when the ownership level increases, the downward pressure on output leads to a higher pass-through rate, a more elastic derived demand curve, and a lower input price, *ceteris paribus*. Finally, in the case of constant curvature, this channel does not come into play and the ownership increase has no effect on the equilibrium input price.

As discussed above, popular demand forms such as linear demand and constant elasticity demand have constant curvatures (see also Bulow and Pfleiderer, 1983). This property is sometimes imposed also on general functional forms for tractability reasons (e.g., López and Vives, 2019). Yet, Fabinger and Weyl (2016) have recently characterized the curvature properties of various demand forms according to their underlying distributions of consumer valuations and shown that many commonly used specifications (e.g.,

[13] Note that, as $\partial\sigma(Q)/\partial Q$ may itself vary with output, the conditions in Proposition 1 are local conditions that apply at the equilibrium point.

[14] Intuitively, if demand is generated from an underlying distribution of consumer valuations, a more convex demand function corresponds to a greater heterogeneity, i.e., many consumers with high valuations and many consumers with low valuations. In this case, if the input price increases, the downstream firms may find it profitable to abandon the low-valuation consumers and sell only to high-valuation consumers which leads to a large reduction in total quantity. See also Weyl and Fabinger (2013) and Miller et al. (2017).

logit demand) have curvatures that are increasing in price, i.e., decreasing in quantity. For such demands, OOAs reduce input prices in my model. A simple example that encompasses all three cases is $P(Q) = (1-Q)^x$, with $x > 0$. For this demand function, we have $\sigma(Q) = -Q(1-x)/(1-Q)$ and $\partial\sigma(Q)/\partial Q = (x-1)/(1-Q)^2$. Thus, demand curvature here is decreasing if $x < 1$, increasing if $x > 1$, and constant if and only if $x = 1$.

The above conditions on curvature also have two alternative interpretations. First, demand curvature being decreasing (respectively, increasing, constant) in output is equivalent to the pass-through rate of input prices to the market *price*, i.e., $\partial P/\partial w$, being increasing (resp., decreasing, constant) in w. To see this, recall that $\partial P/\partial w = 1/[1+\alpha-\alpha\sigma(Q)]$, which yields

$$\frac{\partial^2 P}{\partial w^2} = \underbrace{\left[\frac{\alpha}{P'(Q)[1+\alpha-\alpha\sigma(Q)]^3}\right]}_{<0} \frac{\partial\sigma(Q)}{\partial Q}. \tag{19}$$

Second, demand curvature being decreasing (respectively, increasing, constant) in output is also equivalent to the curvature of consumer demand being greater than (resp., smaller than, equal to) the curvature of derived demand.[15] This can be shown by using (1), (9), (11), (15), and (17) to write the difference between the two curvatures as follows:

$$\sigma(Q) - \tau(Q) = \underbrace{\left[\frac{-\alpha Q}{1+\alpha-\alpha\sigma(Q)}\right]}_{<0} \frac{\partial\sigma(Q)}{\partial Q}. \tag{20}$$

5.1.1. Further remarks on related results

The derivative of demand curvature has previously been identified as a key factor also with respect to other questions in the literature. Of particular interest is a paper by Gaudin (2016) on pass-through rates in vertical supply chains. He shows (see his Corollary 1 at p. 3) that the upstream pass-through rate (i.e., $\partial w/\partial c$) is larger (respectively, smaller) than the downstream pass-through rate (i.e., $\partial P/\partial w$) if demand curvature is decreasing (resp., increasing) in price.[16] This result is closely related to the relationship between $[\sigma(Q) - \tau(Q)]$ and $\partial\sigma(Q)/\partial Q$ noted in (20) above. However, in Gaudin's (2016) model there is only a single downstream firm. Thus, he does not analyze any questions related to downstream competition and ownership, which are the focus in my model.

Proposition 1 also sheds new light on some findings in the recent literature on downstream OOAs. First, Hu et al. (2022a, p. 782) briefly consider the case without upstream R&D and find then that the equilibrium input price does not depend on the degree of ownership. In a similar vein, Li and Shuai (2022, p. 56) note that in their model, input prices become independent of the ownership level when the ownership stakes are symmetric. However, neither of these papers connected these neutrality results to demand curvature. My analysis contributes in this respect by clarifying that the neutrality results occur because of the restriction to linear and constant elasticity demands with constant curvatures.

5.2. Output and welfare

In this section, I consider the effect of overlapping ownership on output and welfare, starting with output. From the analysis thus far, it is clear that the net effect of overlapping ownership on total output can be decomposed into two parts. First, there is a *direct* effect in the downstream market. This effect is always negative: For a given input price, overlapping ownership reduces downstream competition and thereby output (see (3)). Second, there is an *indirect* effect working through the input price. The sign of this indirect effect depends on the derivative of demand curvature as explained above. The sign of the net effect on output is determined by the direct and indirect effects together.

The sign of the net effect is obvious in two of the cases in Proposition 1. First, the net effect is clearly negative if demand curvature is constant because the indirect effect then is zero (i.e., $\mathrm{d}w/\mathrm{d}\lambda = 0$). Second, the net effect is clearly negative also if curvature is increasing, as in this case the indirect effect is negative (i.e., OOAs raise the input price, $\mathrm{d}w/\mathrm{d}\lambda > 0$, which in turn further reduces output because $\partial Q/\partial w < 0$). However, in the third case with decreasing curvature, the sign of the net effect is *a priori* ambiguous. This follows because the indirect effect here is positive (since $\mathrm{d}w/\mathrm{d}\lambda < 0$) which counteracts the negative direct effect.

To gain more insight, we can return to the equation system given by (12) and (13) and solve for $\mathrm{d}Q/\mathrm{d}\lambda$:

$$\frac{\mathrm{d}Q}{\mathrm{d}\lambda} = \underbrace{\left[\frac{(n-1)Q}{n[1+\alpha-\alpha\sigma(Q)][2-\tau(Q)]}\right]}_{>0} [\sigma(Q) - 2]. \tag{21}$$

In (21), the term in brackets is positive under Assumption 1 and Assumption 2. This means that $\mathrm{d}Q/\mathrm{d}\lambda$ has the same sign as $[\sigma(Q) - 2]$. We then have:

[15] These two equivalences were also recognized by Gaudin (2018) and Ghosh et al. (2022).

[16] Chen and Schwartz (2015) found the derivative of demand curvature with respect to price to be the key determinant of the welfare effects of differential pricing when a monopolist sells directly to end users.

Proposition 2. *An increase in the degree of overlapping ownership reduces total output if $\sigma(Q) < 2$ and raises total output if $\sigma(Q) > 2$.*

Thus, for demand functions that are log-concave ($\sigma(Q) < 1$) or "moderately" log-convex (i.e., $1 < \sigma(Q) < 2$), a higher degree of overlapping ownership reduces total output and raises the market price. This is in line with the conventional wisdom and the main policy concern. However, if demand is extremely convex (i.e., $\sigma(Q) > 2$) and has a decreasing curvature ($\partial\sigma(Q)/\partial Q < 0$), we could in fact see a reversal of the standard results. In that case, a small increase in overlapping ownership instead *raises* output and *reduces* price.

Why is a very high degree of demand convexity necessary to observe $\mathrm{d}Q/\mathrm{d}\lambda > 0$? As noted in Section 5.1, the more convex is demand, the more responsive is Q to changes in w (i.e., the higher is the pass-through rate $|\partial Q/\partial w|$). Therefore, when demand is very convex and the marginal increase in λ lowers the input price, the positive knock-on effect on output will also be quite large. If $\sigma(Q)$ exceeds the critical value of 2, this indirect effect through the input price is in fact so strong that it outweighs the direct negative effect. In this case, we therefore obtain the result that $\mathrm{d}Q/\mathrm{d}\lambda > 0$.[17]

As with the above conditions on the derivative of demand curvature, the condition $\sigma(Q) > 2$ is also familiar from the literature. For example, in a seminal paper, Seade (1985, p. 16) demonstrated in the Cournot model that firms' profits increase in their marginal costs precisely when $\sigma(Q) > 2$.[18] Formally, the condition can be related to the slope of the market-wide marginal revenue curve. This curve is given by $P(Q) + QP'(Q)$, so its slope is:

$$\frac{\partial}{\partial Q}\left[P(Q) + QP'(Q)\right] = 2P'(Q) + QP''(Q) = P'(Q)\left[2 - \sigma(Q)\right].$$

Thus, $\sigma(Q) < 2$ (respectively, $\sigma(Q) > 2$) corresponds to the market-wide marginal revenue curve being decreasing (resp., increasing) in output. As is well known, decreasing marginal revenue is needed to satisfy the second-order condition of a (downstream) monopolist (e.g., Mrázová and Neary, 2017, p. 3838). This is easily seen in my model by letting $\lambda \to 1$, in which case $\alpha \to 1$ for any n and Assumption 1 reads as $\sigma(Q) < 2$. However, when λ is smaller (or n is greater), Assumption 1 becomes less strict and $\sigma(Q) > 2$ is permissible. For example, if $\lambda = 0.1$ and $n = 3$, Assumption 1 requires only that $\sigma(Q) < 3.5$.

Proposition 2 also has straightforward welfare implications. In particular, with homogeneous final goods and symmetric costs, total output is a sufficient statistic for consumer surplus and total welfare (the latter of which is given by the sum of consumer surplus and industry profits). Thus, if $\sigma(Q) < 2$ and thereby $\mathrm{d}Q/\mathrm{d}\lambda < 0$, a slight increase in overlapping ownership reduces consumer surplus and welfare. Conversely, if $\sigma(Q) > 2$ and $\partial\sigma(Q)/\partial Q < 0$, a higher degree of overlapping ownership raises consumer surplus and total welfare. Furthermore, note that while overlapping ownership may raise welfare, it unambiguously reduces industry profits, given by $\Pi = [P(Q) - c]Q$. Formally, it can be shown using (1), (4), (5), (9), (10), and (21) that $\mathrm{d}\Pi/\mathrm{d}\lambda = \alpha(n-1)\left[QP'(Q)\right]^2[2-\sigma(Q)]^2 / \left\{n\left[\partial\mu(Q)/\partial Q\right][2-\tau(Q)]\right\} < 0$. This also means that if OOAs raise welfare, the gain in consumer surplus must outweigh the loss of industry profits.

Finally, I comment on how my comparative statics results on λ compare to comparative statics on other parameters. First, it is straightforward to show in the above model that $\mathrm{d}w/\mathrm{d}c > 0$ and $\mathrm{d}Q/\mathrm{d}c < 0$ always hold under Assumption 2. Moreover, if we introduce a constant per-unit downstream cost $k > 0$, it can be shown that $\mathrm{d}Q/\mathrm{d}k < 0$ always holds under Assumption 2 and that $\mathrm{d}w/\mathrm{d}k$ has the same sign as $\tau(Q) - 1$. Thus, the comparative statics on the ownership level are qualitatively different from those on upstream and downstream costs. On the other hand, the comparative statics on λ are qualitatively similar (although not quantitatively identical) as those on the number of firms n. See Appendix A.2 for a formal analysis of the comparative statics on k and n.

6. Conclusion

The existing literature on overlapping ownership has typically considered the effects of this practice in oligopoly models, holding the firms' input costs fixed. This paper has studied the effects of overlapping ownership when input prices are endogenous and set strategically by an upstream supplier. I found that overlapping ownership often affects input prices and that the direction of this effect depends on the relationship between demand curvature and total output. Moreover, this indirect input price effect interacts with the standard direct effect in the product market and the sign of the net effect depends on the convexity of demand.

There is an active debate about the appropriate policy response to OOAs. One proposal has been to impose a cap on the level of common ownership allowed in a given industry (Elhauge, 2015; Posner et al., 2016). Another proposal has been to strengthen the enforcement of non-controlling minority shareholding and acquisitions within the framework of merger control (e.g., OECD, 2017). In either case, it will be important for policy makers and antitrust authorities to evaluate the magnitude of the effects from overlapping ownership and how they depend on market characteristics.

My model illustrates that the impact of OOAs in one market can depend on the strategic response of firms in other, vertically related markets. Moreover, to the extent that decreasing curvature is a reasonable property of actual demand functions, the analysis

[17] What can be an example of a demand function that satisfies the conditions for $\mathrm{d}Q/\mathrm{d}\lambda > 0$? For instance, following Ghosh et al. (2022, p. 112), consider $P(Q) = e^{\frac{1}{Q}}$, for which $P'(Q) = -e^{\frac{1}{Q}}/Q^2$, $P''(Q) = (1+2Q)e^{\frac{1}{Q}}/Q^4$, and thus $\sigma(Q) = 2 + 1/Q > 2$. Further, this demand function also leads to $\mathrm{d}w/\mathrm{d}\lambda < 0$ since $\partial\sigma(Q)/\partial Q = -1/Q^2 < 0$. For iso-elastic demand $P(Q) = Q^{\frac{-1}{\varepsilon}}$, one can show that $\sigma(Q) = 1 + 1/\varepsilon$, which exceeds 2 if $\varepsilon < 1$. However, this demand function has $\partial\sigma(Q)/\partial Q = 0$, so only the negative direct effect would be at work and thus $\mathrm{d}Q/\mathrm{d}\lambda < 0$ nevertheless.

[18] Relatedly, Anderson et al. (2001, pp. 184-185) show (under differentiated-goods price competition) that demand convexity exceeding 2 implies that profits rise with a unit sales tax, whereas an even higher degree of convexity is needed for profits to rise with an *ad valorem* tax.

provides some theoretical support for the idea that an increase in overlapping ownership may give rise to a countervailing effect in the form of reduced input prices. All else being equal, such an effect would alleviate some of the anti-competitive concerns around OOAs in markets where firms buy inputs from powerful upstream suppliers.[19] However, my paper only takes a small step in understanding this effect and there are several open questions that should be addressed in future work.

First, given the issues discussed in Section 3, it would be interesting to extend the model to allow for bargaining over supply terms. Second, one could examine the impact of product differentiation and price competition in the downstream market. Third, given the findings of Levy (2024), it seems worthwhile to explore different mechanisms of corporate control in my framework. It could also be interesting to analyze how the impact of partial ownership on relationship-specific investments as studied by Levy (2024) interacts with the effects on pricing incentives, as studied in the present paper.

CRediT authorship contribution statement

Teis Lunde Lømo: Writing – review & editing, Writing – original draft, Formal analysis, Conceptualization.

Data availability

No data was used for the research described in the article.

Appendix A

A.1. Second-order conditions

Downstream level. Starting from the first-order condition (3) and then applying (1) and (6), it can be shown that $\partial^2\phi_i/\partial q_i^2 = P'(Q)[2-\alpha\sigma(Q)]$ and $\partial^2\phi_i/\partial q_i\partial q_k = P'(Q)[1+\lambda-\alpha\sigma(Q)]$. This gives

$$\frac{\partial^2\phi_i}{\partial q_i^2}+(n-1)\frac{\partial^2\phi_i}{\partial q_i\partial q_k}<0 \iff \sigma(Q)<\frac{n+1+\lambda(n-1)}{1+\lambda(n-1)}=1+\frac{1}{\alpha}, \tag{A.1}$$

as stated in Assumption 1. Further, we have

$$\frac{\partial^2\phi_i}{\partial q_i^2}<0 \iff \sigma(Q)<\frac{2}{\alpha}. \tag{A.2}$$

Note that $2/\alpha > 1+1/\alpha \iff 1/\alpha>1$, which holds $\forall\alpha<1$. Thus, Assumption 1, which ensures $\sigma(Q)<1+1/\alpha$, also ensures $\partial^2\phi_i/\partial q_i^2<0$.

Upstream level. From (7) and (8), we have $\partial\pi_U/\partial w = Q+(w-c)\left[1/(\partial\mu/\partial Q)\right]$. Starting from this, and using (8) again as well as $w-c=-Q(\partial\mu/\partial Q)$ from (10), we obtain

$$\frac{\partial^2\pi_U}{\partial w^2}=\frac{\partial Q}{\partial w}+\frac{1}{\frac{\partial\mu}{\partial Q}}+(w-c)\left[\frac{-\frac{\partial^2\mu}{\partial Q^2}\frac{\partial Q}{\partial w}}{\left(\frac{\partial\mu}{\partial Q}\right)^2}\right]=\frac{2\frac{\partial\mu}{\partial Q}+Q\frac{\partial^2\mu}{\partial Q^2}}{\left(\frac{\partial\mu}{\partial Q}\right)^2}. \tag{A.3}$$

Thus, given (11), we have $\partial^2\pi_U/\partial w^2<0 \iff 2(\partial\mu/\partial Q)+Q\left(\partial^2\mu/\partial Q^2\right)<0 \iff \tau(Q)<2$, as stated in Assumption 2.

A.2. Additional comparative statics

Downstream costs. Suppose that downstream firms have symmetric and constant per-unit costs $k>0$. Their profit functions are then $\pi_i=[P(Q)-w-k]q_i$. The equilibrium at $t=2$ is characterized by $\mu(Q)=w+k$, where $\mu(Q)$ is given by (5). Further, the equilibrium input price at $t=1$ is still $w=c-Q(\partial\mu/\partial Q)$ as in (10). Differentiating these equilibrium conditions with respect to k yields the system

$$\frac{\partial\mu}{\partial Q}\frac{\mathrm{d}Q}{\mathrm{d}k}=\frac{\mathrm{d}w}{\mathrm{d}k}+1,$$

$$\frac{\mathrm{d}w}{\mathrm{d}k}=-\frac{\mathrm{d}Q}{\mathrm{d}k}\left(\frac{\partial\mu}{\partial Q}+Q\frac{\partial^2\mu}{\partial Q^2}\right).$$

By solving this system and using (9) and (11), we find

[19] Within merger control, antitrust authorities have long considered whether a transaction can lead to greater buyer power and lower input prices that offset the reduction in product market competition, see, e.g., the 2010 US Horizontal Merger Guidelines, Section 8.

$$\frac{\mathrm{d}w}{\mathrm{d}k} = \frac{\tau(Q)-1}{2-\tau(Q)}, \tag{A.4}$$

$$\frac{\mathrm{d}Q}{\mathrm{d}k} = \frac{1}{P'(Q)\left[1+\alpha-\alpha\sigma(Q)\right]\left[2-\tau(Q)\right]}. \tag{A.5}$$

Thus, under Assumption 1 and Assumption 2, $\mathrm{d}w/\mathrm{d}k$ has the same sign as $\tau(Q)-1$ and $\mathrm{d}Q/\mathrm{d}k<0$ always.

Note also that if $\partial\sigma(Q)/\partial Q=0$, then $\tau(Q)=\sigma(Q)$ by (20) and $\mathrm{d}w/\mathrm{d}k=[\sigma(Q)-1]/[2-\sigma(Q)]$ by (A.4). Thus, while $\partial\sigma(Q)/\partial Q=0 \iff \mathrm{d}w/\mathrm{d}\lambda=0$ for a change in ownership, a similar neutrality result does not hold for a change in downstream cost. Instead, we here have $\mathrm{d}w/\mathrm{d}k=0$ only when $\sigma(Q)=0$ (e.g., for linear demand), and otherwise (given that $\partial\sigma(Q)/\partial Q=0$) $\mathrm{d}w/\mathrm{d}k<0$ if demand is log-concave and $\mathrm{d}w/\mathrm{d}k>0$ if demand is log-convex.

The number of firms. Differentiating (4) and (10) with respect to n yields

$$\frac{\partial\mu}{\partial Q}\frac{\mathrm{d}Q}{\mathrm{d}n} = \frac{\mathrm{d}w}{\mathrm{d}n} - \frac{(\lambda-1)}{n^2}QP'(Q),$$

$$\frac{\mathrm{d}w}{\mathrm{d}n} = -\frac{\mathrm{d}Q}{\mathrm{d}n}\left(\frac{\partial\mu}{\partial Q}+Q\frac{\partial^2\mu}{\partial Q^2}\right) - \left\{\frac{(\lambda-1)}{n^2}Q\left[P'(Q)+QP''(Q)\right]\right\},$$

where $(\lambda-1)/n^2=\partial\alpha/\partial n<0$. Solving this system and using (9), (11), (15), and (17) gives

$$\frac{\mathrm{d}w}{\mathrm{d}n} = \underbrace{\left[\frac{(1-\lambda)Q^2P'(Q)}{n^2\left[1+\alpha-\alpha\sigma(Q)\right]\left[2-\tau(Q)\right]}\right]}_{<0}\frac{\partial\sigma(Q)}{\partial Q} \tag{A.6}$$

$$\frac{\mathrm{d}Q}{\mathrm{d}n} = \underbrace{\left[\frac{(\lambda-1)Q}{n^2\left[1+\alpha-\alpha\sigma(Q)\right]\left[2-\tau(Q)\right]}\right]}_{<0}\left[\sigma(Q)-2\right]. \tag{A.7}$$

By comparing (A.6) to (18), we see that the sign of $\mathrm{d}w/\mathrm{d}n$ depends on $\partial\sigma(Q)/\partial Q$ in exactly the opposite way of $\mathrm{d}w/\mathrm{d}\lambda$, e.g., $\partial\sigma(Q)/\partial Q<0 \iff \mathrm{d}w/\mathrm{d}n>0$ whereas $\partial\sigma(Q)/\partial Q<0 \iff \mathrm{d}w/\mathrm{d}\lambda<0$. This is natural since a higher n amounts to an increase in downstream competition, whereas a higher λ amounts to a reduction of competition. When considering instead a marginal ***reduction*** in n, the effect on w is qualitatively the same as for a rise in λ. (However, the effects are quantitatively different since $(\lambda-1)/n^2 \neq -(n-1)/n$.) A parallel argument can be made for $\mathrm{d}Q/\mathrm{d}n$ by comparing (A.7) to (21).

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