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Price Regulation, Quality Competition and Location Choice with Costly Relocation*

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Abstract

We consider a model where for-profit providers compete in quality in a price-regulated market that has been opened to competition, and where the incumbent is located at the center of the market, facing high costs of relocation. The model is relevant in markets such as public health care, education and schooling, or postal services. We find that, when the regulated price is low or intermediate, the entrant strategically locates towards the corner of the market to keep the incumbent at the low monopoly quality level. For a high price, the entrant locates at the corner of the market and both providers implement higher quality compared to a monopoly. In any case, the entrant implements higher quality than the incumbent provider. Social welfare is always higher in a duopoly if the cost of quality is low. For higher cost levels welfare is non-monotonic in the price and it can be optimal to the regulator not to use its entire budget. Therefore, the welfare effect of entry depends on the price and the size of the entry cost, and the regulator should condition the decision to allow entry on an assessment of the entry cost.

JEL codes: D43, L13, L51

Keywords: Quality competition; Price regulation; Location choice; Product differentiation

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1 Introduction

The purpose of this paper is to characterize the market solution and social welfare when price-regulated markets are opened to competition. This issue is relevant in markets such as public health care, education and schooling, or postal services. These markets are typically characterized by a former public monopoly where the monopolist is already centrally located in the market and faces high relocation costs.

One example is the Swedish market for compulsory education. This market was opened up for competition in the early 1990s, with an aim of increasing the educational performance of all children. The market reform (of 1992) had three pillars: the right for private actors, including for-profit companies, with adequate qualifications to be officially recognized as education providers; voucher-based financing of compulsory and upper-secondary education; and an acknowledgement of the parents' right to select the private or publich school they want their children to attend. The share of students attending private schools increased after the reform, reaching approximately 11% in 2009 (Böhlmark and Lindahl, 2015).

Wondratschek et al. (2013) analyze the effects of the reform, including students in both private and public school, using detailed geographical information on the location of schools and students' residence to construct measures of competition. Specifically, the higher the number of schools is within a given radius around a student's home, the stronger is the competition.[1] The authors find that having one more school within the radius had a positive but small effect on final grades from compulsory school, and that tightening the radius increases the effect slightly.[2]

In this paper, we investigate a Hotelling model of price-regulated quality competition that captures the main characteristics of the market reforms and is capable of explaining the results of Wondratschek et al. (2013). First, we consider a case where a monopoly market is opened to a for-profit provider. Second, depending on the parameters of the model, different types of equilibrium result. These differ in qualities and in the location of the entrant. Quality is always higher for the entrant, and in equilibria, where the entrant locates in the interior of the market, the incumbent sticks to the monopoly quality level.

Our model has three stages and three active players: the regulator, the incumbent provider, and the entrant. At stage 1, the regulator sets the price aiming to maximize welfare subject to a constrained budget. At stage 2, in case of entry, the entrant decides where to locate on the Hotelling line, taking the central location of the incumbent as given. Then, both providers choose non-verifiable quality at stage 3, not being able to discriminate among customers.

[1] The authors use the median commuting distance within the municipality as their preferred measure, but also estimate the effects of a tighter radius (2 km).

[2] Studies of the reform find mixed results of whether competition, measured as a higher share of students in the municipality attending *private schools*, raises educational outcomes; Böhlmark and Lindahl (2007; 2015); Edmark et al. (2014); Hennerdal et al. (2018).

Finally, customers decide where to buy, trading off the differences in quality and travel costs. Except from locations, the model is kept symmetric; i.e., both providers maximize profit and face the same cost structure.

We find that the entrant chooses a location such that the incumbent sticks with the monopoly quality level when the regulated price is low or intermediate. For a high price, the incumbent increases quality and the entrant locates at the corner of the market. For all cases, the entrant implements higher quality. Our analysis shows that social welfare is non-monotone in the regulated price. For low and high prices, social welfare strictly increases in the regulated price. For intermediate prices, the change in social welfare depends on the marginal cost of quality. For a high marginal cost, social welfare eventually decreases in the regulated prices. In this case, the regulator will not use the full budget. This result follows since the regulator balances the effect of a higher price on the customers' increased travel cost and their benefit of higher quality. For all other cases, the benefit of higher quality from a higher price outweighs the increase in travel cost and hence social welfare increases in price. However, it might still be optimal to retain a part of the budget.

We complete our analysis with exploring the welfare consequences of entry. It turns out that entry always increases welfare if the cost of quality is small. In this case, welfare is higher in a duopoly whenever entry is profitable, since the travel cost is lower and quality (weakly) higher in a duopoly than in a monopoly. For intermediate or large levels of this cost, however, the welfare effect of entry depends on the price and the size of the entry cost. In this case, the customers' net benefit from treatment is smaller, while the entrant's profit is larger. As a consequence, the entrant also enters the market for large levels of the entry cost that exceed the net benefit from treatment.

The assumption of a centrally located incumbent is key to our analysis. We impose this assumption, since public markets such as schools or hospitals used to be regulated to be a monopoly. A central location is optimal in such a monopoly when customers are distributed uniformly in space. Moreover, these incumbents generally chose their location before discussions about market deregulation started. Providers in these markets often exhibit a substantial relocation cost, since highly specialized buildings containing school laboratories or operating theatres are involved. Therefore, a forward-looking entrant will take the incumbent's central location as given.

We consider the case where the incumbent operates as a profit-maximizing provider. Hence, either he converts its ownership status from not-for-profit to for-profit, or act as a profit-maximizer even when he may retain its status as a not-for-profit provider. That is, we follow Easley and O'Hara (1983) and Glaeser and Shleifer (2001), who use the assumption of noncontractible quality to motivate the existence of not-for-profit firms. The mechanism is that the nondistribution constraint on not-for-profit firms limits the ability of not-for-profit firms to distribute profits to the owner. Therefore, customers may prefer to purchase from not-for-profit firms. However, if the government does not enforce the nondistribution constraint, not-for-profit firms might maximize profit, which is what Weisbrod (1988) calls "for-profits-in-disguise."

Our paper is related to three strands of theoretical literature on competition. The first strand examines the role of competition in improving the performance of schools, see Epple et al. (2017) for an overview. An early contribution on voucher programs is Nechyba (1999). In his model, he introduces a private school market into a local public good economy and shows the importance of household mobility and general equilibrium effects in predicting the outcomes for voucher programs. There are, however, no strategic effects of competition in this model. Barseghyan et al. (2019) consider the effects of competition and peer preferences on school quality. In the absence of peer preferences, schools provide higher quality in a competitive environment. However, both schools choose the same quality level and no students exercise choice in equilibrium. Unlike in our model, there is no competition along the spatial dimension.

The second strand of literature considers the effects of competition in price-regulated (hospital) markets. Most of these papers consider models where firms are located along a Hotelling line. Brekke et al. (2006) and Bardey et al. (2012) consider profit-maximizing hospitals. Both these papers model competition in symmetric locations and quality, and analyze the equilibrium outcomes of markets where the product price is exogenous. Using an extended version of the Hotelling model, they assume that firms choose their locations and the quality of the product they supply. In Brekke et al. (2006) a welfarist regulator sets the optimal price, and the results depend on whether the regulator can commit to a price prior to hospitals choosing locations. If this is the case, and transportation costs are high, the optimal (second-best) price causes overinvestment in quality and results in an insufficient degree of horizontal differentiation. If the regulator is not able to make a commitment prior to hospitals choosing locations, the optimal price induces first-best quality, but horizontal differentiation is inefficiently high. The model of Bardey et al. (2012) differs from Brekke et al. (2006) in that quality increases the variable cost incurred by providers and that the regulator sets both the regulated price and a cost reimbursement rate. Giving the regulator an extra instrument (viz. the cost reimbursement rate) will improve the allocative efficiency in cases with low transportation costs. Besley and Malcomson (2018) explore the implications of entry by for-profit providers in public service provision, when there are two (non-verifiable) quality dimensions where one is also unobserved. In their mixed duopoly, entry by a for-profit provider is beneficial for providers given that the incumbent not-for-profit provider remains active. The intuition is that the for-profit-provider supplies markedly higher quality in the observable dimension to offset lower unobserved quality. Furthermore, they allow the (regulated) prices given to the providers to differ and calculate which provider should be given the highest price.

Our paper is also related to Hehenkamp and Kaarboe (2020), who analyze the location choice and quality competition in mixed hospital markets. More specifically, they consider the case in which a private hospital considers entering a market where a public hospital already resides at the center of the market. The public hospital cares about patients' utility (altruism). An implication of this is that the non-negative profit constraint can become binding in equilibrium. Depending on the degree of altruism, the public hospital might implement higher quality than the private hospital. Furthermore, for sufficiently large budgets, the equilibrium outcome corresponds to the constrained welfare optimum; i.e., the maximum welfare obtainable when quality is non-verifiable and the regulator can write a contract on the entrant's location. Our paper differs from Hehenkamp and Kaarboe (2020) in that both providers maximize profit. One implication of this assumption is that in the subgame perfect equilibrium, the entrant always provides higher quality than the incumbent does. Moreover, we provide a complete welfare analysis, covering all parameter configurations, while Hehenkamp and Kaarboe (2020) only consider the case of a small or large budget. As it turns out, welfare is decreasing for intermediate levels of the price if the cost of quality is intermediate or large. It follows that in the range of intermediate prices, entry does not always raise welfare. In this case, the regulator would need to assess the size of the entry cost to determine the welfare effect of entry. Our paper differs from the papers of Brekke et al. (2006) and Bardey et al. (2012) as we examine equilibria in which providers are located asymmetrically in the market. Finally, the paper differs from Besley and Malcomson (2018) as they do not consider a model of product differentiation, as we consider quality to be observable, and as we do not allow for differential prices among the providers.

The third strand of literature was initiated by Launhardt (1885) and Hotelling (1929) and investigates how competition affects locational choice. While Hotelling (later shown to be wrong) argues that firms will locate closely to each other, d'Aspremont et al. (1979) proved that, under some assumptions, firms will locate at a maximal distance from each other.[3] In both these papers, firms first choose locations and then prices to compete for consumers' demand. In contrast to this literature, we assume prices are regulated and providers compete in quality. The basic trade-off, however, is the same. From a provider's perspective, a larger distance softens competition, while moving closer to the competitor results in an increase in demand, as a provider "steals" customers away from its competitor. The first effect has been called *competition effect*, the second *demand-stealing effect.* Taking the incumbent's location at the center as given, the entrant balances these two effects when deciding where to locate in the market. The regulator in our model sets a uniform price to influence the entrant's location and the quality levels of the providers in order to maximize the social welfare subject to a constrained budget. To the best of our knowledge, the literature has not examined this issue. As it turns out, it can be optimal to the regulator not to use the full budget, in order to avoid too much differentiation and too high a transportation cost. Finally, observe that, by the duality result in Crémer and Thisse (1991), our analysis also bears implications for models of vertical differentiation where quality is two-dimensional. However, while the center location of the incumbent has a clear meaning and interpretation in our model, a fixed quality level in the middle along one of the two quality dimensions bears an unclear interpretation.

[3] In contrast, minimum differentiation results in the set-up of d'Aspremont et al. (1979) if the survival of firm strategies is analyzed under an economic evolutionary approach (Hehenkamp and Wambach, 2010).

2 The model

We examine a price-regulated market with both horizontal and vertical product differentiation that has been opened to allow for competition. There are three active players in the market: the regulator, an incumbent provider ($i = 1$), and a potential entrant ($i = 2$). The incumbent provider used to hold a monopoly position, which he took at the center of the market. Moreover, he faces a substantial relocation cost, which is why both the incumbent provider and the potential entrant commonly know that the incumbent provider 1 will not change its location in the case of entry.

We consider the following three-stage market game. At stage 1, the regulator decides whether or not to open the market and allow for entry. If he chooses to do so, he sets the price P given to the two providers. At stage 2, provider 2 decides whether to enter and where to locate, taking provider 1's location in the middle of the market as given. Finally, at stage 3, the active providers in the market choose their quality level, and customers decide where to receive one unit of the service. We solve this market game for its subgame perfect equilibrium.

Customers are uniformly distributed over the unit interval. Living at $x \in [0, 1]$, a customer seeking service at provider i ($i = 1, 2$) gains a utility

$$u = s + q_i - t\,(x - x_i)^2\,,$$

where $s > 0$ denotes the exogenous gross utility from the service, $q_i \geq \underline{q}$ the utility of the observable quality of the service at provider i, $\underline{q}$ the minimum quality level each provider has to offer (set to zero), where $t > 0$ stands for a transportation cost parameter, and where x_i represents the location of provider i. Without loss of generality, we assume the entrant locates to the right of the incumbent, i.e., $x_1 = 1/2 \leq x_2$. Observing the location and quality levels of the providers, customers choose one of the two providers to maximize their utility. We assume $s > t > 0$, which ensures that all customers request the service no matter where the private provider locates and no matter which quality levels the providers implement,

Providers earn profit from serving customers. Let π_i denote provider i's profit, gross of the entry cost $f > 0$, and $d_i \in [0, 1]$ provider i's demand. Provider i's objective is to choose quality (for $i = 1, 2$) and location (for $i = 2$) to maximize π_i subject to profit and quality being non-negative, i.e., $\pi_i \geq 0$ and $q_i \geq 0$, respectively. Let $P > 0$ be the reimbursed price and let parameters $k \geq 0$ and $c \in (0, 1)$ represent the cost of providing quantity and quality,[4] respectively. Provider i's gross profit π_i is then given by

$$\pi_i = (P - k - cq_i)\,d_i.$$

Since the incumbent provider is already present in the market, his entry cost is sunk. Without loss of generality, we set $k = 0$. Correspondingly, in our

[4] Notice that for $c \geq 1$ the marginal cost of quality would weekly exceed its marginal benefit. In this case, $q_i = 0$ would be socially optimal and low quality no longer a concern.

analysis the price P is to be interpreted as the mark-up on the marginal cost k of quantity. We assume that the cost of quality is related to the services that customers receive. Other aspects of provider quality, such as investments in technology and training programs for the staff, will typically involve a fixed cost, which does not fit our specification of the providers' cost function. Taking into account the above, we rewrite the profit function of provider i as

$$\pi_i = (P - cq_i)\, d_i. \tag{1}$$

While the cost $f > 0$ determines the entry decision of provider 1 at stage 2, it is sunk upon entry. Accordingly, at stage 3, the non-negative profit constraint of each provider is given by $q_i \leq P/c$. As a consequence, the price-cost margin is non-negative. We can hence write provider i's constrained maximization problem as

$$\max_{q_i \in \left[0, \frac{P}{c}\right]} (P - cq_i)\, d_i.$$

The demand of the two providers depends on locations and quality levels. If the entrant decides to locate at $x_2 = \frac{1}{2}$, then $x_1 = x_2$ and all customers select the provider with the highest quality. If both qualities coincide, customers distribute evenly. Correspondingly, the demand of the incumbent and the entrant are given by

$$d_1 = \begin{cases} 0 & \text{if } q_1 < q_2 \\ \frac{1}{2} & \text{if } q_1 = q_2 \\ 1 & \text{if } q_1 > q_2 \end{cases} \tag{2}$$

and $d_2 = 1 - d_1$, respectively. On the other hand, if the entrant locates away from the incumbent, $x_1 \neq x_2$, then the customer $\check{x}$ that is indifferent between seeking treatment at the incumbent or the entrant is characterized by:

$$\begin{aligned} s + q_1 - t\,(\check{x} - x_1)^2 &= s + q_2 - t\,(\check{x} - x_2)^2 \\ \iff \quad \check{x} &= \frac{x_1 + x_2}{2} + \frac{q_1 - q_2}{2t\,(x_2 - x_1)} \\ \iff \quad \check{x} &= \frac{S}{2} + \frac{q_1 - q_2}{2t\Delta}, \end{aligned} \tag{3}$$

where $S \equiv x_1 + x_2$ and $\Delta \equiv x_2 - x_1$. The demand of providers 1 and 2 is then

$$\begin{aligned} d_1 &= \max\{0, \min\{1, \check{x}\}\} \\ &= \min\{1, \max\{0, \check{x}\}\} \end{aligned}$$

and $d_2 = 1 - d_1$, respectively. The demand of provider i increases in own quality, q_i, and decreases in the quality level of the competitor, q_j, for $j \neq i$.

The regulator aims at maximizing social welfare. We define (gross) welfare, as the customers' utility plus profit, minus reimbursement and gross of the entry cost f. It is assumed that the regulator is given a fixed budget $B > 0$ by the central government and that quality is observable, but non-verifiable.[5]

[5] Quality is non–contractible or non–verifiable if it is observable by the contracting parties but not verifiable to outsiders, in particular the courts (Grossman and Hart (1986) and Hart and Moore(1988)).

Correspondingly, the regulator chooses the reimbursement price, $P \in [0, B]$, to maximize *welfare in the duopoly*,

$$W^d = \int_0^{d_1} \left(q_1 - t(x_1 - x)^2\right) dx + \int_{d_1}^1 \left(q_2 - t(x_2 - x)^2\right) dx \\ +s + \sum_i (P - cq_i)\, d_i - P, \tag{4}$$

subject to a constrained budget B, i.e., $P \leq B$. In addition, the regulator decides whether to open the market or not. Opening the market is optimal when entry raises welfare. Accordingly, for a given entry cost $f > 0$, the market will be opened if net welfare in the duopoly, $W^d - f$, exceeds that of the original monopoly W^m, i.e., if $W^d - f \geq W^m$.

3 Quality competition

Solving the model by backward induction, we start with deriving the Nash equilibrium for each quality subgame, proving its existence and uniqueness. Subsequently, we classify the set of Nash equilibria that arise across different subgames according to the levels of quality implemented by the two providers in the respective Nash equilibrium. We conclude this section by providing a two-part comparative static analysis on (1) how the equilibrium type changes with increasing price (for a given location) and (2) how the equilibrium type depends on the location of the entrant (for a given price).

Let the price $P > 0$ and the entrant's location $x_2 \geq 1/2$ be given, and consider first the case of minimum differentiation, i.e., $x_2 = 1/2$. In this case, the unique Nash equilibrium has both providers implementing[6]

$$q_1^* = q_2^* = \frac{P}{c},$$

and earning zero (running) profit.[7]

Now consider the case in which the entrant does not locate at the center, i.e., $1/2 < x_2$. In this case we have

$$\begin{aligned} \pi_1 &= (P - cq_1)\, d_1 \\ &= \frac{1}{2t\Delta}(P - cq_1)(q_1 - q_2 + t\Delta S). \end{aligned}$$

Maximizing π_i subject to the non-negative profit constraint for providers 1 and 2 yields the following first order conditions for interior candidates:

$$\frac{\partial \pi_i}{\partial q_i} = \frac{\partial (P - cq_i)}{\partial q_i} d_i + (P - cq_i)\frac{\partial d_i}{\partial q_i} = 0,$$

[6] As usual we use the superscript $*$ to denote equilibrium values of the various variables.

[7] Notice that the entrant's fixed cost of entry is sunk at this point.

for $i, j = 1, 2$, and $i \neq j$. Solving $\partial \pi_i / \partial q_i = 0$ for q_i, we obtain the best reply functions of the two providers for interior candidates[8]:

$$\begin{aligned} q_1^{\text{FOC}}(q_2) &= \frac{P}{2c} - \frac{t\Delta S}{2} + \frac{1}{2} q_2 \\ q_2^{\text{FOC}}(q_1) &= \frac{P}{2c} - \frac{t\Delta(2-S)}{2} + \frac{1}{2} q_1. \end{aligned} \tag{5}$$

Taking the constraints of non-negative quality into account, we get

$$q_i^{\text{BR}}(q_j) = \max\left\{0, q_i^{\text{FOC}}(q_j)\right\} < \frac{P}{c}, \tag{6}$$

for all $q_j \in [0, P/c]$, for $i, j = 1, 2$, and for $j \neq i$. Observe that a non-central location of the entrant always induces a positive profit to both the incumbent and the entrant.[9] Building on (5) and (6), we are ready to prove our first result:

Proposition 1 (Existence and uniqueness of a quality equilibrium) *For any $P > 0$ and any $x_2 \in [\frac{1}{2}, 1]$, there exists a unique Nash equilibrium (q_1^*, q_2^*) of the quality subgame. If $x_2 = \frac{1}{2}$, then we have $q_1^* = q_2^* = \frac{P}{c}$; if $x_2 \in \left(\frac{1}{2}, 1\right]$, then $q_i^* < \frac{P}{c}$ is obtained for both providers $i = 1, 2$.*

Proof. See the Appendix.

While the Nash equilibrium is unique for any quality subgame (P, x_2), different types of Nash equilibria occur across different quality subgames. We classify these Nash equilibria as follows:

Definition 1 (Equilibrium types and regions) *Let $P > 0$ and $x_2 \in [\frac{1}{2}, 1]$ be given and suppose (q_1^*, q_2^*) represents a Nash equilibrium of the quality subgame. Then we distinguish the following* equilibrium types*:*
Type I: *Both providers choose minimum quality, i.e., $q_1^* = q_2^* = 0$.*
Type II: *Only the entrant implements positive quality, i.e., $q_1^* = 0 < q_2^* < P/c$.*
Type III: *The quality equilibrium is interior, i.e., $q_i^* \in (0, P/c)$ for both providers $i = 1, 2$.*
Type IV: *The quality level of both providers is constrained by the non-negative profit condition: $q_1^* = q_2^* = P/c$.*
We say a location x_2 lies in equilibrium region *$\tau \in \mathcal{T} \equiv \{\text{I, II, III, IV}\}$, denoted by $x_2 \in X_\tau$, if location x_2 gives rise to a quality equilibrium of the corresponding type $\tau \in \mathcal{T}$.*

Intuitively, a higher type number corresponds to a higher level of quality. Equilibrium type IV occurs if and only if the entrant locates at the center, $x_2 = \frac{1}{2}$, that is $X_{IV} = \{1/2\}$.

Figure 1 displays the various types of quality equilibrium; parts (a) to (c) of the figure correspond to equilibrium types I, II, and III, respectively.

[8] The second order condition for a profit maximum is satisfied for all $x_2 > 1/2$ because of $\partial^2 \pi_i / \partial q_i^2 < 0$. Moreover, notice that the location of the entrant x_2 and the price P only affect the interceptions of the best reply functions, but not the slope.

[9] This holds because $x_2 \in (1/2, 1]$ implies $\Delta > 0$ and $S \leq 3/2$ and hence $q_2^{\text{FOC}}(P/c) < P/c$.

Figure 1 to be included here (see pp. 41-43)

We continue with a comparative static analysis of the quality equilibria across subgames. Taking the price $P > 0$ as given, Proposition 2 below examines how the equilibrium type varies with the entrant's location $x_2 \in (1/2, 1]$.

Proposition 2 (Equilibrium types by price and location) *Let $P > 0$ be given. If $x_2 = 1/2$, then the quality equilibrium is of type IV. For $x_2 \in \left(\frac{1}{2}, 1\right]$, the equilibrium type is as follows:*

(a) If $P > 7ct/12$, then type III results for all $x_2 \in \left(\frac{1}{2}, 1\right]$.

(b) If $ct/4 < P \leq 7ct/12$, then type III entails for $x_2 < \xi_{III,II}$ and type II for $x_2 \in \left[\xi_{III,II}, 1\right]$, where

$$\xi_{III,II} = -1 + \frac{\sqrt{3}}{2t}\sqrt{\frac{t}{c}(4P + 3ct)} \in \left(\frac{1}{2}, 1\right]. \tag{7}$$

(c) If $P \leq ct/4$, then type III occurs for $x_2 < \xi_{III,II}$, type II for $x_2 \in \left[\xi_{III,II}, \xi_{II,I}\right)$, and type I for $x_2 \in \left[\xi_{II,I}, 1\right]$, where

$$\xi_{II,I} = 1 - \frac{1}{2ct}\sqrt{c^2t^2 - 4ctP} \in \left(\frac{1}{2}, 1\right). \tag{8}$$

According to Proposition 2, the normal intuition applies. Quality competition is the more intense the closer the entrant locates to the incumbent. The entrant can avoid a positive quality level of both providers by locating sufficiently close to the corner, i.e., at $x_2 > \xi_{II,I}$. It implements a positive quality, but keeps the incumbent at zero quality by locating in the intermediate range, i.e., for $x_2 \in \left[\xi_{III,II}, \xi_{II,I}\right)$. Competition becomes intense and both providers implement positive levels of quality when the entrant locates too close to the center, i.e., for $x_2 < \xi_{III,II}$. Moreover, both boundary locations, $\xi_{III,II}$ and $\xi_{II,I}$, are increasing in price, similarly reflecting that, for a given location of the entrant, competition becomes more intense at a higher price.

The three cases of Proposition 2 motivate the following definition:

Definition 2 *We call the price P*

(a) low for $P \leq ct/4$,

(b) intermediate for $ct/4 < P \leq 7ct/12$, and

(c) high for $P > 7ct/12$.

A similar terminology applies to the budget B in (4).

By construction, if the price is low, with varying locations $x_2 \in \left(\frac{1}{2}, 1\right]$ all equilibrium types I, II and III occur. If the price is intermediate, type I does not occur for any location $x_2 \in \left(\frac{1}{2}, 1\right]$. Finally, if the price is high, this implies an equilibrium of type III no matter where the entrant locates.

4 Entry and location

We continue with analyzing stage 2. At this stage, the entrant decides whether to enter and, if so, where to locate. When making this decision, the entrant takes the price $P > 0$ and the incumbent's location at the center of the market as given and anticipates the consequences of the location choice for the quality competition at stage 3.

We start with deriving the incumbent's optimal location, taking entry as given. To this end, we first determine the optimal location within each equilibrium region. We then compare the optimal locations across equilibrium regions. Subsequently, we analyze the conditions for which entry actually occurs.

The incumbent chooses its location $x_2 \in [1/2, 1]$ in order to maximize profit:

$$\pi_2 = (P - cq_2)\, d_2.$$

Hence, marginal profit is given by[10]

$$\begin{aligned} \frac{\partial \pi_2}{\partial x_2} &= \frac{\partial}{\partial x_2}\left((P - cq_2)\, d_2\right) \\ &= -c\frac{\partial q_2}{\partial x_2} d_2 + (P - cq_2)\frac{\partial d_2}{\partial x_2}, \end{aligned}$$

where q_2, d_2, and the derivatives depend on the equilibrium region under consideration.

As an example, consider the location choice within *equilibrium region I*. In this case, we have $q_1^* = q_2^* = 0$ and hence $d_1^* = S/2$ and $d_2^* = 1 - S/2$. Therefore, a change in the location of the entrant only has a direct effect on its demand, but no indirect effect via a change in quality. As the entrant's demand decreases in x_2, we have

$$\frac{\partial \pi_2}{\partial x_2} = P\frac{\partial d_2}{\partial x_2} = -\frac{P}{2} < 0.$$

Thus, within equilibrium region I, the entrant will locate as close to the center as possible.

The other two cases are analyzed in the Appendix. Proposition 3 below summarizes our findings.

Proposition 3 (Location choice within equilibrium regions) *The profit of the entrant strictly increases (strictly decreases) with its location* x_2*,*

$$\frac{\partial \pi_2}{\partial x_2} > (<) 0,$$

in equilibrium region III (in equilibrium regions I and II).

[10] Even though the (reduced) profit function of the entrant at stage II represents a function of a single variable, we use the notation for partial derivatives here to avoid misunderstandings, given that the symbol d has already been introduced to denote demand.

Proof. See the Appendix.

Like in the standard Hotelling model, such as analyzed in d'Aspremont et al. (1979), our model exhibits a trade-off between moving away from the competitor to soften competition (the *competition effect*) and getting closer to steal demand (the *demand effect*). To provide the intuition of Proposition 3, consider a marginal shift of the entrant's location to the left, i.e., a small decrease in x_2.

In an equilibrium of type I or II, the incumbent does not respond by changing its quality. That is, the competition effect is weak. Correspondingly, the entrant moves closer to the center to steal demand from the incumbent. In contrast, in an equilibrium of type III, the incumbent does respond by raising quality when the entrant moves closer. Since quality competition is costly, the entrant locates further away from the center to dampen quality competition.

We continue our analysis by comparing locations across equilibrium types. First, consider the case of a *high* price such that all locations $x_2 \in (1/2, 1]$ give rise to an equilibrium of type III. In this case, the above proposition shows that the entrant optimally locates at $x_2^* = 1$, entailing a positive profit of the entrant. In contrast, profit would be zero at $x_2 = 1/2$. Similarly, if the price is *low or intermediate* then the monotonicity properties established in the proposition above suggest that the entrant optimally locates at the boundary of regions II and III. We thus have:

Proposition 4 (Location choice across equilibrium regions)
(a) If the price is low or intermediate, then the entrant locates at the boundary of regions II and III, i.e., $x_2^* = \xi_{III,II}$.
(b) If the price is high, then the entrant locates at the corner $x_2^* = 1$.

Proof. See the Appendix.

Figure 2 displays the entrant's location choice for the three price ranges that can occur.

Figure 2 to be included here (see p. 44)

When *the price is high*, the standard intuition of d'Aspremont et al. (1979) applies: The competition effect dominates the demand effect and maximum differentiation results because the equilibrium region III encompasses (almost) the entire set of the entrant's feasible locations. When *the price is low or intermediate*, maximum differentiation no longer occurs because the competition effect (as compared to the demand effect) is strong only in region III, but not in regions I and II.

We complete the analysis of stage 2 with examining the conditions such that market entry ex ante generates nonnegative profit for provider 2 in equilibrium. In this case, it is optimal to provider 2 to enter the market. Obviously, ex ante profit depends on the price P and on the entry cost f. The next proposition characterizes how the equilibrium profit of provider 2 varies with the price P. It also gives upper limits of the entry cost above which no entry occurs.

Proposition 5 (The entrant's profit) *The equilibrium profit of provider 2,* $\pi_2^*(P)$, *is continuous in* P. *Let the entry cost* $f > 0$ *be given.*
(a) For low and intermediates prices, $\pi_2^*(P)$ *strictly increases in* P *and we have* $\lim_{P\to 0^+} \pi_2^*(P) = 0$ *and* $\pi_2^*(7ct/12) = 25ct/144$. *Provider 2 enters the market if* $f \leq \pi_2^*(P)$ *and stays out otherwise.*
(b) For high prices, the profit of provider 2 is constant; we have $\pi_2^*(P) = 25ct/144$ *for all* $P \geq 7ct/12$. *Provider 2 enters for* $f \leq 25ct/144$ *and abstains from entry otherwise.*

Proof. See the Appendix.

If the price is low or intermediate then, for a given level of the entry cost f, by continuity and positive monotonicity of $\pi_2^*(P)$, there exists a critical price $\widehat{P}$ such that entry only occurs above this price. For clarity of the exposition, we contend ourselves with providing an implicit characterization of this price, $\pi_2^*(\widehat{P}) = f$.[11] On the other hand, if the price is high, the entrant locates at the corner of the market such that any increase in the price does not affect its location. Furthermore, any increase in the price affects both providers quality choices in the same way so that the difference in qualities and hence demand remain constant. Finally, the raise in the price induces the qualities to increase exactly to the extent that the increase in the cost of quality matches the increase in the price. Therefore, profit remains constant for high prices.

How can the regulator induce entry? By Proposition 5, the entrant's equilibrium profit, $\pi_2^*(P)$, is continuous and strictly increasing from 0 to $25ct/144$ for low and intermediate prices $P \in (0, 7ct/12]$. Therefore, for any level of the entry cost $f' \in (0, 25ct/144]$, there exists a price $P' \in (0, 7ct/12]$ such that provider 2 enters for all prices $P \geq P'$, but not for any price $P < P'$. Equivalently, for any low or intermediate level of the price, $P' \in (0, 7ct/12]$, there exists a level of the entry cost $f' \in (0, 25ct/144]$ such that entry occurs for any entry cost $f \leq f'$, while no entry occurs for any entry cost $f > f'$. Since for high prices, $P > 7ct/12$, the entrant's profit remains constant at its maximum level $\pi_2^*(P) = 25ct/144$, no entry occurs for any entry cost $f > 25ct/144$ at any price $P > 0$.

5 Welfare analysis

The regulator should open the market to the entrant if welfare in a duopoly exceeds the welfare in a monopoly. In the case of a *monopoly*, the incumbent provider implements zero quality no matter how large the price is. This follows from the assumption that quality is non-verifiable. Therefore, welfare in the

[11] Since profit, considered as a function of the price P, can be rewritten as a cubic polynomial, the explicit expression of $\widehat{P}$ looks cumbersome.

case of a monopoly amounts to[12]

$$W^m = s - t\int_0^1 \left(\frac{1}{2} - x\right)^2 dx = s - \frac{t}{12}. \tag{9}$$

We continue with the *duopoly* case assuming entry occurs. Further below, we integrate the entry decision into our analysis. Recall net welfare (4), which is to be maximized subject to a constrained budget B, i.e., $P \leq B$. We rewrite (4) as

$$W^d = \sum_{i=1,2} (1-c)\, q_i d_i - \int_0^{d_1} t\left(\frac{1}{2} - x\right)^2 dx - \int_{d_1}^1 t\,(x_2 - x)^2\, dx + s. \tag{10}$$

The first term on the right hand side represents the social benefit from quality provision net of the social cost of providing a positive level of quality. The second and third term stand for the transportation cost of customers served by providers 1 and 2, respectively. Finally, the gross utility of the service, s, is constant and hence is neither relevant for maximizing (4) nor is it relevant for the comparison with welfare in case of a monopoly.

In a duopoly, *quality* in equilibrium is strictly positive for those customers served by the entrant. For customers served by the incumbent provider, quality will be zero for both a low and an intermediate price $P \leq 7ct/12$, since in this case the entrant locates at $x_2^* = \xi_{III,II} \leq 1$ and the resulting quality equilibrium is of type II. In contrast, quality is strictly positive for both the incumbent's and the entrant's customers when prices are high, $P > 7ct/12$, such that the boundary location $x_2^* = 1$ gives rise to a quality equilibrium of type III. Thus, for both a low and an intermediate price, the entrant's customers benefit from increased quality, while for a high price, also the incumbent's customers receive higher quality.

For *low and intermediate prices* $P \leq 7ct/12$, an increase in the price P has two opposing effects on the *social net benefit of quality*, i.e., on $\sum_{i=1,2} (1-c)\, q_i^* d_i^*$. On the one hand, it pushes the entrant to raise his quality q_2^* (competition effect). On the other hand, it induces the entrant to locate further away from the center of the market, implying a lower market share d_2^* (demand effect). A change in the price affects the trade-off between the competition effect and the demand effect, strengthening the former. Hence, the social net benefit of quality is increasing in the price P.[13]

For a low price $P \in (0, ct/4]$, the entrant locates to the left of 3/4. Therefore, *transportation costs* are decreasing in price since an entrant's customers to his right face a larger transportation cost than customers to his left. It thus follows that welfare is increasing for low prices.

For intermediate prices $P \in (ct/4, 7ct/12]$, an increase in the price induces the entrant to locate further towards the right corner of the market. Hence,

[12] Observe that the center location minimizes transportation cost and hence represents the welfare-optimal location in this case.

[13] Technically, a raise in the price has a linear effect on quality, while the effect on demand is of order 1/2.

a trade-off between the social net benefit of quality and transportation cost arises. It turns out that, for a low marginal cost of quality $c \leq 5/12$, the marginal social net benefit of quality exceeds the marginal transportation cost for all intermediate prices. In contrast, for a higher marginal cost of quality $c \in (5/12, 1)$, marginal transportation costs eventually exceed the marginal social net benefit of quality. In these cases, welfare will eventually decrease in price.

For *high prices* $P > 7ct/12$, the entrant always locates at the boundary $x_2^* = 1$. While both providers raise quality in response to higher prices, their market shares do not change. Therefore, *transportation costs* remain constant. Since the *social net benefit of quality* increases in price so does welfare.

The following proposition summarizes our findings.

Proposition 6 (Welfare in duopoly) *Let the budget $B > 0$ be strictly positive and let $W(P)$ denote welfare in a duopoly as a function of the price $P > 0$.*

(a) If the budget is low, then welfare strictly increases in price for all $P \leq B$. Spending the full budget maximizes welfare $W(P)$, i.e., $P^* = B$.

(b) Suppose the budget is intermediate. Then welfare $W(P)$ is continuous at $P = ct/4$, i.e., $\lim_{P \to (ct/4)^+} W(P) = W(ct/4)$. Moreover, we have:
(i) If $c \leq \frac{5}{12}$, then welfare $W(P)$ strictly increases in price P for all $P < B$. Spending the full budget maximizes welfare, i.e., $P^* = B$.
(ii) If $c > \frac{5}{12}$, then there exists a unique price $\widehat{P}_1 < 7ct/12$ such that $W(\widehat{P}_1) > W(P)$ for all $P \leq 7ct/12, P \neq \widehat{P}_1$. The welfare-maximizing price is $P^* = B$ for budgets $B \leq \widehat{P}_1$ and it is $P^* = \widehat{P}_1 < B$ for budgets $B \in (\widehat{P}_1, \frac{7ct}{12}]$ (in short, $P^* = \min\{B, \widehat{P}_1\}$). In the latter case, it is welfare-optimal not to spend the full budget.

(c) Suppose the budget is large. Then welfare $W(P)$ constitutes an affine, strictly increasing function of the price for all $P > 7ct/12$. Welfare $W(P)$ is continuous at $P = 7ct/12$, i.e. $\lim_{P \to (7ct/12)^+} W(P) = W(7ct/12)$. Moreover, we have:
(i) If $c \leq \frac{5}{12}$, then spending the full budget maximizes welfare W, i.e., $P^* = B$.
(ii) If $c > 5/12$, then there exists a unique $\widehat{P}_2 > 7ct/12$ such that $W(\widehat{P}_2) = W(\widehat{P}_1)$, where $\widehat{P}_1$ constitutes the locally optimal price defined in part (b)(ii). For budgets $B \in (7ct/12, \widehat{P}_2)$, the welfare-optimal price is $P^* = \widehat{P}_1$, for budgets $B > \widehat{P}_2$ it amounts to $P^* = B$.

Figure 3 illustrates the welfare-optimal decision of the regulator for the case of $c > 5/12$. The graph depicts welfare as a function of the price P. For low prices $P \leq ct/4$, welfare is strictly increasing. For intermediate prices such that $P/ct \in (1/4, 7/12]$, welfare first increases and then decreases in price if the cost is large ($c > 5/12$). Welfare is maximal at $\widehat{P}_1$. For large prices $P > 7ct/12$, welfare is linear and strictly increasing in price. Therefore, welfare eventually reaches the maximum welfare $W(\widehat{P}_1)$ of the intermediate range, which occurs at $\widehat{P}_2$. Accordingly, for low and intermediate budgets $B \leq \widehat{P}_1$, it is optimal to spend the full budget, i.e., $P^* = B$. In contrast, for intermediate and large budgets $B \in (\widehat{P}_1, \widehat{P}_2)$, the regulator sets $P^* = \widehat{P}_1$ and optimally retains $B - \widehat{P}_1$ of the budget. Finally, for large budgets $B \geq \widehat{P}_2$ spending the full budget is again optimal.

Figure 3 to be included here (see p. 45)

While Figure 3 suggests that the range between the two critical prices $\widehat{P}_2$ and $\widehat{P}_1$ can be quite substantial if the marginal cost of quality is large $c \in (5/12, 1)$, this need not be the case. As Proposition 7 shows, the range disappears in the limit $c \to (5/12)^+$, but strictly increases in c for all levels of marginal cost $c \in (5/12, 1)$. Finally, in the limit of $c \to 1$, the range becomes arbitrarily large.

Proposition 7 (Range of non-optimal prices) *Let $c \in (5/12, 1)$ and $t > 0$ be given arbitrarily. Then the range $\widehat{P}_2(c) - \widehat{P}_1(c)$ strictly increases with $c \in (5/12, 1)$. Moreover, we have*

$$\lim_{c \to \frac{5}{12}^+} \left(\widehat{P}_2(c) - \widehat{P}_1(c) \right) = 0$$

and

$$\lim_{c \to 1^-} \left(\widehat{P}_2(c) - \widehat{P}_1(c) \right) = \infty.$$

As a last step of our analysis, we integrate the entry decision of provider 2 into our welfare analysis. The welfare effect of entry depends on the fixed cost of entry, $f > 0$, which in itself represents a welfare cost. To explore the welfare effect of entry, it is sufficient to restrict attention to those levels of the entry cost f that allow provider 2 to earn a net profit given the price P set by the regulator, i.e. to $f \leq \pi_2^*(P)$. This observation motivates the following definition:

Definition 3 *We say* entry *(of provider 2)* always raises welfare at price $P > 0$ *if, and only if, for all levels of the entry cost $f > 0$, we have*[14]

$$f \leq \pi_2^*(P) \quad \Longrightarrow \quad W^d(P) - f \geq W^m. \tag{11}$$

According to Definition 3, it suffices to compare the social welfare in a monopoly and the social welfare in a duopoly net of the maximum feasible entry cost of $f = \pi_2^*(P)$. Whenever entry increases welfare for this maximum feasible entry cost, it also increases welfare for any entry cost lower than that, i.e., for $f \leq \pi_2^*(P)$. Equivalently, when entry does not always raise welfare then

[14] Recall that $W^d(P)$ denotes social welfare at price P *gross* of the entry cost f.

welfare in a duopoly falls below welfare in a monopoly for the maximum feasible entry cost $f = \pi_2^*(P)$, i.e., we have $W^d(P) - \pi_2^*(P) < W^m$. In the proof of the following theorem, we draw on this insight to explore the welfare consequences of entry.[15]

Theorem *Let $t > 0$ be given arbitrarily.*

(a) *If $c \leq 5/14$ then entry always raises welfare at any price $P > 0$.*

(b) *Suppose that $5/14 < c < 1/2$. Then there exist price thresholds $0 < \check{P}_1 < 7ct/12$ and $\check{P}_2 > 7ct/12$ such that (i) entry always raises welfare at any price $P \in \left(0, \check{P}_1\right] \cup \left[\check{P}_2, \infty\right)$, while (ii) entry does not always raise welfare at prices $P \in \left(\check{P}_1, \check{P}_2\right)$.*

(c) *Suppose that $c \geq 1/2$. Then there exists a price threshold $\check{P}_2 > 7ct/12$ such that (i) entry always raises welfare at any price $P \in \left[\check{P}_2, \infty\right)$, while (ii) entry does not always raise welfare at prices $P \in \left(0, \check{P}_2\right)$.*

To provide the intuition underlying the above theorem, observe that equilibrium profit is linear and strictly increasing in the cost c, once it is considered as function of a standardized price $p \equiv P/(ct)$. This holds for all levels of the price $P > 0$.[16] Moreover, we can decompose welfare in a duopoly, W^d, into variable patient benefit $\beta(p)$ and transportation cost $\tau(p)$ as follows:

$$W^d = (1 - c)\,\beta(p) - \tau(p) + s.$$

Accordingly, a change in the cost c directly affects the social net benefit of quality, while it influences the transportation cost only indirectly via the standardized price p. Consequently, if the cost c is sufficiently low, then the equilibrium profit of the entrant will be small in comparison to the social net benefit of quality for all levels of the price, since the direct effect of a change in cost dominates the indirect effects through the price. Since welfare in monopoly does not depend on the price, entry always raises welfare in this case (part (a)). In contrast, if the cost is sufficiently large, then the social net benefit of quality becomes quite small and the entrant's profit large. In this case, entry does not always raise welfare for low and intermediate prices. However, for large prices, equilibrium profit of the entrant is constant in p, while the social net benefit of quality $\beta(p)$ is affine and strictly increasing in p. Therefore, for prices sufficiently large, the welfare difference between duopoly and monopoly exceeds the entrant's profit eventually. Hence, entry always raises welfare for large prices (part (c)). Finally, if the cost c is intermediate, then equilibrium profit of the entrant falls below the welfare difference between duopoly and monopoly for

[15] Notice that the critical prices derived in the theorem do not coincide with those of Propositions 6 and 7.

[16] See equations (22) and (26) in the appendix.

small prices (like in part (a)). For intermediate prices, the entrant's profit exceeds this welfare difference. Hence, for levels of the entry cost in between, entry occurs but reduces welfare. For larger prices the intuition of part (c) applies.[17]

To sum up, if the cost is low, opening the market always increases welfare. If the cost is intermediate or large, then the regulator can ensure that entry always raises welfare by setting the price sufficiently large. In case the budget would not allow to set the price accordingly, it depends on the actual entry cost, whether entry raises welfare. In this case, the regulator should condition the decision to allow entry on an assessment of the entry cost. It should permit entry for levels of the entry cost $f \in \left(0, W^d\left(P\right) - W^m\right]$, while for larger levels of f entry should be prohibited.

6 Conclusion

In this paper, we introduce a model where for-profit providers compete in quality in a price-regulated market that has been opened to competition and where the incumbent is located at the center of the market and faces high costs of relocation. We find that the entrant locates away from the incumbent to soften quality competition. For low and intermediate prices (budgets), the incumbent will not raise quality, which is in contrast to what politicians typically will expect from such a reform. Excluding the entry cost, however, social welfare in the duopoly is higher than in the original monopoly. This result follows since entry reduces customers' transportation costs and since the entrant implements strictly higher quality than the incumbent. To obtain the positive welfare effect of competition, the regulator sometimes has to retain a part of the budget, which can be challenging to defend in the public domain.

Once the fixed entry costs are taken into account, a more complex picture emerges. For low levels of the variable quality cost, the social benefit from treatment exceeds the entrant's profit (gross of the entry cost). Therefore, entry entails an increase in social welfare. For intermediate and large levels of the quality cost, the entrant's profit surpasses the social benefit from treatment for certain ranges of the price. Then, opening the market reduces social welfare if the entry cost is large, but just falls below the entrant's profit. In this case, the regulator would optimally condition the admission of market entry on an estimate of the entry costs.

We have assumed that the incumbent operates as a profit-maximizing provider. Hence, either the provider operates as a for-profit in disguise or the incumbent converts its ownership from not-for-profit to a for-profit status. In the US, such conversions of ownership are not uncommon. For example, 700 conversions of hospital ownership took place between 1985-99, most of these from non-for-profit to for-profit (Shen, 2002) and 237 hospitals converted from not-for-profit to for-profit between 2003-10 (Joynt et al., 2014). In the educational

[17] As the proof of the theorem reveals, the interval $\left(0, \check{P}_1\right]$ collapses for $c \geq 1/2$. In this case, the welfare difference between duopoly and monopoly first falls below the entrant's profit for all prices $P \in \left(0, \check{P}_2\right)$, and it exceeds the entrant's profit for all prices $P > \check{P}_2$.

sector, 40 of the 58 conversions of US postsecondary educational institutions that occurred from 2004-09 were from not-for-profit to for-profit (Fox Garrity and Fiedler, 2016). What motivates these conversions? One reason might be that a legal or a market constraint that a provider faces is changing, and that the change implies heterogeneous effects on the ownership types. For example, lowering the tax rate for-profit hospitals faces reduces the financial benefit of maintaining a non-for-profit status. On the other hand, increased competition or changes in the financing system might lead to financial losses and a need for not-for-profit providers to increase the focus on the financial constraints.

Joynt et al. (2014) consider the effects of hospital conversion to for-profit status on financial performance and quality of care for Medicare inpatient care. For these patients Medicare pays hospitals a fixed price per patient discharge using the Inpatient Prospective Payment System[18]. Using a difference-in-difference approach the authors find that hospitals that converted to for-profit status improved their financial performance and that their quality of care including measures of mortality rates, remained unchanged.[19] The hospitals that converted to for-profit status had very poor financial performance prior to the conversion. The rationale for the conversion was most likely the need to improve the financial performance.

As mentioned in the introduction, the empirical studies of the Swedish educational reform find mixed results of whether competition, measured either by the share of students in the municipality attending private schools or as the number of schools within a given radius around a student's home, raise educational outcomes (Böhlmark and Lindahl, 2007, 2015; Edmark et al., 2014; Hennerdal et al., 2020; Wondratschek et al., 2013). Specifically, they find either no or a small positive effect on educational outcomes. Our results show that this is to be expected as the entrant locates away from the incumbent to soften quality competition. This often results in no quality improvements for the incumbent firm and only a slightly higher quality level of the entrant. Only when the price is high, our model predicts higher equilibrium quality of both providers. Finally, our model is consistent with the evidence that the private educational providers implement higher quality (Böhlmark and Lindahl, 2007).

A limitation of our model is that we do not include a fixed cost of quality provision. Expanding the model's relevance by including such a fixed cost would cause problems with the existence of pure strategy equilibria in the quality competition subgame when the entrant locates close to the incumbent. From the literature on the Bertrand model, we know that mixed strategy equilibria still exist when there is such a fixed cost. However, when there is a substantial cost of relocation it is hard to imagine that providers follow mixed strategies (Dastidar, 1995).

[18] See e.g. Cubanski et al. (2015).

[19] Studies of the conversions in the 1990 find that quality is reduced after conversions to for-profit status (Picone et al, 2002; Shen, 2002). The regulatory framework is however different between the 1990s and the latter years, with current policies focusing on monitoring, reporting and rewarding quality.

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Appendix

Proof of Proposition 1. Consider $x_2 = 1/2$ first. In this case, by discontinuity of the provider-specific demand function (2), a standard Bertrand type of argument shows the existence and uniqueness of the Nash equilibrium $(q_1^*, q_2^*) = (P/c, P/c)$.

Now consider $x_2 \in (1/2, 1]$. Notice that the best reply curves (6) are weakly increasing. Therefore, Tarski's fixed-point theorem yields the existence of a Nash equilibrium (q_1^*, q_2^*) (Tarski, 1955). To show that this equilibrium is unique, the following Lemma turns out helpful.

Lemma 1 *Let $q_i^{BR}(q_j)$ be given by (6) and $q', q'' \in Q \equiv [0, \infty)^2$ such that $q' = (q_1', q_2')$ and $q'' = (q_1'', q_2'')$. Then we have:*
(i) If $q_1' = q_1^{BR}(q_2')$ and $q_2' \leq q_2''$, then $q_1^{BR}(q_2'') \leq q_1' + (q_2'' - q_2')/2$.
(ii) If $q_2' = q_2^{BR}(q_1')$ and $q_1' \leq q_1''$, then $q_2^{BR}(q_1'') \leq q_2' + (q_1'' - q_1')/2$.

Proof of Lemma 1. Choose $q', q'' \in Q$ arbitrarily.

To prove part (i), let $q_1' = q_1^{BR}(q_2')$ and $q_2' \leq q_2''$. We need to show that

$$q_1^{BR}(q_2'') \leq q_1^{BR}(q_2') + \frac{1}{2}(q_2'' - q_2'). \tag{12}$$

First, consider the case $q_1^{FOC}(q_2'') \leq 0$. If $q_1^{FOC}(q_2'') \leq 0$, then $q_2' \leq q_2''$ implies that $q_1^{FOC}(q_2') \leq q_1^{FOC}(q_2'') \leq 0$ and hence $q_1^{BR}(q_2'') = q_1^{BR}(q_2') = 0$. Therefore, the inequality is satisfied.

Second, consider $q_1^{FOC}(q_2'') > 0$. On the one hand, if $q_1^{FOC}(q_2') > 0$ then $q_1^{BR}(q_2'') - q_1^{BR}(q_2') = q_1^{FOC}(q_2'') - q_1^{FOC}(q_2') = \frac{1}{2}(q_2'' - q_2')$ and inequality (12) holds with equality. On the other hand, if $q_1^{FOC}(q_2') \leq 0$ then we have $q_1^{BR}(q_2'') = q_1^{FOC}(q_2'')$ and $q_1^{BR}(q_2') = 0$. In this case, inequality (12) reduces to

$$q_1^{FOC}(q_2'') = \frac{P}{2c} - \frac{tS\Delta}{2} + \frac{1}{2}q_2'' \leq \frac{1}{2}(q_2'' - q_2'),$$

which is equivalent to $q_1^{FOC}(q_2') \leq 0$ and hence satisfied.

Part (ii) can be shown similarly. ■

We continue with proving uniqueness. Let $q', q'' \in Q$ both be Nash equilibria of the quality subgame and suppose that $q_1' \leq q_1''$ (w.l.o.g.). On the one hand, as $q_2^{BR}(q_1)$ is weakly increasing, it follows that $q_2' \leq q_2''$. On the other hand, the above Lemma implies that

$$\begin{aligned} q_1'' &\leq q_1' + \frac{1}{2}(q_2'' - q_2') \qquad \text{and} \\ q_2'' &\leq q_2' + \frac{1}{2}(q_1'' - q_1'). \end{aligned}$$

Rewriting the first inequality as

$$\frac{1}{2}(q_1'' - q_1') \leq \frac{1}{4}(q_2'' - q_2')$$

and combining it with the second inequality, we obtain

$$q_2'' \leq q_2' + \frac{1}{4}\left(q_2'' - q_2'\right),$$

which is equivalent to $q_2'' \leq q_2'$. We have thus shown that $q_2' = q_2''$. Since the best reply correspondences are single-valued, it moreover follows that $q_1' = q_1^{BR}(q_2') = q_1'' = q_1^{BR}(q_2'')$ and hence $q' = q''$, which shows uniqueness of the Nash equilibrium. ■

Proof of Proposition 2. Let $P > 0$ be arbitrary and let $q^* = (q_1^*, q_2^*)$ denote the unique Nash equilibrium of the quality game. The case $x_2 = 1/2$ is fully covered by Proposition 1. Therefore, consider $x_2 \in (1/2, 1]$. The proof is divided into two parts. We first derive three conditions on the price and the entrant's location such that, for each of these conditions, one of the equilibrium types I, II or III results, respectively. Subsequently, we establish parts (a) to (c) of the proposition.

The table below summarizes the relationship between the conditions and the equilibrium types. For brevity, we set $p := P/(ct) > 0$.

Type	**Condition**	**Quality equilibrium**
I	$p \leq \Delta(2-S)$	$q_1^* = q_2^* = 0$
II	$\Delta(2-S) < p \leq \Delta(S+2)/3$	$q_2^* > q_1^* = 0$
III	$\Delta(S+2)/3 < p$	$q_2^* > q_1^* > 0$

Notice that the conditions partition the set of possible cases, because $x_2 \in (1/2, 1]$ implies $\Delta(2-S) < \Delta(S+2)/3$.

First, consider $p \leq \Delta(2-S)$. In this case, the first order conditions (5) imply

$$q_2^{\mathrm{FOC}}(0) = \frac{P}{2c} - \frac{t\Delta(2-S)}{2} = \frac{t}{2}\left(p - \Delta(2-S)\right) \leq 0.$$

By (6), it hence follows that $q_2^{\mathrm{BR}}(0) = 0$. Moreover, $S > 2 - S$ implies

$$q_1^{\mathrm{FOC}}(0) = \frac{P}{2c} - \frac{t\Delta S}{2} = \frac{t}{2}\left(p - \Delta S\right) < \frac{t}{2}\left(p - \Delta(2-S)\right) \leq 0$$

and hence $q_1^{\mathrm{BR}}(0) = 0$. Thus, the equilibrium is of *type I.*

Second, let $\Delta(2-S) < p \leq \Delta(S+2)/3$. By the above, the first inequality implies

$$q_2^{\mathrm{FOC}}(0) = \frac{t}{2}\left(p - \Delta(2-S)\right) > 0 \tag{13}$$

and hence $q_2^{\mathrm{BR}}(0) = q_2^{\mathrm{FOC}}(0) > 0$. In addition, we have

$$\begin{aligned} q_1^{\mathrm{FOC}}\left(q_2^{\mathrm{FOC}}(0)\right) &= \frac{P}{2c} - \frac{t\Delta S}{2} + \frac{1}{2}q_2^{\mathrm{FOC}}(0) \\ &= \frac{t}{2}\left(p - \Delta S\right) + \frac{t}{4}\left(p - \Delta(2-S)\right) \\ &= \frac{3t}{4}\left(p - \frac{\Delta(S+2)}{3}\right) \leq 0, \end{aligned}$$

which implies $q_1^{\mathrm{BR}}\left(q_2^{\mathrm{FOC}}(0)\right)=0$. Thus, the equilibrium is of *type II*.

Finally, consider $p>\Delta(S+2)/3$. Observe that the solution to the first order conditions (5) is given by

$$\begin{aligned} q_1^{\mathrm{FOC}} &= \frac{P}{c}-\frac{t\Delta(S+2)}{3} \\ q_2^{\mathrm{FOC}} &= \frac{P}{c}-\frac{t\Delta(4-S)}{3}. \end{aligned} \tag{14}$$

Moreover, $x_2>1/2$ implies $q_2^{\mathrm{FOC}}>q_1^{\mathrm{FOC}}$ and $p>\Delta(S+2)/3$, in turn, implies $q_1^{\mathrm{FOC}}=t(p-\Delta(S+2)/3)>0$. Thus, the equilibrium is of *type III*.

Part (a): Let $P>7ct/12$ or, equivalently, $p>7/12$. Notice that $\Delta(S+2)/3$ is increasing in x_2 and that $\Delta(S+2)/3\leq 7/12$ because of $x_2\leq 1$. It hence follows that $p>\Delta(S+2)/3$ for all $x_2\in(1/2,1]$. Thus, equilibrium type III results in this case.

Part (b): Suppose that $ct/4<P\leq 7ct/12$, which is equivalent to $1/4<p\leq 7/12$. Because of $\Delta(2-S)\leq 1/4$ for all $x_2\in(1/2,1]$, it follows that $\Delta(2-S)<p$, which implies that the equilibrium can be either of type II or of type III. Moreover, since $\Delta(S+2)/3\in(0,7/12]$ is increasing in x_2, there exists a $\xi_{III,II}\in(1/2,1]$ such that

$$\frac{1}{3}\left(\xi_{III,II}-\frac{1}{2}\right)\left(\xi_{III,II}+\frac{5}{2}\right)=p. \tag{15}$$

For $x_2\geq\xi_{III,II}$ we have $\Delta(S+2)/3\geq p$, which implies that the equilibrium is of type II. In contrast, for $x_2<\xi_{III,II}$ we have $\Delta(S+2)/3<p$ and the equilibrium is of type I. Solving (15) for $\xi_{III,II}\in(1/2,1]$ and inserting $p=P/(ct)$, we obtain

$$\xi_{III,II}=-1+\frac{\sqrt{3}}{2t}\sqrt{\frac{t}{c}(4P+3ct)},$$

which assumes values in $\left(-1+\sqrt{3},1\right]$ for $P\in(ct/4,7ct/12]$.

Part (c): Finally, consider $P\leq ct/4$, i.e., $p\leq 1/4$. In this case, the equilibrium is of type I if $p\leq\Delta(2-S)$. Notice that $\Delta(2-S)\in(0,1/4]$ is increasing in x_2. Consequently, there exists a $\xi_{II,I}\in(1/2,1]$ such that

$$\left(\xi_{II,I}-\frac{1}{2}\right)\left(\frac{3}{2}-\xi_{II,I}\right)=p. \tag{16}$$

For $x_2\geq\xi_{II,I}$, we have $\Delta(2-S)\geq p$. Hence the equilibrium is of type I. For $x_2<\xi_{II,I}$ the equilibrium is either of type II or of type III. By part (b), it is of type II for $x_2\in\left[\xi_{III,II},\xi_{II,I}\right)$ and of type III for $x_2\in\left(1/2,\xi_{III,II}\right)$. Solving (16) for $\xi_{II,I}\in(1/2,1]$ and inserting $p=P/(ct)$, we obtain

$$\xi_{II,I}=1-\frac{1}{2ct}\sqrt{c^2t^2-4ctP},$$

which assumes values in $(1/2, 1)$ for $P \in (0, ct/4]$. Moreover, observe that $\xi_{II,I} > \xi_{III,II}$ in this range of P.

■

Proof of Proposition 3. Let $P > 0$ and $x_2 \in (1/2, 1]$ be given arbitrarily and let $q^* = (q_1^*(x_2), q_2^*(x_2))$ denote the corresponding quality equilibrium that results at stage 3. Correspondingly, let $\pi_2(x_2) \equiv \pi_2(q_1^*(x_2), q_2^*(x_2))$ denote the reduced profit function of the entrant at stage 2. The entrant chooses x_2 in order to maximize profit, $\pi_2(x_2) = (P - cq_2^*(x_2)) d_2^*(x_2)$, where equilibrium demand $d_2^*(x_2)$ is given by

$$d_2^*(x_2) = 1 - \frac{1/2 + x_2}{2} - \frac{q_1^*(x_2) - q_2^*(x_2)}{2t(x_2 - 1/2)}. \tag{17}$$

Hence, marginal profit $\pi_2'(x_2)$ is given by[20]

$$\begin{aligned} \pi_2'(x_2) &= \frac{\partial}{\partial x_2}((P - cq_2^*) d_2^*) \\ &= -c\frac{\partial q_2^*}{\partial x_2} d_2^* + (P - cq_2^*) \frac{\partial d_2^*}{\partial x_2}. \end{aligned}$$

In the following, we derive the sign of the entrant's marginal profit, $\pi_2'(x_2)$, for each of the different equilibrium regions separately. For any $\tau \in \mathcal{T} \equiv \{I, II, III\}$, let X_τ and Q_τ denote the set of locations x_2 and the set of quality equilibria $q^*(x_2) = (q_1^*(x_2), q_2^*(x_2))$ corresponding to equilibrium type $\tau \in \mathcal{T}$, respectively.

Type I: Consider a location in equilibrium region I, i.e., $x_2 \in X_I$. Then, we have $q_1^*(x_2) = q_2^*(x_2) = 0$ in the corresponding quality equilibrium, which implies $\partial q_2^*(x_2)/\partial x_2 = 0$, $d_2^*(x_2) = 3/4 - x_2/2$, and hence

$$\pi_2'(x_2) = P\frac{\partial d_2^*}{\partial x_2} < 0.$$

Type II: Consider $x_2 \in X_{II}$. By definition, we have $q_1^*(x_2) = 0$ and $q_2^*(x_2) = q_2^{BR}(0) = P/(2c) - t\Delta(2 - S)/2 > 0$. Inserting $q_1^*(x_2)$ and $q_2^*(x_2)$ in (17), we get

$$\begin{aligned} d_2^*(x_2) &= 1 - \frac{S}{2} + \frac{q_2^*(x_2)}{2t\Delta} \\ &= \frac{P + ct\Delta(2 - S)}{4ct\Delta}. \end{aligned}$$

Consequently, profit $\pi_2(x_2)$ reduces to

$$\begin{aligned} \pi_2(x_2) &= \left(P - c\left(\frac{P}{2c} - \frac{t\Delta(2 - S)}{2}\right)\right) \frac{P + ct\Delta(2 - S)}{4ct\Delta} \\ &= \frac{(P + ct\Delta(2 - S))^2}{8ct\Delta}. \end{aligned}$$

[20] For boundary locations x_2, $\pi_2'(x_2)$ denotes the relevant one-sided derivative: e.g. for $x_2 = 1$, we have $\pi_2'(x_2) \equiv \pi_2'(x_2^-) = \lim_{h \to 0^-} (\pi_2(x_2 + h) - \pi_2(x_2))/h$.

Calculating $\pi_2'(x_2)$, we obtain

$$\begin{aligned}\frac{\partial\pi_2}{\partial x_2} &= \frac{\partial\pi_2}{\partial S}\underbrace{\frac{\partial S}{\partial x_2}}_{=1}+\frac{\partial\pi_2}{\partial\Delta}\underbrace{\frac{\partial\Delta}{\partial x_2}}_{=1}\\ &= \frac{\partial\pi_2}{\partial S}+\frac{\partial\pi_2}{\partial\Delta}<0,\end{aligned}$$

where the strict inequality follows from

$$\begin{aligned}\frac{\partial\pi_2}{\partial\Delta} &= -\frac{P^2-t^2\Delta^2c^2(2-S)^2}{8t\Delta^2c}<0\\ \text{and}\quad \frac{\partial\pi_2}{\partial S} &= -\frac{1}{4}(P+(2-S)t\Delta c)<0.\end{aligned}$$

The first expression is strictly negative, since we have $P>ct\Delta(2-S)$ in equilibrium region II. The second expression is strictly negative because of $P>0$, $S\le 3/2$ and $\Delta>0$. Thus, $\pi_2'(x_2)<0$.

Type III: Consider $x_2\in X_{III}$. In equilibrium region III, the quality equilibrium is given by the solution to (5),

$$\begin{aligned}q_1^*(x_2) &= \frac{P}{c}-\frac{t\Delta(S+2)}{3}\\ q_2^*(x_2) &= \frac{P}{c}-\frac{t\Delta(4-S)}{3}.\end{aligned}$$

This solution yields a quality gap of

$$q_1^*(x_2)-q_2^*(x_2)=-\frac{2t\Delta(S-1)}{3}.$$

Inserting the quality gap in (17), we get

$$d_2^*(x_2)=1-\frac{S}{2}-\frac{q_1^*(x_2)-q_2^*(x_2)}{2t\Delta}=\frac{2}{3}-\frac{S}{6}$$

and hence

$$\begin{aligned}\pi_2(x_2) &= (P-cq_2^*(x_2))\,d_2^*(x_2)\\ &= \left(P-c\left(\frac{P}{c}-\frac{t\Delta(4-S)}{3}\right)\right)\left(\frac{2}{3}-\frac{S}{6}\right)\\ &= \frac{ct\Delta}{18}(4-S)^2.\end{aligned}$$

To establish $\pi_2'(x_2)>0$, we derive

$$\begin{aligned}\frac{\partial\pi_2}{\partial x_2} &= \frac{\partial\pi_2}{\partial S}\frac{\partial S}{\partial x_2}+\frac{\partial\pi_2}{\partial\Delta}\frac{\partial\Delta}{\partial x_2}=\frac{\partial\pi_2}{\partial S}+\frac{\partial\pi_2}{\partial\Delta}\\ &= -\frac{ct\Delta}{9}(4-S)+\frac{ct}{18}(4-S)^2\\ &= \frac{ct}{18}(4-S)(4-2\Delta-S), \qquad (18)\end{aligned}$$

which is strictly positive for any location $x_2 \in (1/2, 1]$ and the corresponding quality equilibrium of type III. ■

Proof of Proposition 4. Let $P > 0$ and suppose $\xi_{III,II}$ is given by (7). Furthermore, let $x_2 \in [1/2, 1]$ be given arbitrarily and let $q^* = (q_1^*(x_2), q_2^*(x_2))$ denote the corresponding quality equilibrium that results at stage 3. Correspondingly, let $\pi_2(x_2) = \pi_2(q_1^*(x_2), q_2^*(x_2))$ denote the reduced profit function of the entrant at stage 2. The entrant chooses x_2 in order to maximize profit, $\pi_2(x_2) = (P - cq_2^*(x_2))\, d_2^*(x_2)$, where equilibrium demand $d_2^*(x_2)$ is given by (17). Let $x_2^* \in [1/2, 1]$ denote this profit maximizing location.

As a preliminary, observe that $\pi_2(1/2) = 0$ and $\pi_2(x_2) > 0$ for all $x_2 \in (1/2, 1]$. Consequently, the entrant never finds it optimal to locate at the center, i.e., $x_2^* > 1/2$.

Part (a): Suppose the price is low or intermediate, i.e., $P \leq 7ct/12$. In this case, we have $\xi_{III,II} \in (1/2, 1]$. Consider $x_2 < (1/2, \xi_{III,II})$ first. By Proposition 2, the quality equilibrium is of type III. Hence, by Proposition 3, we have $\pi_2'(x_2) > 0$. Now, consider $x_2 \in (\xi_{III,II}, 1]$. By Proposition 2, the quality equilibrium is either of type I or of type II. Hence, by Proposition 3, we have $\pi_2'(x_2) < 0$. By continuity of the reduced profit function $\pi_2(x_2)$ at $x_2 = \xi_{III,II}$ we thus obtain $\pi_2(\xi_{III,II}) \geq \pi_2(x_2)$ for all $x_2 \in (1/2, 1]$.

Part (b): Suppose the price is high, i.e., $P > 7ct/12$. Then Proposition 2(a) implies that the quality equilibrium is of type III for all $x_2 \in (1/2, 1]$. By Proposition 3, it hence follows that $\pi_2'(x_2) > 0$ for all $x_2 \in (1/2, 1]$. Thus, $x_2^* = 1$ maximizes the entrant's profit in this case. ■

Proof of Proposition 5. Let $f > 0$ be given arbitrarily. Further below, we derive explicit expressions of $\pi_2^*(P)$, first for the case of low and intermediate prices, then for the case of high prices (see equations (22) and (26), respectively). From these expressions, it can be easily seen that $\pi_2^*(P)$ is continuous in P.

Part (a): Let P be low or intermediate, that is, $p := P/(ct) \leq 7/12$. It follows from Proposition 4 that the entrant locates at $x_2^* = \xi_{III,II}$ given by (7), which yields a quality equilibrium of type II. We hence have $q_1^* = 0$ and, from (13), we obtain

$$\begin{aligned} q_2^* &= \frac{t}{2}\left(p - \Delta(2 - S)\right) \\ &= \frac{t}{2}\left(\frac{P}{ct} - \left(\xi_{III,II} - \frac{1}{2}\right)\left(\frac{3}{2} - \xi_{III,II}\right)\right) \\ &= t\left(3 - \sqrt{9 + 12\frac{P}{ct}} + 2\frac{P}{ct}\right). \end{aligned} \tag{19}$$

Inserting x_2^*, q_1^* and q_2^* into (17), we get

$$d_2^* = \frac{3}{4} - \frac{1}{12}\sqrt{\left(12\frac{P}{ct}+9\right)} \quad \text{and} \tag{20}$$

$$d_1^* = \frac{1}{4} + \frac{1}{12}\sqrt{\left(12\frac{P}{ct}+9\right)}. \tag{21}$$

From (19) and (20), it hence follows that

$$\begin{aligned}
\pi_2^*(P) &= (P - cq_2^*)\, d_2^* \\
&= \left(P - c\left(3t - \sqrt{3}\sqrt{3t^2 + 4\frac{P}{c}t} + 2\frac{P}{c}\right)\right)\left(\frac{3}{4} - \frac{1}{12}\frac{\sqrt{3}}{t}\sqrt{3t^2 + 4\frac{P}{c}t}\right) \\
&= ct\left(\sqrt{9 + 12\frac{P}{ct}} - 3 - \frac{7}{4}\frac{P}{ct} + \frac{1}{12}\frac{P}{ct}\sqrt{9 + 12\frac{P}{ct}}\right).
\end{aligned} \tag{22}$$

Strict monotonicity of $\pi_2^*(P)$: Consider the derivative of (22),

$$\pi_2^{*\prime}(P) = ct\frac{\sqrt{4\frac{P}{ct}+3}}{4\left(4P+3ct\right)}\left(2\sqrt{3}\frac{P}{ct} - 7\sqrt{4\frac{P}{ct}+3} + 9\sqrt{3}\right).$$

Because of $c > 0$ and $t > 0$, the first factors are strictly positive. We show that the term in parantheses is strictly positive as well. To this end, consider the following equivalent transformations:

$$\begin{aligned}
2\sqrt{3}\frac{P}{ct} - 7\sqrt{4\frac{P}{ct}+3} + 9\sqrt{3} &> 0 \\
\sqrt{3}\frac{\rho^2 - 3}{2} - 7\rho + 9\sqrt{3} &> 0 \\
\rho^2 - \frac{14}{\sqrt{3}}\rho + 15 &> 0,
\end{aligned} \tag{23}$$

where, from the second to the third line, we have deployed the transformation $\rho \equiv (4P/(ct)+3)^{1/2}$ (which is equivalent to $P/(ct) = (\rho^2 - 3)/4)$ for $P \in (0, 7ct/12]$ or $\rho \in (\sqrt{3}, 4/\sqrt{3}])$. We show that the last inequality indeed holds true. Set $f(\rho) = \rho^2 - 14\rho/\sqrt{3} + 15$. It turns out that $f(\cdot)$ is strictly decreasing for $\rho \in (\sqrt{3}, 4/\sqrt{3}]$, because we have

$$f'(\rho) = 2\rho - \frac{14}{3}\sqrt{3} \le 2\cdot\frac{4}{\sqrt{3}} - \frac{14}{3}\sqrt{3} = -2\sqrt{3} < 0$$

for $\rho \le 4/\sqrt{3}$. Then the claim follows from $f(\rho) \ge f(4/\sqrt{3}) = 5/3 > 0$.

Limit behavior: First, taking the limit $P \to 0^+$ of $\pi_2^*(P)$, we obtain

$$\lim_{P\to 0^+}\left(\sqrt{3}c\sqrt{3t^2 + 4\frac{P}{c}t} - 3ct - \frac{7}{4}P + \frac{1}{12}\sqrt{3}\frac{P}{t}\sqrt{3t^2 + 4\frac{P}{c}t}\right) = 0.$$

Second, inserting $P = 7ct/12$ in $\pi_2^*(P)$, we get

$$\left[\sqrt{3}c\sqrt{3t^2 + 4\frac{P}{c}t} - 3ct - \frac{7}{4}P + \frac{1}{12}\sqrt{3}\frac{P}{t}\sqrt{3t^2 + 4\frac{P}{c}t}\right]_{P=7ct/12} = \frac{25}{144}ct$$

Part (b): Now, suppose the price is high, i.e. $p = P/(ct) > 7/12$. Proposition 4 implies that $x_2^* = 1$, which entails $\Delta = 1/2$ and $S = 3/2$. By Proposition 2(a), the quality equilibrium is of type III and, from (14), we hence obtain

$$\begin{aligned} q_1^{\text{FOC}} &= \frac{P}{c} - \frac{t\Delta(S+2)}{3} = \frac{P}{c} - \frac{7}{12}t \qquad \text{and} \\ q_2^{\text{FOC}} &= \frac{P}{c} - \frac{t\Delta(4-S)}{3} = \frac{P}{c} - \frac{5}{12}t. \end{aligned} \tag{24}$$

Using these expressions, demand reduces to

$$\begin{aligned} d_1 &= \frac{S}{2} + \frac{q_1^{\text{FOC}} - q_2^{\text{FOC}}}{2t\Delta} = \frac{7}{12} \quad \text{and} \\ d_2 &= 1 - d_1 = \frac{5}{12}, \end{aligned} \tag{25}$$

respectively. Observe that demand does not depend on the price P, while quality levels do so. Consequently, the entrant's profit reduces to

$$\pi_2^* = (P - cq_2^*)\, d_2^* = \frac{5ct}{12}\frac{5}{12} = \frac{25}{144}ct \tag{26}$$

■

Proof of Proposition 6. Let $B > 0$ be arbitrary and let $W(P)$ denote welfare as a function of the price $P > 0$. To prepare our analysis of parts (a) and (b), we first investigate the properties of the welfare function when the price is small or intermediate. Subsequently, we prove parts (a) to (c).

Let $p := P/(ct) \leq 7/12$ denote the transformed price. In this case the entrant's location $x_2^* = \xi_{III,II}$ and the quality level q_2^* are given by (7) and (19), respectively, yielding demands (21) and (20). With slight abuse of notation, we rewrite welfare (10) as a function of the transformed price p and decompose it into (variable) *patient benefit* $\beta(p)$ and *transportation cost* $\tau(p)$ as follows:

$$\begin{aligned} W(p) &= (1-c)\, q_2^* d_2^* - \left(\int_0^{d_1^*} t\,(1/2-\theta)^2\, d\theta + \int_{d_1^*}^1 t\,(x_2^*-\theta)^2\, d\theta\right) + s \\ &= (1-c)\,\beta(p) - \tau(p) + s, \end{aligned} \tag{27}$$

where we set

$$\beta(p) = q_2^* d_2^* \quad \text{and} \tag{28}$$

$$\tau(p) = \left(\int_0^{d_1^*} t\,(1/2-\theta)^2\, d\theta + \int_{d_1^*}^1 t\,(x_2^*-\theta)^2\, d\theta\right). \tag{29}$$

To explore the properties of welfare, we first examine those of patient benefit and transportation cost separately. Inserting (19) and (20), patient benefit (28) reduces to

$$\beta(p) = t\left(3 + \frac{5}{2}p - \sqrt{12p+9} - \frac{1}{6}p\sqrt{12p+9}\right). \tag{30}$$

We show that patient benefit $\beta(p)$ is strictly increasing and strictly convex in p for $p \leq 7/12$. To this end, consider the first and second derivative,

$$\begin{aligned} \beta'(p) &= -\frac{1}{6}t\frac{\sqrt{3}\,(4p+3) + 2\sqrt{3}p - 15\sqrt{4p+3} + 12\sqrt{3}}{\sqrt{4p+3}} \quad \text{and} \qquad (31)\\ \beta''(p) &= t\left(\frac{4\sqrt{3}}{(4p+3)^{\frac{3}{2}}} - \frac{2\sqrt{3}}{3\sqrt{4p+3}} + \frac{2\sqrt{3}p}{3\,(4p+3)^{\frac{3}{2}}}\right) = t\frac{2\sqrt{3}\,(1-p)}{\left(\sqrt{4p+3}\right)^3}. \end{aligned}$$

For a low price, $p \leq 7/12$, we have $\beta''(p) > 0$, that is, patient benefit is strictly convex. Hence, $\beta'(p)$ is strictly increasing for $p \leq 7/12$. To show $\beta'(p) > 0$ for all $p \in (0, 7/12]$ it is therefore sufficent to see that $\lim_{p\to 0^+}\beta'(p) = 0$. Thus, patient benefit is strictly increasing and strictly convex for $p \in (0, 7/12]$.

We continue with establishing that *transportation cost* is strictly decreasing for $p \leq 1/4$ and strictly convex for $p \in (0, 7/12]$. To this end, we first simplify (29):

$$\begin{aligned} \tau(p) &= \left(\int_0^{d_1} t\,(1/2-\theta)^2\,d\theta + \int_{d_1}^{1} t\,(x_2-\theta)^2\,d\theta\right)\\ &= \frac{t}{12}\left(12d_1^2x_2 - 6d_1^2 - 12d_1x_2^2 + 3d_1 + 12x_2^2 - 12x_2 + 4\right)\\ &= \frac{t}{12}\left(\frac{81}{2}p - 18\sqrt{12p+9} - \frac{5}{2}p\sqrt{12p+9} + 55\right), \end{aligned} \tag{32}$$

where, in the last row, we have inserted (21) and $x_2 = \xi_{III,II}$ given by (7). As to the first and second derivative of $\tau(p)$, we obtain

$$\begin{aligned} \tau'(p) &= -\frac{t}{12}\left(\frac{36\sqrt{3}}{\sqrt{4p+3}} + \frac{5\sqrt{3}}{2}\sqrt{4p+3} + \frac{5\sqrt{3}p}{\sqrt{4p+3}} - \frac{81}{2}\right), \qquad (33)\\ \tau''(p) &= \frac{t}{2}\frac{\sqrt{3}\,(7-5p)}{(4p+3)^{\frac{3}{2}}}. \end{aligned}$$

Clearly, we have $\tau''(p) > 0$ for $p \in (0, 7/12]$. Hence, $\tau(p)$ is strictly convex in this range of p. To show $\tau'(p) < 0$ for small prices $p \in (0, 1/4]$, it is therefore sufficient to see that $\tau'(1/4) = 27t/8 - 63\sqrt{3}t/32 < 0$ for $t > 0$.

It follows that, for $p \in (1/4, 7/12]$, transportation cost $\tau(p)$ assumes its maximum at the upper boundary, i.e., for $p = 7/12$. By strict convexity of $\tau(\cdot)$, it is sufficient to compare $\tau(1/4)$ and $\tau(7/12)$. Calculating these values, we get

$$\lim_{p\to\frac{1}{4}^+}\tau(p) = \left(\frac{521}{96} - \frac{149}{48}\sqrt{3}\right)t \quad \text{and} \quad \tau\left(\frac{7}{12}\right) = \frac{19}{288}t.$$

Since the second value is larger, it follows that

$$\tau(p) \leq \frac{19}{288}t, \tag{34}$$

where the inequality holds strictly for all $1/4 < p < 7/12$.

Finally, we insert (30) and (32) into (27), to simplify welfare net of the gross benefit from treatment s:

$$\begin{aligned}
& W(p) - s \\
= \;& (1-c)\,\beta(p) - \tau(p) \qquad (35)\\
= \;& (1-c)\,t\left(3 + \frac{5}{2}p - \sqrt{12p+9} - \frac{1}{6}p\sqrt{12p+9}\right) \\
& -\frac{t}{12}\left(\frac{81}{2}p - 18\sqrt{12p+9} - \frac{5}{2}p\sqrt{12p+9} + 55\right) \\
= \;& \frac{t}{24}\Big(\left(p\,(1+4c) + 12\,(1+2c)\right)\sqrt{12p+9} - (21+60c)\,p - 72c - 38\Big).
\end{aligned}$$

Calculating the first and second derivative of welfare with respect to the transformed price p, we obtain

$$\begin{aligned}
W'(p) &= t\frac{\sqrt{3}\,(20c + 9 + (2+8c)\,p) - (7+20c)\sqrt{4p+3}}{8\sqrt{4p+3}} \qquad (36)\\
W''(p) &= t\frac{\sqrt{3}}{2}\frac{((p-1)\,(1+4c) - 2)}{(4p+3)^{\frac{3}{2}}},
\end{aligned}$$

the latter of which is strictly negative for all $p \leq 7/12$. Thus, welfare is strictly concave for small and intermediate prices.

Part (a): Suppose B is small, i.e., $B \leq ct/4$. By the above, it follows that $\tau'(p) < 0$ and $\beta'(p) > 0$, since $P \leq B$ implies $P \leq ct/4$. Thus, $W'(p) > 0$ for all $p \leq 1/4$, that is, $P^* = B$ maximizes welfare.

Part (b): Suppose B is intermediate, i.e., $ct/4 < B \leq 7ct/12$, and consider $c \in (0, 5/12]$ first. Then welfare is strictly increasing for all $p < 7/12$, since $W(p)$ is strictly concave and because of $W'(p) \geq W'(7/12) = \sqrt{3}t\left(5\sqrt{3}/6 - 2\sqrt{3}c\right)/32$, which is nonnegative for $c \leq 5/12$. Therefore, $P^* = B$ maximizes welfare.

Second, consider $c \in (5/12, 1)$. In this case, we have $\lim_{p\to 0^+} W'(p) = t/4 > 0$ and $W'(7/12) < 0$. Since $W'(p)$ is continuous, by the intermediate value theorem, there exists $p_1 \in (0, 7/12)$ such that $W'(p_1) = 0$. We can set $\hat{P}_1 \equiv p_1 ct$. By strict concavity of $W(p)$, the solution p_1 and hence $\widehat{P}_1$ are unique and the claim follows.

Part (c): Suppose B and $P \leq B$ are large, i.e., $B, P > 7ct/12$. In this case, it follows from Proposition 4 that $x_2^* = 1$, $\Delta = 1/2$ and $S = 3/2$. Moreover, equilibrium levels of quality and demand are given by (24) and (25), respectively.

Inserting these into welfare (10), we obtain

$$\begin{aligned} W(P) &= \sum_{i=1,2} (1-c) q_i d_i - \int_0^{d_1} t\left(\frac{1}{2}-x\right)^2 dx - \int_{d_1}^1 t(x_2-x)^2 dx + s \\ &= (1-c)\left(\left(\frac{P}{c}-\frac{7}{12}t\right)\frac{7}{12}+\left(\frac{P}{c}-\frac{5}{12}t\right)\frac{5}{12}\right) \\ &\quad -t\left(\int_0^{\frac{7}{12}}\left(\frac{1}{2}-x\right)^2 dx + \int_{\frac{7}{12}}^1 (1-x)^2 dx\right)+s \\ &= (1-c)\,t\left(\frac{P}{ct}-\frac{37}{72}\right)-\frac{19}{288}t+s, \end{aligned} \tag{37}$$

where the last equality follows from

$$\left(\int_0^{\frac{7}{12}}\left(\frac{1}{2}-x\right)^2 dx + \int_{\frac{7}{12}}^1 (1-x)^2 dx\right) = \frac{19}{288}.$$

Observe that welfare (37) is an (affine-)linear and increasing function of both the price P and the transformed price $p = P/(ct)$. Therefore, there exists a unique $\widehat{P}_2 > 7ct/12$ such that $W(\widehat{P}_2) = W(\widehat{P}_1)$. By the above, welfare is strictly lower for all $P \in (\widehat{P}_1, \widehat{P}_2)$, i.e., $W(P) < W(\widehat{P}_1)$. For budgets $B \in (7ct/12, \widehat{P}_2)$, the optimal price is $P^* = \widehat{P}_1$. For budgets $B > \widehat{P}_2$, the optimal price is $P^* = B$. ■

Proof of Proposition 7. Let $c \in (5/12, 1)$ be arbitrary. In parts (i) and (ii) we determine $\widehat{P}_1$ and $\widehat{P}_2$ as functions of the parameters (c, t), respectively. Subsequently, we show in part (iii) that the range, $\widehat{P}_2(c) - \widehat{P}_1(c)$, is strictly increasing in $c \in (5/12, 1)$. Finally, in parts (iv) and (v), we complete the proof examining the range in the limit of $c \to 5/12$ and $c \to 1$, respectively.

Part (i): To determine the *lower boundary* $\widehat{P}_1$, we maximize (35) within the range of intermediate transformed prices, $\widetilde{p} \in (1/4, 7/12]$. Accordingly, we set marginal welfare (36) to zero and solve the resulting equation for $\widetilde{p}$ to obtain

$$\widetilde{p} = \frac{80c^2+56c+11-(20c+7)\sqrt{16c^2+4c+1}}{3(4c+1)^2} =: \widehat{p}_1(c).$$

Observe that $\lim_{c\to(5/12)^+} \widehat{p}_1(c) = 7/12$. Consequently, the lower boundary $\widehat{P}_1$ is given by $\widehat{P}_1(c) = ct\widehat{p}_1(c)$. As we will show next, $\widehat{p}_1(c)$ is strictly decreasing in c. It hence follows that $\widehat{p}_1(c) < 7/12$ for $c > 5/12$.

To show that $\widehat{p}_1(c)$ is strictly decreasing in c, we utilize the implicit characterization of $\widehat{p}_1(c)$ by the first order condition

$$(1-c)\,\beta'(\widehat{p}_1(c)) = \tau'(\widehat{p}_1(c)), \tag{38}$$

where $\beta'(\widetilde{P})$ and $\tau'(\widetilde{P})$ are given by (31) and (33), respectively. Taking the derivative of (38) with respect to c, we get

$$-\beta'(\widehat{p}_1(c)) + (1-c)\,\beta''(\widehat{p}_1(c))\frac{\partial \widehat{p}_1(c)}{\partial c} = \tau''(\widehat{p}_1(c))\frac{\partial \widehat{p}_1(c)}{\partial c}. \tag{39}$$

We solve (39) for $\partial\widehat{p}_1(c)/(\partial c)$ to obtain

$$\frac{\partial\widehat{p}_1(c)}{\partial c}=\frac{\beta'(\widehat{p}_1(c))}{(1-c)\beta''(\widehat{p}_1(c))-\tau''(\widehat{p}_1(c))},$$

which is strictly negative, since the numerator is strictly positive by strict positive monotonicity of patient benefit and since the denominator is strictly negative by strict concavity of welfare for $p\leq 7/12$.

Moreover, notice that welfare W, evaluated at $\widehat{p}_1(c)$, is strictly decreasing in c. To see this, consider maximum welfare as a function of c, i.e., $W=W(\widehat{p}_1(c),c)$. Applying the envelope theorem, we obtain that

$$\begin{aligned}\frac{\partial W(\widehat{p}_1(c),c)}{\partial c} &= \underbrace{\left.\frac{\partial W(\widetilde{p},c)}{\partial\widetilde{p}}\right|_{\widetilde{p}=\widehat{p}_1(c)}}_{=0}\frac{\partial\widehat{p}_1(c)}{\partial c}+\left.\frac{\partial W(\widetilde{p},c)}{\partial c}\right|_{\widetilde{p}=\widehat{p}_1(c)}\\ &= -\beta(\widehat{p}_1(c))<0, \qquad (40)\end{aligned}$$

where the last equality follows from differentiating (27) with respect to c.

Part (ii): We continue with determining the *upper boundary* $\widehat{P}_2>7ct/12$. For prices $P>7ct/12$, we can also rewrite (37) as a function of the transformed price $p=P/(ct)$:

$$W(p)=t\left((1-c)p+\frac{37}{72}c-\frac{167}{288}\right)+s.$$

The upper boundary $\widehat{P}_2$ and its transformed value $\widehat{p}_2=\widehat{P}_2/(ct)$ are then implicitly characterized by

$$t\left((1-c)\widehat{p}_2+\frac{37}{72}c-\frac{167}{288}\right)+s=W(\widehat{p}_1(c)),$$

where the right-hand side represents welfare, evaluated at the lower transformed boundary $\widehat{p}_1(c)$. Solving this equation for $\widehat{p}_2$, we obtain

$$\widehat{p}_2(c)=\frac{(167-148c)t+288(W(\widehat{p}_1(c))-s)}{t(1-c)288} \qquad (41)$$

for the transformed value. We show that $\widehat{p}_2'(c)>0$. To this end, we differentiate (41) with respect to c to obtain

$$\begin{aligned}\frac{\partial\widehat{p}_2(c)}{\partial c} &= \frac{\left(-148t+288\frac{\partial W(\widehat{p}_1(c),c)}{\partial c}\right)(1-c)}{288t(1-c)^2}\\ &\quad+\frac{(167-148c)t+288(W(\widehat{p}_1(c))-s)}{288t(1-c)^2} \qquad (42)\\ &= \frac{19t+288(W(\widehat{p}_1(c))-s-(1-c)\beta(\widehat{p}_1(c)))}{288t(1-c)^2}, \qquad (43)\end{aligned}$$

where, in the last row, we have inserted marginal welfare (40). The numerator of (43) is strictly positive, since it follows from (27) that

$$W\left(\widehat{p}_1\left(c\right)\right)-s-\left(1-c\right)\beta\left(\widehat{p}_1\left(c\right)\right)=-\tau\left(\widehat{p}_1\left(c\right)\right)$$

and since (34) implies

$$-\tau\left(\widehat{p}_1\left(c\right)\right)\geq-\tau\left(\frac{7}{12}\right)=-\frac{19}{288}t.$$

Moreover, the inequality holds strictly because $c>5/12$ implies $\widehat{p}_1\left(c\right)<7/12$ and since $\tau\left(\cdot\right)$ is strictly convex. Thus, $\widehat{p}_2\left(c\right)$ is strictly increasing in c. It immediately follows that the upper boundary, which is given by

$$\widehat{P}_2\left(c\right)=ct\widehat{p}_2\left(c\right)=c\frac{\left(167-148c\right)t+288\left(W(\widehat{p}_1\left(c\right))-s\right)}{288\left(1-c\right)},\tag{44}$$

is strictly increasing as well.

Part (iii): To examine the *range,* $\widehat{P}_2\left(c\right)-\widehat{P}_1\left(c\right)$, observe that

$$\widehat{P}_2\left(c\right)-\widehat{P}_1\left(c\right)=\left(\widehat{p}_2\left(c\right)-\widehat{p}_1\left(c\right)\right)ct.\tag{45}$$

It hence follows that

$$\frac{\partial}{\partial c}\left(\widehat{P}_2\left(c\right)-\widehat{P}_1\left(c\right)\right)=\underbrace{ct\frac{\partial}{\partial c}\left(\widehat{p}_2\left(c\right)-\widehat{p}_1\left(c\right)\right)}_{>0}+\underbrace{\left(\widehat{p}_2\left(c\right)-\widehat{p}_1\left(c\right)\right)t}_{\geq0}>0.\tag{46}$$

The first term of (46) is strictly positive since $\widehat{p}_1\left(c\right)$ is strictly increasing and $\widehat{p}_2\left(c\right)$ is strictly decreasing in c. The second term of (46) is non-negative because of (45) and since, by construction of $\widehat{p}_2\left(c\right)$ and $\widehat{p}_1\left(c\right)$, we have $\widehat{p}_2\left(c\right)\geq\widehat{p}_1\left(c\right)$. It thus follows that the gap $\widehat{P}_2\left(c\right)-\widehat{P}_1\left(c\right)$ is strictly increasing in c.

Part (iv): We have already seen that $\lim_{c\to(5/12)^+}\widehat{p}_1\left(c\right)=7/12$ in part (i). Therefore, showing that $\lim_{c\to(5/12)^+}\widehat{p}_2\left(c\right)=7/12$ proves our claim. Notice first that, since welfare (27) is continuous with respect to c and $\widetilde{P}$, as $c\to\left(5/12\right)^+$, welfare converges to

$$\begin{aligned}&\lim_{c\to(5/12)^+}\left(W(\widehat{p}_1\left(c\right))-s\right)\\=&\lim_{c\to(5/12)^+}\left(\left(1-c\right)\beta\left(\widehat{p}_1\left(c\right)\right)-\tau\left(\widehat{p}_1\left(c\right)\right)\right)\\=&-\frac{11}{432}t,\end{aligned}\tag{47}$$

where the last equality follows from

$$\begin{aligned}\lim_{c\to(5/12)^+}\beta\left(\widehat{p}_1\left(c\right)\right)&=\lim_{\widetilde{P}\to(7/12)^-}\beta\left(\widetilde{P}\right)=\frac{5}{72}t\qquad\text{and}\\\lim_{c\to(5/12)^+}\tau\left(\widehat{p}_1\left(c\right)\right)&=\lim_{\widetilde{P}\to(7/12)^-}\tau\left(\widetilde{P}\right)=\frac{19}{288}t.\end{aligned}$$

Setting (37) equal to (47) and solving it for $\widehat{p}_2(c) \equiv P/(ct)$, we obtain $\widehat{p}_2(c) = (444c - 479)/(864c - 864)$, which implies

$$\lim_{c \to (5/12)^+} \widehat{p}_2(c) = \frac{7}{12}.$$

Part (v): To examine the limit $c \to 1^-$, we solve

$$(1-c)\,t\left(\widehat{p}_2 - \frac{37}{72}\right) - \frac{19}{288}t + s = W(\widehat{p}_1(c))$$

for $\widehat{p}_2$ to obtain

$$\begin{aligned} \widehat{p}_2(c) &= \frac{\frac{19}{288}t + \frac{37}{72}t(1-c) + W(\widehat{p}_1(c)) - s}{t(1-c)} \\ &= \frac{37}{72} + \frac{\frac{19}{288}t + W(\widehat{p}_1(c)) - s}{t(1-c)}. \end{aligned} \tag{48}$$

Because of

$$\begin{aligned} \lim_{c \to 1^-} \widehat{p}_1(c) &= \frac{49 - 9\sqrt{21}}{25} \qquad \text{and} \\ \lim_{c \to 1^-} W(\widehat{p}_1(c)) &= \frac{14\sqrt{21} - 79}{300} + s \end{aligned}$$

the numerator of the third term in (48) converges to $\frac{19}{288}t + \left(14\sqrt{21} - 79\right)t/300 > 0$ as $c \to 1^-$. It thus follows that $\lim_{c \to 1^-} \widehat{P}_2(c) = \lim_{c \to 1^-} tc\widehat{p}_2(c) = \infty$. ∎

Proof of the Theorem. Let $c \in (0,1)$, $t > 0$ and $P > 0$ be given arbitrarily. Consider $W^d(P) - W^m - \pi_2^*(P)$, where W^m is given by (9), while $\pi_2^*(P)$ and $W^d(P)$ depend on the price range and are given by (22) and (35) for $P \in (0, 7ct/12]$ and by (26) and (37) for $P > 7ct/12$, respectively. Set $p :\equiv P/(ct) > 0$ and define the corresponding transformation

$$\begin{aligned} \gamma(p,c,t) &= \omega^d(p,c,t) - \omega^m(t) - \pi_2^d(p,c,t) \\ &= \omega^d(p,c,t) - \pi_2^d(p,c,t) + s - t/12, \end{aligned} \tag{49}$$

where we have inserted $\omega^m(t) = s - t/12$, while $\omega^d(p,c,t)$ and $\pi_2^d(p,c,t)$ depend on the price range and will be specified further below. According to Definition 3, entry always raises welfare at price P if $\gamma(p,c,t) \geq 0$ for $p = P/(ct)$. We therefore start our analysis with characterizing the solutions $p_1 \in (0, 7/12)$ and $p_2 > 7/12$ to $\gamma(p,c,t) = 0$. Subsequently, we establish parts (a) to (c) of the theorem.

Case 1: Consider $p \in (0, 7/12]$. In this case, it follows from (35) and (22) that

$$\omega^d(p,c,t) = \frac{t\left((p(1+4c) + 12(1+2c))\sqrt{12p+9} - (21+60c)p - 72c - 38\right)}{24} + s$$

$$\text{and} \qquad \pi_2^d(p,c,t) = ct\left(\sqrt{9+12p} - 3 - \frac{7}{4}p + \frac{1}{12}p\sqrt{9+12p}\right),$$

respectively. Inserting $\omega^d(p,c,t)$ and $\pi_2^d(p,c,t)$ in (49), we obtain

$$\gamma(p,c,t) = -\frac{t\left(21p - 12\sqrt{3}\sqrt{4p+3} + 18cp - \sqrt{3}p\sqrt{4p+3} - 2\sqrt{3}cp\sqrt{4p+3} + 36\right)}{24}.$$

Lemma 2 below collects useful properties of $\gamma(p,c,t)$. Subsequently, Lemma 3 characterizes the existence of solutions $p \in (0, 7/12)$ to $\gamma(p,c,t) = 0$.

Lemma 2 *(i) Boundaries:*

$$\lim_{p\to 0^+} \gamma(p,c,t) = 0 \tag{50}$$

$$\gamma\left(\frac{7}{12}, c, t\right) = (5 - 14c)\frac{5t}{288} \tag{51}$$

(ii) Partial derivative w.r.t. p:

$$\frac{\partial\gamma(p,c,t)}{\partial p} = \frac{t\left(9\sqrt{3} - 7\sqrt{4p+3} - 6c\sqrt{4p+3} + 2\sqrt{3}c + 2\sqrt{3}p + 4\sqrt{3}cp\right)}{8\sqrt{4p+3}} \tag{52}$$

$$\lim_{p\to 0^+} \frac{\partial\gamma(p,c,t)}{\partial p} = \frac{t}{4}(1 - 2c) \tag{53}$$

$$\left[\frac{\partial\gamma(p,c,t)}{\partial p}\right]_{p=7/12} = \frac{t}{64}(5 - 22c) \tag{54}$$

(iii) $\gamma(\cdot,c,t)$ *is strictly concave on* $(0, 7/12]$ *for* $c \le 1/2$.
(iv) Strict monotonicity w.r.t. c:

$$c' < c'' \quad \Longrightarrow \quad \gamma(p,c',t) > \gamma(p,c'',t)$$

Proof of Lemma 2. We confine ourselves with proving parts (iii) and (iv), since parts (i) and (ii) are immediate.

Ad (iii): Differentiating (52) with respect to p, we get

$$\begin{aligned}\frac{\partial^2}{\partial p^2}\gamma(p,c,t) &= \frac{\sqrt{3}t(2c+p+2cp-3)}{2(4p+3)^{\frac{3}{2}}} \\ &\le \left[\frac{\sqrt{3}t(2c+p+2cp-3)}{2(4p+3)^{\frac{3}{2}}}\right]_{c=1/2} = \frac{\sqrt{3}t(p-1)}{(4p+3)^{\frac{3}{2}}},\end{aligned}$$

where the inequality follows from $c \le 1/2$. Notice that the right hand side is strictly negative for $p \le 7/12$.

Ad (iv): Fix $c', c'' \in (0,1)$ such that $c' < c''$. First, it follows from (27) that $\omega^d(p,c',t) > \omega^d(p,c'',t)$. Monotonicity is strict because of $\beta(p) > 0$ by (28) and since the equilibrium is of type III by Propositions 2 and 4. Second, we obtain from (22) that $\pi_2^d(p,c',t) < \pi_2^d(p,c'',t)$. Combining the two inequalities, we get

$$\begin{aligned}\gamma(p,c',t) &= \omega^d(p,c',t) - \omega^m(t) - \pi_2^d(p,c',t)\\ &> \omega^d(p,c'',t) - \omega^m(t) - \pi_2^d(p,c'',t) = \gamma(p,c'',t),\end{aligned}$$

which proves the claim. ■

Lemma 3 *Let $c \in (0,1)$ and $t > 0$. Then we have:*
(i) A solution $p_1 \in (0, 7/12)$ to $\gamma(p_1,c,t) = 0$ exists if and only if

$$\gamma\left(\frac{7}{12}, c, t\right) < 0 \tag{55}$$

and

$$\lim_{p\to 0^+} \frac{\partial}{\partial p}\gamma(p,c,t) > 0 \tag{56}$$

(ii) The solution $p_1 \in (0, 7/12)$ to $\gamma(p_1,c,t) = 0$ is unique whenever it exists.

Proof of Lemma 3. Fix $t > 0$. *Part (i):* If conditions (55) and (56) are satiesfied, existence follows from the intermediate value theorem, since $\gamma(p,c,t)$ is continuous in p by Proposition 5 and because of (50).

To show necessity of the two conditions, suppose one of the two conditions is not satisfied. First, if $\gamma(7/12,c,t) \geq 0$, then (51) implies $c \leq 5/14$. We show that $\gamma(p,c,t) > 0$ for all $p \in (0,7/12)$, which contradicts the existence of a root $p_1 \in (0, 7/12)$. Observe that $c < 5/14$ implies $\gamma(p,c,t) > \gamma(p,5/14,t)$ by negative monotonicity of $\gamma(p,c,t)$ in c. It is therefore sufficient to establish that $\gamma(p,c,t) > 0$ for $c = 5/14$ and for all $p \in (0,7/12)$.

Define

$$\widetilde{\gamma}(p) := \frac{\gamma\left(p, \frac{5}{14}, t\right)}{t} = \frac{1}{2}\sqrt{3}\sqrt{4p+3} - \frac{8}{7}p + \frac{1}{14}\sqrt{3}p\sqrt{4p+3} - \frac{3}{2}.$$

We show that $\widetilde{\gamma}(p)$ is strictly concave in p for $p < 7/12$. To see this, we calculate the first and second derivative,

$$\begin{aligned}\widetilde{\gamma}'(p) &= \frac{1}{14\sqrt{4p+3}}\left(6\sqrt{3}p - 16\sqrt{4p+3} + 17\sqrt{3}\right) \qquad \text{and}\\ \widetilde{\gamma}''(p) &= \frac{2}{7}\sqrt{3}\frac{3p-4}{(4p+3)^{\frac{3}{2}}},\end{aligned}$$

respectively. Clearly, we have $\widetilde{\gamma}''(p) < 0$ for all $p < 7/12$. Moreover, notice that $\widetilde{\gamma}(0) = \widetilde{\gamma}(7/12) = 0$. Strict concavity of $\widetilde{\gamma}(\cdot)$ hence implies $\gamma(p,5/14,t) > 0$ for all $p \in (0,7/12)$.

Second, if $\lim_{p\to 0^+}(\partial\gamma(p,c,t)/\partial p) \leq 0$, then $c \leq 1/2$ by (53) and the strict concavity of $\gamma(p,c,t)$ in $p \in (0,7/12]$ implies $\partial\gamma(p,c,t)/\partial p < 0$ for all $p \in$

$(0, 7/12]$. It hence follows from (50) that $\gamma(p, c, t) < 0$ for all $p \in (0, 7/12]$, which contradicts the existence of a solution $p_1 \in (0, 7/12)$ to $\gamma(p_1, c, t) = 0$.

Part (ii): Suppose two solutions $p', p'' \in (0, 7/12)$ exist, that is, $\gamma(p', c, t) = \gamma(p'', c, t) = 0$. W.l.o.g., let $p' < p''$. Hence, by Lemma 3, the conditions (51) and (53) both hold true, which, by Lemma 2, implies that $c \in (5/14, 1/2)$. By Lemma 2(iii), $\gamma(p, c, t)$ is strictly concave in $p \in (0, 7/12]$. Consider the extension of $\overline{\gamma}(\cdot, c, t)$ defined by

$$\overline{\gamma}(p, c, t) := \begin{cases} 0 & p = 0 \\ \gamma(p, c, t) & p > 0 \end{cases}. \tag{57}$$

By Lemma 2(i) and by continuity of $\gamma(\cdot, c, t)$ on $(0, 7/12]$, the extension $\overline{\gamma}(\cdot, c, t)$ is continuous on $[0, 7/12]$. Moreover, it inherits the strict concavity of $\gamma(\cdot, c, t)$. Set $p = 0$ and choose $\lambda \in (0, 1)$ such that $p' = \lambda p + (1 - \lambda) p''$. We have $p < p' < p''$. Then strict concavity implies

$$\overline{\gamma}(p', c, t) > \lambda \overline{\gamma}(p, c, t) + (1 - \lambda) \overline{\gamma}(p'', c, t).$$

By assumption we have $\overline{\gamma}(p', c, t) = \overline{\gamma}(p'', c, t) = 0$ and from (57), it follows that $\overline{\gamma}(p, c, t) = 0$. Consequently, both the left and the right hand side are zero, which yields a contradiction. Thus, a solution $p_1 \in (0, 7/12)$ to $\gamma(p_1, c, t) = 0$ must be unique in $(0, 7/12]$. ■

Case 2: Now consider $p > 7/12$. In this case, we deploy $\pi_2^d(p, c, t) = 25ct/144$ from (26) and

$$\omega^d(p, c, t) = (1 - c) t \left(p - \frac{37}{72}\right) - \frac{19}{288} t + s$$

from (37) and insert these expressions in (49) to obtain

$$\begin{aligned} \gamma(p, c, t) &= \omega^d(p, c, t) - \omega^m(t) - \pi_2^d(p, c, t) \\ &= (1 - c) t \left(p - \frac{37}{72}\right) - \frac{19}{288} t + s - \left(s - \frac{t}{12}\right) - \frac{25}{144} ct \\ &= \frac{t}{288} (98c - 143 + 288p - 288cp) \\ &= \frac{5t}{288} (5 - 14c) + \frac{t}{12} (12p - 7)(1 - c). \end{aligned} \tag{58}$$

Observe that $\gamma(p, c, t)$ is linear and strictly increasing in p. Moreover, we have

$$\lim_{p \to \frac{7}{12}} \gamma(p, c, t) = \frac{5t}{288} (5 - 14c),$$

which is nonnegative if and only if $c \leq 5/14$. This establishes Lemma 4 below.

Lemma 4 *A* solution $p_2 > 7/12$ *to* $\gamma(p_2, c, t) = 0$ *exists if and only if* $c > 5/14$. *Moreover, this solution is unique and it is given by*

$$p_2 = \frac{7}{12} + \frac{5}{288} \frac{14c - 5}{1 - c}.$$

We are now in the position to establish parts (a) to (c) of the Theorem.

Part (a): Fix $c \leq 5/14$. First, by Lemma 4 there does not exist any solution $p > 7/12$ to $\gamma(p, c, t) = 0$. In addition, it follows from (58) that $\gamma(p, c, t) > 0$ for all $p > 7/12$. Second, because of $c \leq 5/14$ it follows from (51) that $\gamma(7/12, c, t) \geq 0$. By Lemma 3 hence no solution $p \in (0, 7/12)$ exists to $\gamma(p, c, t) = 0$. Moreover, (50) and (53) imply that $\gamma(p, c, t) > 0$ for all $p \in (0, 7/12)$ and hence $\gamma(7/12, c, t) \geq 0$, by continuity of $\gamma(p, c, t)$ in $p \in (0, 7/12]$. We have thus shown that $\gamma(p, c, t) \geq 0$ for all $p > 0$, that is, entry always raises welfare at any price $P = pct > 0$ if $c \leq 5/14$.

Part (b): Fix $c \in (5/14, 1/2)$. First, it follows from (51) and (53) that the sufficient conditions in Lemma 3 are satisfied. Consequently, there exists a unique solution $p_1 \in (0, 7/12)$ to $\gamma(p_1, c, t) = 0$. We set $\check{P}_1 := p_1 ct$. Moreover, it follows from (53) that $\lim_{p \to 0^+} (\partial\gamma(p, c, t) / (\partial p)) > 0$, which, by continuity of $\gamma(\cdot, c, t)$, implies $\gamma(p, c, t) > 0$ for all $p \in (0, p_1)$. Furthermore, $c > 5/14$ implies $\gamma(7/12, c, t) < 0$ by (51), which, again by continuity of $\gamma(\cdot, c, t)$, entails $\gamma(p, c, t) < 0$ for all $p \in (p_1, 7/12)$. Second, by Proposition 4 there exists a unique solution $p_2 > 7/12$ to $\gamma(p_2, c, t) = 0$. We set $\check{P}_2 := p_2 ct$. It thus follows from (58) that $\gamma(p, c, t) > 0$ for all $p > p_2$. Third, $\gamma(7/12, c, t) < 0$ and the continuity of $\gamma(p, c, t)$ in p imply that $\gamma(p, c, t) < 0$ for all $p \in (p_1, p_2)$, since by Lemmas 3 and 4 the solutions $p_1 \in (0, 7/12)$ and $p_2 \in (7/12, \infty)$ are unique in their respective ranges. We have thus shown that entry always raises welfare at any price $P = pct \in \left(0, \check{P}_1\right] \cup \left[\check{P}_2, \infty\right)$, while it does not for prices $P \in \left(\check{P}_1, \check{P}_2\right)$.

Part (c): Fix $c \geq 1/2$. By (53), we have $\lim_{p \to 0^+} (\partial\gamma(p, c, t) / (\partial p)) \leq 0$ and hence, by Lemma 3, no solution $p_1 \in (0, 7/12]$ to $\gamma(p_1, c, t) = 0$ exists, i.e. $\gamma(p, c, t) \neq 0$ for all $p \in (0, 7/12]$. Moreover, it follows from (51) that $\gamma(7/12, c, t) < 0$, and hence, by continuity of $\gamma(\cdot, c, t)$, that $\gamma(p, c, t) < 0$ for all $p \in (0, 7/12]$. Moreover, it follows from Lemma 4 that there exists a unique solution $p_2 > 7/12$ to $\gamma(p_2, c, t) = 0$ and, from (58), it follows that $\gamma(p, c, t) > 0$ for all $p > p_2$, while $\gamma(p, c, t) < 0$ for $p \in (7/12, p_2)$.

Setting $\check{P}_2 := p_2 ct$, we have thus shown that entry raises welfare at any price $P = pct \in \left[\check{P}_2, \infty\right)$, while it does not for prices $P \in \left(0, \check{P}_2\right)$. ■

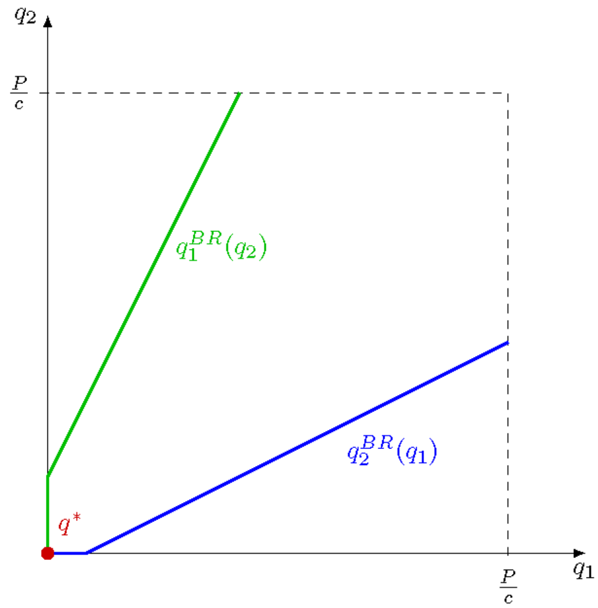

Figure 1(a): Quality equilibrium of type I

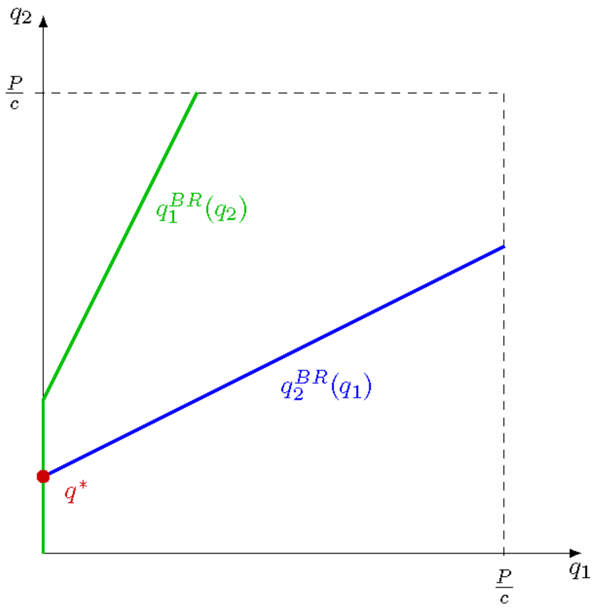

Figure 1(b): Quality equilibrium of type II

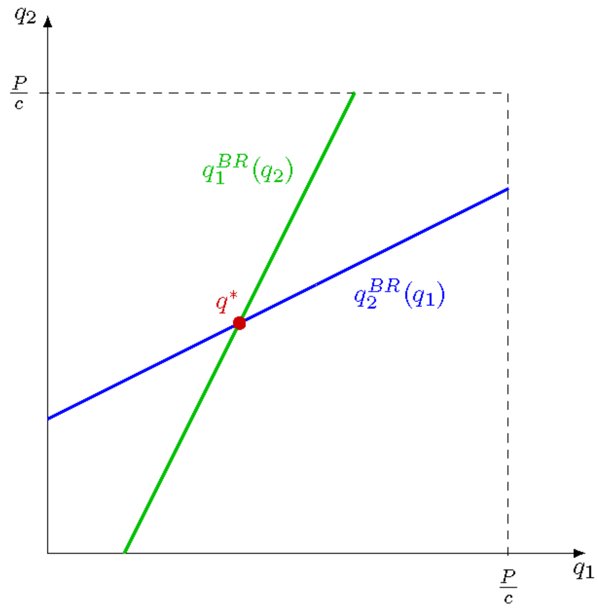

Figure 1(c): Quality equilibrium of type III

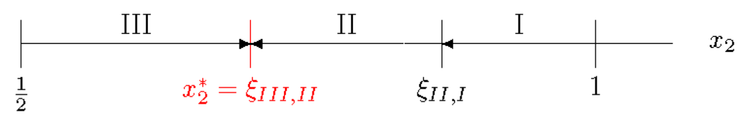

Figure 2(a): Location choice for a low price

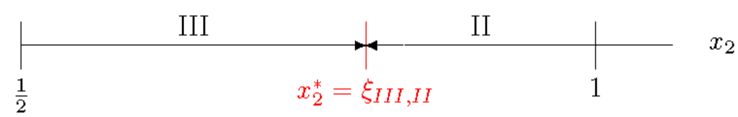

Figure 2(b): Location choice for an intermediate price

Figure 2(c): Location choice for a large price

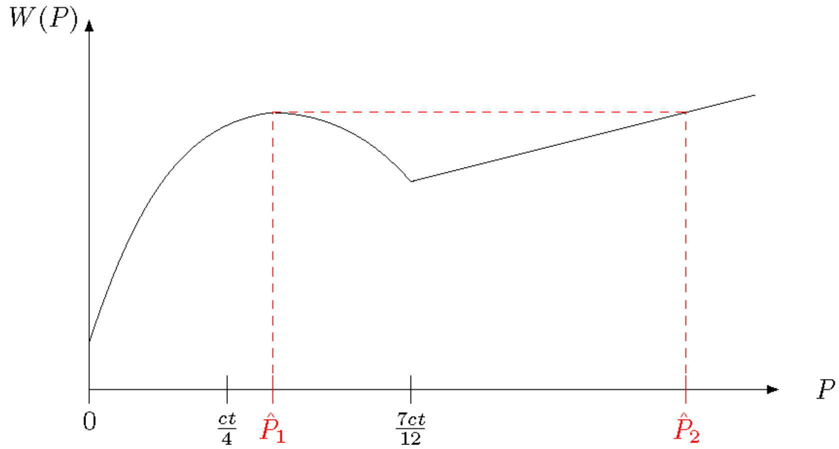

Figure 3: Welfare as a function of the price for $c > 5/12$