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Partially Directed Search for prices

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Partially Directed Search for Prices*

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Abstract

I analyse a model of partially directed search where searchers decide which firm to visit based on correct, but incomplete, information about firms' prices. Firms' pure strategies are allowed to be price distributions and in the unique symmetric pure-strategy equilibrium the price distributions are nondegenerate. The model's results rationalise empirical observations on promotions and changes in consumer prices: the lowest offered prices are unprofitable, the pdf of the price distribution is increasing, and the lowest prices are decreasing in the number of firms and the search cost.

JEL codes: D83, D43, L13
Keywords: price dispersion, partial information, partially directed search, sales, unprofitable promotions

1 Introduction

In 2014 firms worldwide spent about one trillion USD on trade promotions, mostly price discounts (Nielsen, 2015).[1] But about 60% of the promotions did not break even (McKinsey, 2019). Why do firms offer such unprofitable price promotions?

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[1] According to Nielsen (2015), trade promotions are marketing activities other than advertising that retailers and manufacturers use to induce people to buy. These activities include price discounts, bonus programs, and gifts.

One plausible answer is that firms internalise that search is costly and offer promotions to attract consumers to their stores. Because of search costs, some of the attracted consumers buy more products than only those on promotion. Others are attracted by the promotion, but will visit only when it is over. A firm that understands this, thus, takes into account that a distribution of prices that it sets across products or over time are all relevant for its profits.

Even though search costs matter for promotions in reality, search models to date fail to account for unprofitable promotions.[2] My paper fills the gap: I present a model where consumers search for a low price and firms offer unprofitable promotions in equilibrium.

The paper makes three contributions. First, I propose a parsimonious model to think about search markets where the prices that firms set across products or over time are related: one price can determine the demand of another product or at another point in time. My model makes a novel prediction, while confirming some findings in the seminal models of Stahl (1989) and Spiegler (2006).[3] The novel prediction is that the lowest equilibrium prices decrease as search becomes costlier. Second, the model's results rationalise several empirical observations on promotions and consumer prices; I explain them in detail below. Third, I generalise the method in Spiegler (2006) to determine whether a profile of distributions is a pure-strategy equilibrium. The generalised method accommodates different payoff functions and can be used to derive equilibria in distributions in other models.

In my model, homogeneous consumers sequentially search for a product (basket) at a low price. They value each product equally. A fixed number of homogeneous firms sell the product at a common marginal cost. Firms set prices and their pure strategies are *price distributions*.[4] Prior to search, each consumer sees a sample of *price signals* for free: one randomly drawn price from each firm's distribution. By assumption, a consumer *partially directs* his search: he visits the

[2] The literature review is at the end of the Introduction.

[3] The models' comparative statics are compared in Table 1, on p. 15.

[4] Setting a price distribution as a pure strategy is conceptually very different from mixing over singleton prices. For the sake of illustration, let a price distribution be the prices across different days in a month. In an equilibrium where a firm mixes over singleton prices, it must earn the same profit each day. Conversely, in a pure-strategy equilibrium where a firm's strategy is a distribution of prices, the firm optimises its prices over the month: it may be optimal to make a loss on some days and earn a profit on others.

firms in an ascending order according to their price signals in his sample.[5] When the consumer visits a firm, his actual *price offer* is a new price draw from the firm's price distribution. If the consumer does not like his price offer, he continues to search. If he likes the offer, he buys and stops searching. Getting a price offer costs the consumer a constant search cost.

Before describing the model's results, I comment on two of its assumptions. The first assumption is that firms' pure strategies are price distributions. A price distribution at a firm can be seen as a distribution across the firm's broad set of products' prices or across the firm's product's price over time. The assumption is supported by empirical evidence. Retailers offer a very large selection of products, and the prices of individual products change frequently.[6] Also, empirical evidence suggests that firms set prices, including promotions, long in advance.[7]

The second assumption is that consumers get price signals before searching. The signals can be interpreted as stemming from the consumer's own or his friend's past search, for a similar product. Another interpretation is that the consumer learns partial, potentially outdated, information about the price of the product basket that he intends to buy, e.g., that some products that he seeks were on sale. If a firm sets a nondegenerate distribution in my model's equilibrium, a consumer's price signal and offer may differ, which reflects the idea that the consumer might look for a somewhat different product basket today than in the past or that he reaches a firm after its sale ends. But a price signal and offer are equal if the firm sets a degenerate price distribution; e.g., the consumer is offered the regular price for sure if the firm's product is never on sale.

The model's results are as follows. In the unique symmetric pure-strategy equilibrium the price distributions are nondegenerate. A singleton-price equilibrium fails to exist because a firm profits from deviating to two prices if others set one. The firm can set one price a bit below and the other above the proposed equi-

[5]The visiting rule can be seen as a tie-breaking rule in symmetric equilibria. I show that it is a consumer-optimal ordinal tie-breaking rule (see Section 5.1). The paper's results remain valid if only some consumers use this visiting rule and if only firms expect them to do so (see Section 4.2). Alternatively, consumers who use the rule can be thought of as boundedly rational; an interpretation favoured by Spiegler (2006) and in line with Osborne and Rubinstein (1998).

[6]See, e.g., Shaffer (2005) or Quan and Williams (2018) on retailers' product variety and Bils and Klenow (2004) or Dhyne et al. (2006) on frequent price changes.

[7]See, e.g., Sinitsyn (2017) or Anderson and Fox (2019).

librium price, with equal probabilities. The firm attracts half of the consumers, which makes the deviation profitable as long as there are at least three firms.

In a symmetric equilibrium consumers correctly expect the average price to be the same across firms so a natural question is why consumers would partially direct search according to their price signals. I show that the signal-ascending visiting rule is a consumer-optimal ordinal tie-breaking rule because it leads to a symmetric equilibrium with the lowest expected price (see Section 5.1).

The first interesting equilibrium feature is that the lowest offered prices are below the marginal cost. In other words, they resemble unprofitable price promotions. Such prices are never offered in equilibria of models where firms set singleton prices, which contradicts the empirical fact that worldwide a large fraction of price promotions are unprofitable (Nielsen, 2015; McKinsey, 2019).

The second interesting feature is the shape of the price distribution: the density is increasing. If the price distribution is interpreted as a distribution across time, this means that price decreases are less frequent, but larger in magnitude, than price increases. This pattern of consumer price changes holds both in the US and in the euro area (Dhyne et al., 2006; Nakamura and Steinsson, 2008).

The third interesting feature is that the lowest equilibrium prices decrease in the search cost, which is a novel prediction of my model, and in the number of firms. These two predictions rationalise the empirical fact that price decreases are larger in the US than in the euro area (Dhyne et al., 2006).[8] On the one hand, search costs are likely larger in the US because both the GDP per capita is higher and supermarkets are further from households in the US than in Europe (World Bank, 2021; Cant, 2019). On the other hand, the retail sector is more competitive in the US (Dhyne et al., 2006). In sum, my model offers a new explanation based on search costs to why price changes are larger in the US than in the euro area.

I also discuss two extensions and alternative interpretations of the model. In particular, I argue that the assumption that consumers direct their search is a natural one because the visiting rule is a consumer-optimal ordinal tie-breaking rule. I also comment on the importance of an analyst's choice of firms' strategy spaces. Last, I discuss three alternative interpretations of my model. Despite its

[8]In fact, my model also rationalises the fact that price increases are larger in the US than the euro area (Dhyne et al., 2006) because the highest prices increase in the search cost.

parsimony, the model has interesting implications when interpreted as a model of a labour market, of disciplined deception, or of endogenous price stickiness.

Literature. Most of prior search literature assumes that searchers have either no information or perfect information about the price (or wage) offers that they would get at different firms; my model is in between.[9] Others have studied ordered search or partially directed search based on non-price or full price information.[10] In all these models, if a searcher uses price information to direct search, he knows what singleton price he is offered at a firm. In contrast, in my model he can get a different price offer than the one he directs search on.

In most other search models that feature price dispersion, the dispersion is across rather than within firms.[11] An exception is Salop (1977), where a monopolist posts a price distribution. In Salop (1977), in contrast to my model, all prices exceed the marginal cost. In other search models where a firm sets different prices for different units of the same product, the firm discriminates between consumers.[12] In contrast, in my model firms cannot discriminate between consumers.

In Varian (1980) and other models of sales where firms' pure strategies are singleton prices, firms never set prices below the marginal cost, in contrast to my model.[13] But models of loss leaders exist where firms charge prices below the marginal cost to attract consumers and other prices to make profits.[14] These papers do not analyse sequential search, including the effects of the search cost.

The rest of the paper is as follows. Section 2 describes the model. Section 3 derives the equilibrium in price distributions. Section 4 contains two extensions and Section 5, the discussion. Section 6 concludes.

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[9] See Stigler (1961) and Moen (1997) for the seminal papers on random and directed search.

[10] For papers on consumer search see, for example, Weitzman (1979), Arbatskaya (2007), Armstrong et al. (2009), Wilson (2010), Armstrong and Zhou (2011), Haan and Moraga González (2011), Zhou (2011), Armstrong (2017), Choi et al. (2018), Ding and Zhang (2018), García and Shelegia (2018), Haan et al. (2018), Parakhonyak and Titova (2018), Teh and Wright (2020), Anderson and Renault (2017), Choi and Smith (2020), and Obradovits and Plaickner (2020). For early papers on advertising (where an ad reveals a product's feature), see Butters (1977) or Grossman and Shapiro (1984). For search directed by cheap talk messages, see Menzio (2007).

[11] See, for example, Burdett and Judd (1983), Stahl (1989), and the many papers that follow.

[12] For discrimination based on consumer characteristics, see, e.g., Fabra and Reguant (2020) or Mauring (2021), and based on behaviour, Armstrong and Zhou (2016) or Kaplan et al. (2019).

[13] In Sobel (1984), Albrecht et al. (2013), and Dilmé and Garrett (2021) firms use sales to price discriminate over time and in Rudanko (2021) to respond to reductions in production costs.

[14] See Gerstner and Hess (1990), Spiegler (2006), Weinstein and Ambrus (2008), and Chen and Rey (2012).

2 Model

Firms. A number $n \geq 2$ of profit-maximising firms produce a product at an identical marginal cost $c \geq 0$. Firm i's pure strategy is a, potentially degenerate, *price distribution* $F_i(p)$. Technically, a pure strategy of firm i, $F_i(p)$, is a cdf from the space of all cdfs defined on $\mathbb{R}$, which I denote by $\mathcal{F}$: $F_i(p) \in \mathcal{F}$ where the function $F_i(p_1) : \mathbb{R} \mapsto [0, 1]$ is defined by $F_i(p_1) := Pr(p \leq p_1)$. The distribution F_i can be continuous, discrete, or mixed. If a firm sets a nondegenerate distribution in equilibrium, then one interpretation of the price distribution is that the firm asks different prices for different products or product baskets that it sells. Another interpretation is that the firm chooses in advance the price of its product for a time interval.[15]

Consumers. A unit mass of homogeneous consumers with a unit demand each are looking for a product at a low price. A consumer's valuation for the unit of a product is $v = 1$ so if he buys at the price offer p^o, his net utility is $1 - p^o$. Consumers can always get zero utility by not buying. A consumer's strategy specifies at which price offers to buy and at which to continue searching.

Information and price offers. Consumers are *partially informed* about the firms' prices. In particular, before starting to search a consumer gets a sample of n price signals, one signal p^s per firm. Signal i in a consumer's sample, p^s_i, is a random draw from firm i's price distribution $F_i(p)$. The samples of signals are independent across consumers. One interpretation of this information is that the consumer learns partial information about the price of the product basket that he intends to buy, e.g., by learning that some products that he intends to buy are on sale. Another interpretation is that he remembers the prices at the different firms from a time in the past when he sought a similar product or product basket. Finally, he may hear about prices at which his friends bought at the different firms. I assume that consumers *direct* their search based on the price sample: a consumer visits first the cheapest firm in his sample, then the second-cheapest firm, etc.[16] When a consumer contacts firm i, he gets a price offer p^o_i from the

[15]Empirical evidence suggests that many firms choose their prices and plan promotions long in advance; Nielsen (2015), Anderson et al. (2017), Sinitsyn (2017), Anderson and Fox (2019).

[16]I discuss this assumption on p. 7.

firm. The offer is a new random draw from the firm's price distribution $F_i(p)$. If the consumer likes the offer, he buys and leaves the market. If he does not like the offer, he continues to search for a lower price or exits. Getting a price offer costs $s > 0$ for a consumer. Recall is free: the consumer can costlessly return to firm i that he visited before and buy at the price that he was offered at i upon his initial visit. If a consumer's expected value from starting the search process is negative, he does not start searching.

Equilibrium. I look for symmetric pure-strategy equilibria ("equilibria", in what follows) where firms set the same price distribution $F(p)$ and consumers use the same policy.[17] I denote the infimum and the supremum of the support of F by p_{min} and p_{max} respectively. A firm's price distribution F is a best response to the other firms' and consumers' strategies. Consumers' optimal strategy is to accept all prices below their optimal cutoff price $\bar{p}$. I assume that a consumer buys if he is indifferent between buying and continuing to search. Consumers have correct beliefs about the firms' behaviour in equilibrium, as usual. I assume that consumers have passive beliefs out of equilibrium, in line with most consumer search literature.[18]

In a symmetric equilibrium firms post the same price distributions so, for a fixed pricing equilibrium, there is no financial reason for a consumer to visit firms in a particular order. A particular order can be justified by considering an appropriate metagame where consumers can choose a tie-breaking rule, to be used in symmetric pricing equilibria. In particular, I show that the tie-breaking rule that specifies visiting firms in an ascending order of their price signals is a consumer-optimal tie-breaking rule because it leads to a symmetric equilibrium with the lowest expected price (see Section 5.1 for details). It also suffices if only some consumers use this tie-breaking rule (see Section 4.2) or if firms expect that consumers direct search in this manner. A different interpretation of the assumption is that consumers are boundedly rational.

[17]A reason to focus on symmetric equilibria is that only about 10% of consumer price dispersion in the US is driven by price differences across stores (Kaplan and Menzio, 2015).

[18]Passive beliefs mean that after seeing an out-of-equilibrium price signal or offer from firm i, a consumer believes both that firm i has deviated in no other part of its strategy and that no other firm has deviated. This is in line with the original specification of passive beliefs as coined by McAfee and Schwartz (1994) (passive beliefs were implicitly used earlier, e.g., by Cremer and Riordan (1987), Hart and Tirole (1990), and O'Brien and Shaffer (1992)).

3 Equilibrium in price distributions

I first describe a consumer's and then a firm's problem. Then I describe the equilibrium and provide the comparative statics' results.

3.1 A consumer's and a firm's problem

First consider a consumer's problem. Suppose that a consumer has received his set of free price signals and decided to visit firm i first because his lowest price signal was from firm i. When he visits i, he gets a price offer p_i^o which is a new draw from $F_i(p)$. Should he stop and accept price p_i^o or continue to search?

In a symmetric equilibrium and because of free recall, the consumer's optimal stopping rule at firm i is independent of how many firms he has visited before as long as he still has one firm to visit (Kohn and Shavell, 1974). The optimal stopping rule is to accept any first price offer that falls below a constant cutoff price $\bar{p}$. If the consumer gets price offers that exceed $\bar{p}$ at all firms, then after visiting all firms he accepts the lowest price offer among them as long as that is below his valuation v.

By standard arguments (see, e.g., Stahl, 1989), the optimal cutoff $\bar{p}$ solves

$$\int_{p_{min}}^{\bar{p}} (\bar{p} - p) \, \mathrm{d}F(p) = s.$$

On the left-hand side (LHS) of the equation is the consumer's expected benefit from visiting a next firm when his lowest price offer so far is $\bar{p}$, i.e., the expected improvement if the next firm offers him a price below $\bar{p}$. The expectation is taken with respect to the equilibrium distribution F. All p_{min}, $\bar{p}$ and F are determined in equilibrium. On the right-hand side (RHS) is the cost of visiting another firm, i.e., the exogenous search cost s. Because of passive beliefs, the consumer's expected benefit from visiting the next firm does not depend on whether the price offer at the current firm i, p_i^o, is an equilibrium offer or not.

Now consider firm i's problem. The firm faces a tradeoff when considering which distribution of prices $F_i(p)$ to set. Low prices attract more consumers, but high prices generate more revenue. The firm's entire price distribution matters even for profits from a single consumer because the consumer's signal and price offer are independent draws from the price distribution.

To simplify writing down firm i's problem, I assume for now that the prices at all firms $j \neq i$ are weakly below $\bar{p}$ and make an observation about i's best response. If all consumers use the same cutoff price $\bar{p}$ and other firms offer prices weakly below $\bar{p}$, offering prices above $\bar{p}$ is dominated for firm i. Firm i's revenue is unaffected by prices above $\bar{p}$: no consumer buys at such prices at i because a consumer gets a price offer $p_j^o \leq \bar{p}$ at another firm j for sure. But then firm i would benefit by changing F_i: by moving the probability mass from prices above $\bar{p}$ to $\bar{p}$. Moving the mass from higher to lower prices (weakly) increases firm i's expected demand, so the move would be profitable. For now I focus on symmetric equilibria where the equilibrium price distribution $F(p)$ puts positive probability mass only on prices weakly below $\bar{p}$: $p_{max} \leq \bar{p}$. I show in the proof of Proposition 2 that no other symmetric equilibria exist and explain why on p. 12.

In equilibria where $p_{max} \leq \bar{p}$, firm i's expected profit from setting a price distribution $F_i(p)$ is

$$\pi_i = \mathbb{E}_{F_i}[D_i(p)]\mathbb{E}_{F_i}[p' - c],$$

where $D_i(p) := \Pi_{j \neq i} 1 - F_j(p)$ is the probability that a signal $p_i^s = p$ from firm i is the lowest received by a consumer. If p is the lowest price signal in a consumer's sample, the consumer visits firm i first. But upon visiting the firm, his price offer at i, p_i^o, is not p but a new price draw p' from $F_i(p)$ so $p_i^o = p'$. The consumer buys at i if $p' \leq \bar{p}$, which holds with probability one in equilibria where $p_{max} \leq \bar{p}$. Since in these equilibria a consumer accepts all prices that a firm offers, $D_i(p)$ can be interpreted as firm i's expected demand at price p. If the consumer buys at p', the firm's revenue is $p' - c$.

In symmetric equilibria where $p_{max} \leq \bar{p}$, a firm's expected equilibrium profit is

$$\pi = \mathbb{E}_F[D(p)]\mathbb{E}_F[p' - c], \tag{1}$$

and a consumer's optimal cutoff simplifies to

$$\bar{p} = \mathbb{E}_F[p] + s. \tag{2}$$

The expected price measures the negative of the consumers' welfare because all consumers buy and search once in these equilibria.

The expectations and the expected demand $D(p)$ in equations (1) and (2) depend on the equilibrium distribution F, including p_{min} and p_{max}. The derivation of the distribution differs from the standard method used in papers of search with price dispersion where an individual firm mixes across prices, but posts a singleton price. In those models, a firm is indifferent across all the prices in the support of the equilibrium price distribution. Here, a firm is *not necessarily indifferent* across the different prices in the support of its distribution.

3.2 Equilibrium price distribution

In this section, I present the equilibrium and explain its features. The most interesting are that the lowest offered prices are below the marginal cost and that the pdf of the price distribution is increasing.

Before deriving the equilibrium, I show that no pure-strategy equilibria exist where the support of $F(p)$ is a singleton. An analogous result is in Spiegler (2006), but is worth a greater emphasis in the context of search literature.

Proposition 1. *Suppose that the firms' equilibrium price distribution $F(p)$ puts all probability mass on a single price p^*. Then a firm has an incentive to deviate.*

Proof. In the Appendix. □

In the proposed single-price equilibrium each firm expects to get $1/n$ of the consumers and get price p^* from each. One profitable deviation for a firm is to choose F' that assigns equal probabilities to two prices: one marginally above and another below the proposed equilibrium price, i.e., $p^* - \varepsilon$ and $p^* + 2\varepsilon$ for some $\varepsilon > 0$ small. The deviating firm (weakly) increases its expected demand because the low price attracts half of the consumers. The firm also increases its revenue because, on average, the consumers pay $p^* + \frac{\varepsilon}{2}$. The deviation is, thus, profitable.

In the context of search literature, Proposition 1 means that if (firms expect that) consumers direct their search based on price information, single-price pure-strategy equilibria break down if firms are allowed to set nondegenerate price distributions. Partially directed search, thus, solves the Diamond (1971) paradox.

The model's unique symmetric pure-strategy equilibrium is summarised in

Proposition 2. *A firm's equilibrium price distribution is* $F(p) = \left(\frac{p_{max}-p}{p_{max}-p_{min}}\right)^{\frac{1}{n-1}}$ *with support* $[p_{min}, p_{max}]$ *where* $p_{max} = \bar{p} = c + 2s$ *and* $p_{min} = c - s(n-2)$*. The equilibrium exists if* $s \leq \frac{1-c}{2}$*.*

Proof. In the Appendix. □

In the proof, I generalise the method of Spiegler (2006) to solve for the equilibrium. In particular, in Step 2 of the proof, I use the property that if $F(p)$ is optimal, then a firm cannot profitably reallocate a small amount of mass from the neighbourhood of any price within the support of $F(p)$ to any other price. This gives me two necessary conditions on $F(p)$: one that must be satisfied by the expected demand $D(p)$ and another by the highest price in the support of $F(p)$.

This generalisation can be used for ruling out other symmetric equilibria, ruling out asymmetric equilibria more easily, and deriving equilibria in other models where agents' pure strategies are distributions. I use the generalisation to rule out other symmetric equilibria, where profits are more complex than equation (1); see Step 9 in the proof of Proposition 2.

Firms' strategies should be modelled as price distributions rather than singleton prices in many setting where firms set related prices. Prices are related, for example, if the prices that a firm sets for some of its products affect the demand for other products. Supermarkets, shopping centres, and other firms that sell products that are either direct or indirect complements are examples of such firms.[19] Prices are also related if the prices that a firm sets today affect its demand tomorrow. Airlines, hotels, and supermarkets are examples of firms that set time-varying prices and face recurring demand.

In the context of search literature, Proposition 2 means that if (firms expect that) consumers direct their search based on price information, firms set nondegenerate price distributions. In other words, price dispersion is generated in a search model with ex ante homogeneous consumers and firms. A firm prefers dispersed prices to a singleton price because the dispersion allows it to achieve the two goals

[19] Under indirect complement, I mean that a consumer is more likely to buy some products together, for example, because this saves search costs.

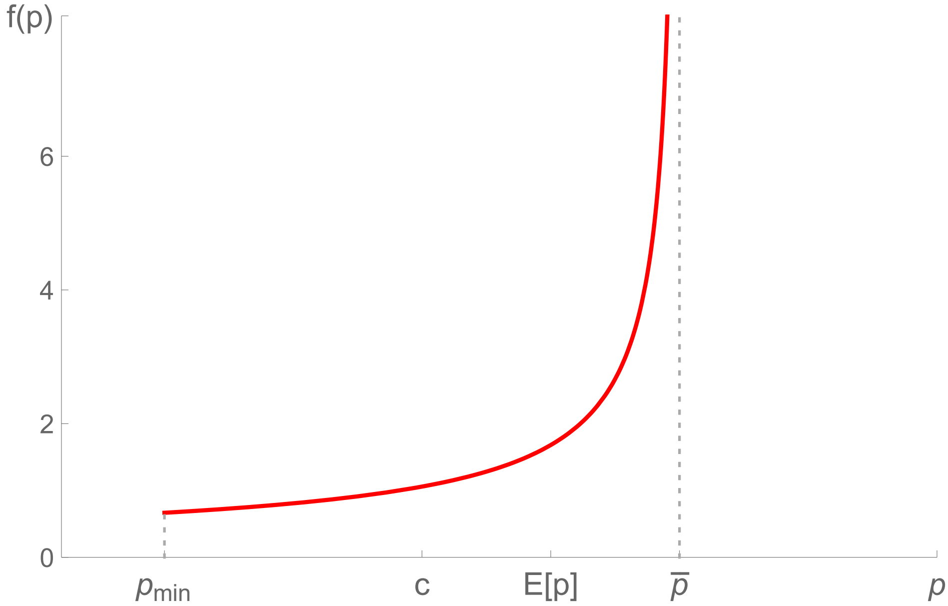

Figure 1: *The equilibrium pdf of prices;* $n = 4$, $s = \frac{1}{8}$, $c = 0$.

of attracting many consumers (with low prices) and generating high revenue (with high prices) without being too predictable for its competitors.

A within-firm price dispersion can be interpreted as a firm asking different prices for different products or product baskets that it sells, as do supermarkets, other retailers, and, in a sense, shopping centres. Another interpretation is that the firm chooses in advance its product's price for a time interval, as do many firms in reality.[20] I illustrate the equilibrium distribution of prices in Figure 1 and now discuss its features in turn.

The highest price in the support of the distribution leaves a consumer just indifferent between buying and continuing to search. A consumer, thus, always buys at the first firm that he visits. The endogenous highest price is one aspect that sets my model apart from Spiegler (2006) (and requires me to specifically prove the uniqueness of the symmetric equilibrium).

The reason why the highest price is equal to the consumers' cutoff price in symmetric equilibria is the following. In short, if other firms price above $\bar{p}$, then a single firm i can reallocate the probability mass in its price distribution mostly from prices that exceed $\bar{p}$ to $\bar{p}$ in a manner that increases its profits. The new distribution can be such that it generates a higher expected demand and at least

[20]Nielsen (2015), Anderson et al. (2017), Sinitsyn (2017), Anderson and Fox (2019).

the same expected price as the proposed equilibrium distribution. Firm i's demand increases because when its prices are weakly below $\bar{p}$, not only does it sell to all consumers that visit it first (whereas some would leave if they were offered a price above $\bar{p}$), but it also sells to all consumers who visit another firm j before i, but are offered a price above $\bar{p}$ at j. Thus, firms optimally set prices weakly below the cutoff price in symmetric equilibria.

An interesting feature of the equilibrium is that the lowest prices are below the marginal cost c (if there are at least three firms), as in Spiegler (2006). These low prices act like price promotions: they attract consumers to a firm and make losses, but the firm knows that consumers rarely end up paying such prices. If the firms' price distribution is interpreted as a distribution across the prices of product baskets, say, in a supermarket, these lowest prices can be interpreted as items on promotion. If the distribution is interpreted as a distribution across time, say, at an airline, these lowest prices can be interpreted as temporary promotions on tickets.

This equilibrium feature rationalises unprofitable price promotions that are observed in reality. Worldwide about 60% and in the US 67% of trade promotions (mostly, price discounts) in the consumer goods' sector do not break even (Nielsen, 2015; McKinsey, 2019). Such unprofitable prices are never offerd in models where firms choose singleton prices, such as the classic papers Varian (1980), Burdett and Judd (1983), and Stahl (1989). In the mixed-strategy equilibria of these models a firm is indifferent between setting any single price in the distribution's support with probability one: unprofitable prices would imply negative equilibrium profits, which cannot be. But unprofitable prices are offered in my model and in Spiegler (2006), because a firm's *pure* strategy is a distribution of prices and the firm is *not* indifferent across all the prices in the distribution's support: the unprofitable prices mainly attract consumers whereas other prices make profits. Figure 2 illustrates the price distributions in my and the other models.

Another interesting feature of the equilibrium distribution is its shape. In my model, as in Spiegler (2006), the density is everywhere increasing and prices above the mean price are more concentrated than below the mean. If the distribution is interpreted as a distribution of prices across time, the shape means that price

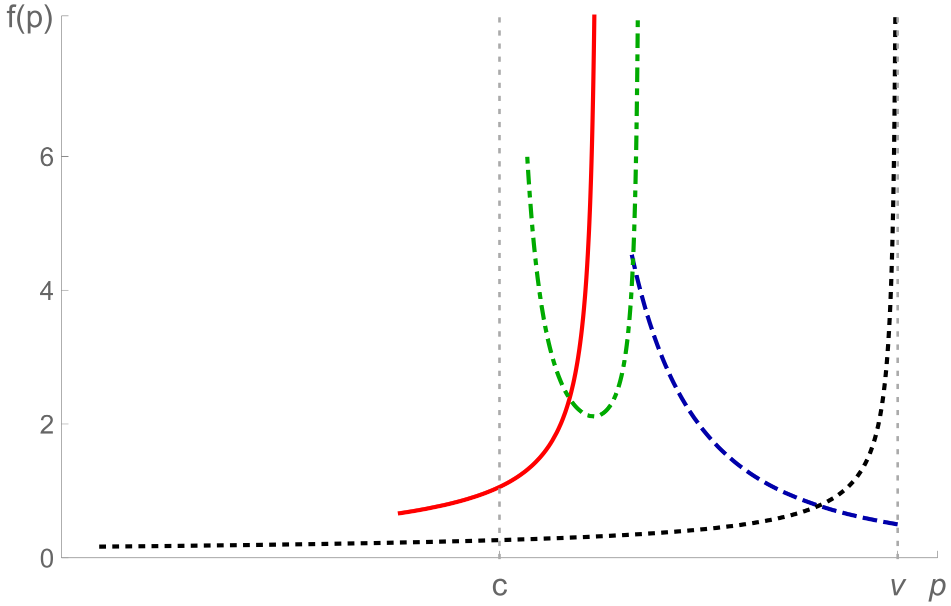

Figure 2: *The equilibrium pdf of prices in my model (red solid), Burdett and Judd (1983) (blue dashed), Stahl (1989) (green dot-dashed), and Spiegler (2006) (back dotted);* $n = 4$, $s = \frac{1}{8}$, $c = 0$.[21]

decreases are less frequent, but larger in magnitude, than price increases. The result rationalises the empirical finding that both in the US and the euro area decreases in consumer goods' prices are less frequent, but larger, than price increases.[22] In contrast to my model, the equilibrium density of prices is increasing in Burdett and Judd (1983) and u-shaped in Stahl (1989) (and Varian, 1980), as illustrated in Figure 2.

3.3 Comparative statics

The comparative statics are summarised in

Corollary 1. *As the number of firms, n, increases,*

[21]Other than c, v and n, I chose the parameters values for Burdett and Judd (1983) ($q = \frac{1}{2}$ so that half of the consumers observe one and the rest two prices) and in the unit-demand version of Stahl (1989) ($\mu = \frac{1}{2}$ so that half of the consumers are shoppers), but these values are not calibrated to match my model so the exact location of the two pdfs should be ignored. I omit Varian (1980) to not clutter the Figure: in Varian (1980) the highest price is equal to v, but the pdf is otherwise similar to that in Stahl (1989).

[22]Both in the US and the euro area about 40% of consumer goods' price changes are price decreases (see Nakamura and Steinsson (2008) and Dhyne et al. (2006) respectively). In the US, the size of an average price decrease is 14.1% and increase is 12.7%, and in the euro area the corresponding numbers are 10% and 8% (Dhyne et al., 2006).

	Stahl (1989)			Spiegler (2006)			My model		
	p_{max}	p_{min}	$\mathbb{E}[p]$	p_{max}	p_{min}	$\mathbb{E}[p]$	p_{max}	p_{min}	$\mathbb{E}[p]$
$n\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$-$	$\downarrow$	$-$	$-$	$\downarrow$	$-$
$s\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	n/a	n/a	n/a	$\uparrow$	$\boldsymbol{\downarrow}$	$\uparrow$

Table 1: Comparative statics with respect to the number of firms and search cost in Stahl (1989), Spiegler (2006) and my model.[23] $\uparrow$ stands for an increase; $\downarrow$ for a decrease; $-$ for no change; n/a for parameters not in the model.

(i) the highest price p_{max} is unaffected and the lowest price p_{min} decreases.

(ii) the expected price is unaffected.

As the search cost, s, increases,

(i) the highest price increases and the lowest price (weakly) decreases.

(ii) the expected price increases.

As the marginal cost, c, increases,

(i) the highest and lowest price increase.

(ii) the expected price increases.

Proof. The comparative statics follow from the proof of Proposition 2. □

My model makes a novel prediction and overturns an unintuitive comparative static in Stahl (1989), but also confirms several comparative static results in Stahl (1989) and Spiegler (2006). I summarise the comparison in Table 1 (the novel prediction is in bold). I comment on the more interesting comparative statics, with respect to the number of firms and the search cost, in turn and present the empirical findings that these results rationalise. The comparative statics with respect to the marginal cost are intuitive and as in earlier literature.

If the number of firms increases in my model, firms offer lower lowest prices due to stiffer competition, but do not change the highest price. The number of firms does not affect the expected price, thus, consumer welfare. If there are more

[23] The comparative statics listed for Stahl (1989) are for the interior solution.

firms, they offer lower lowest prices in equilibrium, but at the same time shift more probability mass to the highest prices. The optimal way of doing this happens to be such that the expected price is unaffected. These comparative statics are as in Spiegler (2006), but different from Stahl (1989). In particular, an unintuitive result in Stahl (1989) is that the highest and expected prices converge to the consumers' valuation if the number of firms becomes large. Here this does not happen and the highest offered price remains below the valuation.

According to Dhyne et al. (2006) the retail sector is more competitive and consumer price decreases are larger in the US than in the euro area. If the price distribution in my model is interpreted as a distribution across time, then lower lowest prices can be interpreted as price decreases being larger. My model's result that price decreases are larger in a more competitive market, thus, rationalises the above finding in Dhyne et al. (2006).

If the search cost increases in my model, firms offer higher highest prices and lower lowest prices. If the equilibrium price distribution did not change, consumers would increase their optimal cutoff price and accept higher prices. This allows firms to increase the highest prices. But then the firms can afford to compete more fiercely for consumers and lower the lowest prices they offer. In line with intuition, consumer welfare decreases as search cost increases.

The novel prediction of my model is that the lowest prices decrease in the search cost, in contrast to Stahl (1989). If we think of the search cost as the opportunity cost of time, the search cost is higher in the US than in the euro area because the GDP per capita is higher in the US.[24] Alternatively, if we think of the search cost as the time to get to a supermarket, the search cost is also higher in the US than in the euro area because some evidence suggests that supermarkets are further from households in the US than in Europe (Cant, 2019).[25] Thus, my model's novel result offers a new explanation to the the empirical finding in Dhyne et al. (2006) that price decreases are larger in the US than in the euro area.[26] In

[24]During the time period studied in Dhyne et al. (2006), 1996-2001, the GDP in the US was 25-81% higher than in the euro area (World Bank, 2021). In particular, the per-capita GDP was about 30 thousand in the US and 23.9 thousand in the euro area in current USD in 1996, and the respective numbers were 37.1 and 20.4 in 2001. The gap increased throughout.

[25]In the US, the closest supermarket to a household was on average 2.2 miles away in 2012 (Ver Ploeg et al., 2015). Unfortunately I could not find data equivalent data for the euro area.

[26]See footnote 22 for the magnitudes of price changes reported in Dhyne et al. (2006).

fact, my model offers an explanation to why both price decreases and increases are larger in the US than the euro area because both the lowest and highest price become more extreme as the search cost increases. In sum, my model offers a new explanation, based on search costs, to why price changes are larger in the US than in the euro area, in addition to confirming the explanation of stiffer competition offered in Dhyne et al. (2006).

4 Extensions

I show that the model's equilibrium is robust to allowing for imperfect matches and for only some consumers receiving price signals.

4.1 Imperfect matches

Suppose that a firm's product matches a consumer's taste with probability $\beta \in (0, 1]$. If the consumer has a match with the product, he gets utility one from buying the good and zero otherwise. The consumer observes if a firm's product is a match when he visits the firm. This modification has no effect on the equilibrium except that a consumer's effective search cost becomes s/β.

A consumer buys only if he has a match with a product and the offered price is below his cutoff price $\bar{p}$. His optimal cutoff price $\bar{p}$ solves

$$\int_{p_{min}}^{\bar{p}} \beta(\bar{p} - p)\ \mathrm{d}F(p) = s.$$

By the same argument as in the main model, a firm never offers prices above the consumer's cutoff price $\bar{p}$. Thus, a consumer accepts any first price offer from a firm with which it has a match and the cutoff price simplifies to

$$\bar{p} = \mathbb{E}_F[p] + \frac{s}{\beta}.$$

As a firm expects to sell to a fraction β of the consumers, its expected profit can be written as

$$\pi = \beta\mathbb{E}[D(p)]\mathbb{E}[p - c].$$

The problem looks like in the main model except that the effective search cost of a consumer is s/β. The equilibrium is described in

Corollary 2. *Suppose that a consumer has a match with a firm's product with probability* $\beta \in (0,1]$. *A firm's equilibrium price distribution is* $F(p) = \left(\frac{p_{max}-p}{p_{max}-p_{min}}\right)^{\frac{1}{n-1}}$ *with support* $[p_{min}, p_{max}]$ *where* $p_{max} = \bar{p} = c + 2\frac{s}{\beta}$ *and* $p_{min} = c - \frac{s}{\beta}(n-2)$. *The equilibrium exists if* $s \leq \frac{\beta(1-c)}{2}$.

Proof. Follows from the proof of Proposition 2. □

The comparative statics with respect to the probability of a match are opposite to those with respect to the search cost, summarised in Corollary 1. As matches become more likely, the prices become more compressed. In line with intuition, consumers benefit from more likely matches.

4.2 Some consumers do not get price signals

Suppose that a fraction $\lambda > 0$ of the consumers are partially informed about the firms' prices as before. The rest of the consumers, fraction $1-\lambda$, are uninformed consumers who do not receive price signals prior to search. All consumers have the same search cost s. I assume that the uninformed consumers randomise with equal probabilities over which firm to visit next. The uninformed consumers can also be interpreted as consumers who, in a symmetric equilibrium, choose to break their indifference by contacting all firms with equal probabilities.

I show below that the presence of these consumers does not have a substantial effect on the firms' price-setting incentives. The only difference is that a firm offers prices below $\bar{p}$ only to attract consumers who direct search. Thus, similar equilibrium to that in Proposition 2 remains an equilibrium. This extension provides another justification to the main model's assumption that all consumers partially direct their search because it explicitly shows that not all consumers are required to direct search for the equilibrium to exist. Furthermore, it is irrelevant for the firms' price-setting incentives whether (some) consumers actually partially direct search or whether firms think that they do so.

The continuation problems of a partially informed and an uninformed consumer look identical. Thus, they optimally use the same cutoff price $\bar{p}$ that solves

$$\int_{p_{min}}^{\bar{p}} (\bar{p} - p)\ \mathrm{d}F(p) = s.$$

If firms offer prices below $\bar{p}$, firm i's expected profit is now

$$\pi = \lambda \mathbb{E}[D(p)]\mathbb{E}[p - c] + \frac{1 - \lambda}{n}\mathbb{E}[p - c],$$

where $D(p) = \Pi_{j \neq i} F_j(p)$ is the probability that the signal p from firm i is the lowest received by a partially informed consumer.

An equilibrium is summarised in

Proposition 3. *Suppose that a fraction $1 - \lambda < 1$ of the consumers are uninformed. An equilibrium exists where a firm's equilibrium price distribution is $F(p) = \left(\frac{p_{max} - p}{p_{max} - p_{min}}\right)^{\frac{1}{n-1}}$ with support $[p_{min}, p_{max}]$ where $p_{max} = \bar{p} = c + \frac{1+\lambda}{\lambda}s$ and $p_{min} = c - s(n - \frac{1+\lambda}{\lambda})$. The equilibrium exists if $s \leq \frac{\lambda(1-c)}{1+\lambda}$.*

Proof. In the Appendix. □

Firms do not offer prices below the marginal cost in equilibrium if partially informed consumers are rare. If these consumers are rare, then firms compete less fiercely to attract them and instead focus on extracting profits from the uninformed consumers. If all consumers were uninformed ($\lambda = 0$), the Diamond paradox would follow. One takeaway of the model is, thus, that even if indifferent, consumers should use price information when choosing which firm to visit rather than randomise with equal probabilities: directing search leads to an equilibrium with lower prices. In fact, I show in Section 5.1 that the buyer-optimal indifference-breaking rule of partially informed buyers is to visit firms in an increasing order of their price signals.

Corollary 3. *As the fraction of partially informed consumers, λ, increases,*

(i) the highest and lowest price decrease.

(ii) the expected price decreases.

Proof. Follows from the proof of Proposition 3. □

The comparative statics' results with respect to the fraction of partially informed consumers are intuitive because these consumers are sensitive to prices: their demand reacts to the firms' prices. Thus, if their amount increases, competition increases, leading to lower prices.

5 Discussion

Here I discuss some assumptions and alternative interpretations of the model.

5.1 Assumption of partially directed search

The assumption that consumers follow price signals to direct their search has multiple interpretations, as described in footnote 5 and on p. 7. I prefer to interpret the assumption as a tie-breaking rule that consumers use in symmetric equilibria. Proposition 4 shows that this tie-breaking rule is a consumer-optimal ordinal tie-breaking rule that consumers use to decide in which order to visit firms in symmetric pricing equilibria. A similar argument to justify the use of a tie-breaking rule has been used, e.g., by Burguet and Che (2004).

Proposition 4. *The tie-breaking that specifies that firms are visited in an ascending order of their price signals is a consumer-optimal ordinal tie-breaking rule.*

Proof. In the Appendix. □

The reason why this tie-breaking rule is optimal for consumers is because it induces the firms to compete for custom as fiercely as possible. Other ordinal rules, that specify visiting the second-highest-priced firm with a positive probability, say, reward also firms that do not set low prices, thus, generate the lowest-priced signals. Such rules would soften price competition among firms and lead to a higher expected equilibrium price.

5.2 Firms' strategy spaces

The equilibrium characterisation in Proposition 2 relies on the assumption that firms' pure strategies are (possibly denegenrate or discrete) price distributions.

If firms' pure strategies, instead, were restricted to singleton prices, as in other models of consumer search, the business-stealing incentives in my model would result in the Bertrand outcome. If firms could choose their strategy space, they would, thus, prefer to be able to set price distributions rather than singleton prices because their expected profit is positive in the equilibrium of Proposition 2. This observation has two general implications: first, the analyst's choice of strategy spaces can be crucial for a model's results. Second, the analyst's choice should perhaps take into account which strategy spaces the players of the game prefer.

5.3 Endogenous level of price stickiness

A possible interpretation of the model is that it is of endogenous price stickiness because the distribution of prices that firms choose can be seen as the distribution of a particular product's price over time. If a firm chooses a singleton price, the firm commits to not changing the price, i.e., the price is perfectly sticky. If a firm chooses a dispersed price distribution, it commits to changing the price according to that schedule. The more dispersed the distribution, the less sticky the prices.

My model predicts that firms choose an intermediate level of price stickiness if the demand they face in one period is a decreasing function of the price that they set previously. The model, thus, generates endogenously sticky prices in the absence of menu costs. If we use the dispersion of prices, $p_{max} - p_{min}$, as the inverse measure of stickiness, then the model predicts that prices are less sticky in markets with more competition (more firms) and more frictions (higher search cost). More search frictions lead to less sticky prices here because a firm that lowers its price one day, knows that it can reap the benefits from another consumer another day since the high search cost prevents the consumer from searching on. The search cost, when interpreted as an opportunity cost of time, and the number of firms are arguably higher in a boom than in a bust. Thus, my model predicts that prices are less sticky in booms than in busts.

5.4 Disciplined deception

An alternative interpretation of the model is that it is of disciplined deception by firms. The distribution of prices that firms choose can be seen as the distribution of promised prices, all of which can be the true price. Then the signal that a consumer sees is like the price that a firm promises, whereas the price offer is the actual price that the firm asks a consumer. The fact that firms commit to a distribution from which both the promised and actual prices are drawn disciplines the deception by firms.

5.5 Labour-market interpretation

Another interpretation of the model is that it is of the labour market. This interpretation provides new insights on wage dispersion. In a labour-market context, workers look for a job with a high wage based on a sample of wage signals. The information contained in wage signals can come from the worker's friends, his own earlier job search (potentially in a different profession or at a different level in the same profession), or a recruitment company's website.[27] In equilibrium, firms set wages $w = 1 - p$ with $w_{min} = 1 - p_{max}$, $w_{max} = 1 - p_{min}$, and $G(w) = 1 - F(p)$, as described in Proposition 2. A within-firm wage distribution can be interpreted as a distribution across wages for different jobs or for a single job, say, because different people get different bonuses.

The equilibrium wage distribution has several interesting characteristics. First, the shape of the distribution reflects the shape of the empirical wage distribution in that the density is decreasing.[28] Second, the highest wages exceed the worker's productivity, which is not the case in models where firms set singleton wages. Third, if the number of firms increases, these highest wages increase. Some empirical evidence suggests that some workers are paid "too much" and that stiffer competition increased their pay.[29]

[27] Some recruitment companies allow people to see a sample of wages earned at different firms, by job; see, for example, Glassdoor (www.glassdoor.com).

[28] In the directed search models with multiple applications of Galenianos and Kircher (2009) and Kircher (2009), the wage distribution is also decreasing, but its support is discrete.

[29] Bivens and Mishel (2013) claim that executives are paid inefficiently much in the US. Also, the CEOs of several financial firms, for example, Merrill Lynch, Bear Stearns and Lehman Brothers, were paid large bonuses after large drops in the firms' values around the financial crisis of 2008 (OECD, 2009; Bebchuk et al., 2010). Stiffer competition increased the pay of executives in the US (Cuñat and Guadalupe, 2009) and Germany (Fabbri and Marin, 2016).

6 Conclusion

Firms in many markets set price distributions rather than singleton prices. For example, prices are dispersed across products at retailers that stock a variety of products. Prices are dispersed across time at firms that change prices of individual products frequently. Faced with dispersed prices, consumers have an incentive to search for a low price. If consumers direct search based on partial price information and visit first the firm with the lowest price signal, then the symmetric pricing equilibrium that ensues indeed features dispersed prices. This visiting rule is the consumer-optimal tie-breaking rule because it induces the lowest expected price among symmetric equilibria induced by all ordinal tie-breaking rules. My model suggests, thus, that people should be encouraged to search based on prices even if there does not seem to be an immediate benefit from doing so.

Some of the prices that firms offer in equilibrium are unprofitable, resembling unprofitable price promotions that are prevalent in reality (McKinsey, 2019). Firms offer such prices in order to lure consumer's to visit, knowing that when there, the consumer may buy also products that are not promoted or arrive only when the promotion has ended. Firms do not offer below-marginal-cost prices in models where they are restricted to setting a singleton price each.

Changes in consumer prices are different in the US and the euro area: price changes are more frequent and larger in the US than in the euro area. In both regions, price cuts are larger but less frequent than price hikes. My model rationalises both regularities. Arguably, competition in the retail sector is stiffer and search costs higher in the US than in Europe. This is consistent with my model's results that prices become more extreme as competition and search cost increase. Larger and less frequent price cuts than hikes is in line with my model's equilibrium price distribution that has a longer and less concentrated left rather than right tail. In sum, my model offers a new explanation, based on search costs, to unprofitable price promotions and to why price changes are larger in the US than in the euro area.

More generally, my paper highlights that the choices that an analyst makes when studying strategic situations are very important. First, it is customary to use the equal-probability tie-breaking rule when agents are indifferent. But this might not be optimal for the agents themselves once they understand that a different tie-breaking rule would lead to an outcome they prefer. Second, when analysing firms' behaviour it is customary to assume that their pure strategies are quite simple; a pure strategy is often a single number, a price or quantity. But these simple strategies may not be optimal for the agents themselves once they understand that a different strategy would lead to an outcome they prefer.

A Appendix

Here are the proofs omitted from the paper. I deal with the net utilities that firms offer to consumers, $u := v - p$ with $v = 1$, and denote the distribution of net utilities that firm i offers by $G_i(u)$. The results in the main part follow when we substitute $p = 1 - u$, $p_{max} = 1 - u_{min}$, $p_{min} = 1 - u_{max}$, and $G(u) = 1 - F(p)$.

Proof. (Proposition 1.) Suppose that G assigns probability one to $u = \hat{u} \leq 1 - c$ in equilibrium. If G assigns probability one to $u = \hat{u}$ in equilibrium, a consumer's expected continuation value, thus, optimal cutoff, is $\bar{u} = \hat{u} - s$ with $\bar{u} < \hat{u}$ so he accepts any first offer. The proposed equilibrium profits are $\hat{\pi} = \frac{1-\hat{u}-c}{n}$.

I show that firm i has an incentive to deviate to a dispersed distribution G_i' such that $P_{G'}(u = \hat{u} - \varepsilon) = P_{G'}(u = \hat{u} + \frac{\varepsilon}{2}) = \frac{1}{2}$ for $\varepsilon > 0$ small.

Firm i's expected profit from this deviation is

$$\pi' = \frac{1}{2}\left(\frac{1 - \hat{u} + \varepsilon}{2} + \frac{1 - \hat{u} - \frac{\varepsilon}{2}}{2} - c\right).$$

Firm i attracts half of the consumers, those who get the signal $u = \hat{u} + \frac{\varepsilon}{2}$ from it. But i delivers utility $\hat{u} - \varepsilon$ to half of them and $\hat{u} + \frac{\varepsilon}{2}$ to the rest. This deviation is profitable since $\pi' > \hat{\pi}$.

Note that the inequality $\pi' > \hat{\pi}$ would hold also if firm i instead deviated to $P_{G'}(u = \hat{u} - \frac{\varepsilon}{2}) = P_{G'}(u = \hat{u} + \varepsilon) = \frac{1}{2}$ as long as ε is small, $\hat{u} < 1 - c$ and $n > 2$. □

Proof. (Proposition 2.) The proof is in nine steps. In Steps 1-8, I restrict my attention to equilibria where $u_{min} \geq \bar{u}$. In Step 1, I derive two conditions on u_{min} and u_{max} that must be satisfied in such equilibria if consumers use the same utility cutoff $\bar{u}$. In Step 2, I derive conditions that u_{min}, u_{max} and $D_i(u)$ must satisfy in a symmetric equilibrium. In Steps 3-6, that follow proofs in Spiegler (2006) and Janssen and Moraga González (2004), I derive the properties of the equilibrium $G(u)$. In Step 7 I explicitly write out the conditions that the symmetric-equilibrium distribution $G(u)$ must satisfy and in Step 8 argue that such an equilibrium exists. Step 9 shows that the restriction I made in Steps 1-8 is without loss: no symmetric equilibria where $u_{min} < \bar{u}$ exist.

Let T_i denote the support of G_i, $u_{min} := \inf(T_i)$ and $u_{max} := \sup(T_i)$. Let the utilities that firms $j \neq i$ set in equilibrium be denoted by z, the lowest utility among those be z_{min}, the highest by z_{max}, and the utility distributions by G_j. Let $D_i(u_i^s)$ denote the probability that firm i's signal u_i^s is the highest utility signal that a consumer sees among his n signals:

$$D_i(u_i^s) = \begin{cases} 0 & \text{for } u_i^s < z_{min}, \\ \Pi_{j\neq i} G_j(u_i^s) & \text{for } z_{min} \leq u_i^s \leq z_{max}, \\ 1 & \text{for } u_i^s > z_{max}. \end{cases} \tag{3}$$

Step 1: If $z_{min} \geq \bar{u}$, then the lowest utility that firm i offers in an equilibrium satisfies $u_{min} \geq \bar{u}$ and the highest utility satisfies $u_{max} \leq z_{max}$.

Firms take the consumers' behaviour as given in equilibrium. For consumers search behaviour, I assumed that a consumer contact firms in a decreasing order of his utility signals. For buying behaviour, I argued in the main part of the paper that a consumer optimally rejects all utility offers below $\bar{u}$ and accepts all offers above it. If no firm offers a utility above $\bar{u}$, he visits all firms and accepts the highest offer among them as long as it is positive.

I first argue that if $z_{min} \geq \bar{u}$, then firm i optimally sets $u_{min} \geq \bar{u}$. If $G_i(u)$ puts positive probability mass on utilities below $\bar{u}$, firm i has an incentive to move all mass from below $\bar{u}$ to $\bar{u}$. The reason is that if $z_{min} \geq \bar{u}$, then utilities below $\bar{u}$ are never accepted at firm i: a utility offer below $\bar{u}$ induces a consumer to continue searching after visiting firm i and, because $z_{min} \geq \bar{u}$, he gets a utility offer that exceeds $\bar{u}$ at another firm for sure. Thus, utilities below $\bar{u}$ do not affect i's revenue and utility signal $u_i^s = \bar{u}$ attracts (weakly, if $z_{min} = \bar{u}$) more consumers than signals $u_i^s < \bar{u}$.

Now I show that if $G_i(u)$ puts positive probability mass on utilities above z_{max}, firm i has an incentive to move all mass from above z_{max} to z_{max}. Utilities higher than z_{max} are more costly than z_{max} for firm i to offer, but utility signals $u_i^s > z_{max}$ attract equally many consumers as signal $u_i^s = z_{max}$. Thus, $u_{max} \leq z_{max}$ in equilibrium.

Step 2: Necessary conditions that u_{min}, u_{max}, and $D_i(u)$ have to satisfy in a symmetric equilibrium.

If firms $j \neq i$ use G_j such that $z_{min} > \bar{u}$, then firm i has an incentive to move all the probability mass from utilities $u \in [u_{min}, z_{min}]$ to $\bar{u}$. The reason is that utilities below z_{min} do not generate demand, but increase the expected revenue. So firm i does best by moving all the mass on utilities below z_{min} to the lowest one that is still acceptable for consumers, which is $\bar{u}$. Thus, any symmetric equilibrium where $z_{min}, u_{min} \geq \bar{u}$ needs to satisfy $z_{min} = u_{min} = \bar{u}$.

Note that the above point together with Step 1 rules out deviations that reallocate probability mass from the proposed symmetric equilibrium distribution to utilities above u_{max} and below $u_{min} = \bar{u}$. To rule out deviations to utilities between u_{min} and u_{max}, I use the property that if $G_i(u)$ is optimal, it must be unprofitable for firm i to reallocate mass from any utility $u \in T_i$ to another utility u'.

Recall that a firm's expected profit in a proposed symmetric equilibrium is

$$\pi = \mathbb{E}[D(u)]\mathbb{E}[1 - c - u],$$

and a consumer's cutoff solves

$$\bar{u} = \mathbb{E}[u] - s.$$

First suppose that $G_i(u)$ assigns positive mass to u_{min}, u_{max}, and some $u \in T_i$. I use the condition that if $G_i(u)$ is optimal, then firm i must be unwilling to move a small amount of mass, $\varepsilon > 0$, from any $u \in T_i$ to some other u'. Note that any u' can be written as $u' = \alpha u_{min} + (1-\alpha)u_{max}$, where $u' \in [u_{min}, u_{max}]$ if $\alpha \in [0,1]$. So I show that, if $D_i(u)$ and u_{max} satisfy certain conditions, it is unprofitable to remove mass $\varepsilon > 0$ from $u \in T_i$ and put some of it, mass $\alpha\varepsilon$, on u_{min} and the rest, mass $(1-\alpha)\varepsilon$, on u_{max} for any finite α.

Removing a small mass $\varepsilon > 0$ from u and putting mass $\alpha\varepsilon$ on u_{min} and mass $(1-\alpha)\varepsilon$ on u_{max} changes firm i's profit by

$$\Delta\pi = \Delta\mathbb{E}[D(u)]\mathbb{E}[1-c-u] + \mathbb{E}[D(u)]\Delta\mathbb{E}[1-c-u]$$

$$= \varepsilon\{[\alpha D(u_{min}) + (1-\alpha)D(u_{max}) - D(u)](1-c-m)$$

$$+d[\alpha(1-c-u_{min}) + (1-\alpha)(1-c-u_{max}) - (1-c-u)]\},$$

where $D := D_i$, $m := \mathbb{E}[u]$ and $d := \mathbb{E}[D(u)]$ for brevity. If $G(u)$ is optimal, $\Delta\pi \leq 0$ has to hold for all small $\varepsilon > 0$. Since $\Delta\pi$ is proportional to ε, $\Delta\pi \leq 0$ if the term in the curly brackets, $\frac{\Delta\pi}{\varepsilon}$, is weakly negative.

Noting that $D(u_{min}) = 0$ and $D(u_{max}) = 1$, we can simplify $\frac{\Delta\pi}{\varepsilon} \leq 0$ to

$$\alpha Q_1 := \alpha[(1-c-m) - d(u_{max} - u_{min})] \tag{4}$$

$$\geq [1 - D(u)](1-c-m) - d(u_{max} - u) =: Q_2.$$

Now Q_1 on the LHS of (4) is just a number and Q_2 on the RHS a function of u with $Q_2 \leq 1$ for all u. Thus, inequality (4) holds for any α only if both $Q_1 = 0$ and $Q_2 = 0$.

Solving $Q_1 = 0$ for u_{max} gives that u_{max} must satisfy

$$u_{max} = u_{min} + \frac{1-c-m}{d}. \tag{5}$$

Solving $Q_2 = 0$ for $D(u)$ and plugging in the expression for u_{max} gives that $D(u)$ must satisfy

$$D(u) = \frac{u - u_{min}}{u_{max} - u_{min}}. \tag{6}$$

Note that $D(u)$ is linearly increasing in u.

Step 3: A Corollary to Step 2, which I use in Step 8, is

Corollary 4. *Given G_{-i}, firm i is indifferent between $G_i(u)$ and $G_i^*(u)$ such that $\mathbb{E}_G[u] = \mathbb{E}_{G^*}[u]$ and $T_i^* \subseteq T_i$.*

Proof. Because $D_i(u)$ is linear, firm i's expected profit is the same under G_i and G_i^* if $\mathbb{E}_G[u] = \mathbb{E}_{G^*}[u]$. □

Step 4: In equilibrium, G is continuous on $[u_{min}, \infty)$.

Suppose instead that G assigns an atom to some $u \in [u_{min}, \infty)$. Then firm i can increase its profits by shifting this mass in its distribution to $u + \varepsilon$, $\varepsilon > 0$ small. Firm i's expected demand would increase by a discrete amount, whereas its expenditure would only increase in the order of ε. Thus, the deviation would be profitable.

Step 5: In equilibrium, $T = [\bar{u}, u_{max}]$.

Suppose instead that in equilibrium firms place no weight on some interval $(u_1, u_2) \in (\bar{u}, u_{max})$. Then firm i can profitably shift weight from $(u_2, u_2+\varepsilon)$, $\varepsilon > 0$ small, to u_1. Firm i's expected demand decreases by an order of magnitude ε, whereas the expected utility that it offers decreases by a discrete amount. Thus, this deviation is profitable.

Step 6: In equilibrium, $\bar{u}$ has the closed-form solution given by equation (10).

First note that in a symmetric equilibrium, conditional on having positive demand, $\mathbb{E}[D(u)] = \frac{1}{n}$. Then rewrite equation (6) as

$$u = u_{max} - (u_{max} - u_{min})(1 - D(u)). \tag{7}$$

Now, borrowing a clever trick in (Janssen and Moraga González, 2004, p. 1097), I define a new variable $z := G(u)$ and express $\mathbb{E}[u]$ as

$$\mathbb{E}[u] = \int_{u_{min}}^{u_{max}} u \, \mathrm{d}G(u) = \int_0^1 u \, \mathrm{d}z. \tag{8}$$

From the consumer's optimisation problem, we know that $\mathbb{E}[u] = \bar{u} + s$ and in a symmetric equilibrium, $D(u) = z^{n-1}$. Thus, I can rewrite (8) using (7) as

$$\bar{u} + s = \int_0^1 \left[u_{max} - (u_{max} - u_{min})(1 - z^{n-1})\right] \, \mathrm{d}z,$$

and because $u_{min} = \bar{u}$,

$$u_{max} - u_{min} = sn, \tag{9}$$

where the right-hand side of (9) depends only on exogenous variables.

Using equations (5) and (9), I can solve for $\bar{u}$:

$$\bar{u} = 1 - c - 2s. \tag{10}$$

A necessary condition for the equilibrium to exist is that the consumers' value from searching is positive, i.e., that $s \leq \frac{1-c}{2}$.

Step 7: In equilibrium, u_{max} has the closed-form solution given by equation (11) and $G(u)$ by (12) for all $u \in [\bar{u}, u_{max}]$.

To get an explicit form for u_{max}, I use the explicit form for $\bar{u}$ in (5):

$$u_{max} = 1 - c + s\,(n-2)\,. \tag{11}$$

Firms offer prices below the marginal cost (or, $u_{max} > 1 - c$) for all $n > 2$.

To get an explicit form for $G(u)$, I use the fact that in a symmetric equilibrium $G(u) = D(u)^{\frac{1}{n-1}}$:

$$G(u) = \left(\frac{u - \bar{u}}{u_{max} - \bar{u}}\right)^{\frac{1}{n-1}}, \tag{12}$$

where $\bar{u} = 1 - c - 2s$.

Step 8: An equilibrium in distributions $G(u)$ as described in equations (10), (11), and (12) exists.

Suppose all firms but i use $G(u)$ as described in equation (12). Firm i's expected demand $D_i(u)$ is determined only by $G_j(u)$, $j \neq i$. Then if $G_i(u) = G(u)$, we know that $D_i(u)$ satisfies (6) and we know from Steps 1 and 2 that firm i cannot improve its profits by reallocating small amounts of mass from $u \in T$. By Corollary 4, firm i is indifferent between G and any G' s.t.

$T' \subseteq T$ and $\mathbb{E}_G[u] = \mathbb{E}_{G'}[u]$ because D is linear. Thus, playing G is a best response for i to other firms playing G.

Step 9: No symmetric equilibria exist where $u_{min} < \bar{u}$.

I prove that the equilibrium derived above is the unique symmetric equilibrium. I show that if in a proposed equilibrium firms use G such that $z_{min} < \bar{u}$, then firm i has a profitable deviation: I show that moving some mass from utilities in the support of G to another utility is profitable. In the proof, I implicitly assume that any symmetric equilibrium G is atomless: the argument is the same as in Step 4 above.

I first show that the expected demand and expected price are no longer separable in a firm's expected profit if the equilibrium G puts mass on utilities below $\bar{u}$. This is because now some consumers get so low utility offers at a firm that they continue searching. In particular, now a consumer buys from a firm i in one of two cases. First, he buys at i immediately upon visiting if $u_i^o \geq \bar{u}$ (let's call such consumers "new customers"). Second, he returns to buy at i after visiting all firms because he got offers below $\bar{u}$ at all firms and the one from i was the highest of them (let's call such consumers "return customers").

Firm i's serves a new customer as a kth firm that he visits if u_i^s is the kth-highest signal among the consumer's signals, he does not stop before, and his offer from i is at least $\bar{u}$. Since the firm does not know which signals the consumer gets from all firms, it has to take an expectation over u_i^s and k. In total, its expected demand from new customers is

$$\mathbb{E}_{G_i}\left[\sum_{k=1}^{n}\binom{n-1}{k-1}(1-G_j(u))^{k-1}G_j(u)^{n-k}G_j(\bar{u})^{k-1}\right](1-G_i(\bar{u})).$$

Inside the sum is the probability that firm i is the kth firm that a consumer plans to visit and that he reaches the firm if his signal from i is u: the probability that $k-1$ of his signals are above u, the rest are below u, and that at the first $k-1$ firms that he visits he gets a utility offer below $\bar{u}$. The last term is the probability that his offer from i is at least $\bar{u}$.

The expected revenue from a new customers is $\mathbb{E}_{G_i}[1 - u - c|u \geq \bar{u}]$. I can simplify the sum above and write firm i's expected profit from new customers as:

$$d_n m_n := \mathbb{E}_{G_i}\left[[G_j(\bar{u}) + G_j(u)(1 - G_j(\bar{u}))]^{n-1}\right] \int_{\bar{u}}^{u_{max}} (1 - u - c)\ \mathrm{d}G_i(u).$$

Firm i's serves a return customer if he gets offers below $\bar{u}$ at all firms and the offer from i, u_i^o, is the highest among them. Note that the expected demand from return customers is only affected by the utility offers at all firms, but not the signals. If the return customer buys from i, he pays $1 - u_i^o$, which generates the non-separability of expected demand and price in a firm's profit function. Given that the customer's offer from i is $u_i^o = u$ and that $u_j^o < \bar{u}$ at all j, the probability that u is higher than u_j^o for all $j \neq i$ is

$$P(u > u_j^o\,\forall j \neq i | u_j^o < \bar{u}\,\forall j) = \frac{G_j(u)^{n-1}}{G_j(\bar{u})^{n-1} G_i(\bar{u})}.$$

Thus, firm i's expected profit from return customers is:

$$\pi_r := G_i(\bar{u}) \int_{u_{min}}^{\bar{u}} \frac{G_j(u)^{n-1}(1 - u - c)}{G_j(\bar{u})^{n-1} G_i(\bar{u})}\ \mathrm{d}G_i(u),$$

where the first term is the probability that his offer from i is below $\bar{u}$.

In total, firm i's expected profit in the proposed equilibrium is

$$\pi = \mathbb{E}_{G_i}\left[[G_j(\bar{u}) + G_j(u)(1 - G_j(\bar{u}))]^{n-1}\right] \int_{\bar{u}}^{u_{max}} (1 - u - c)\ \mathrm{d}G_i(u) \quad (13)$$

$$+ G_j(\bar{u})^{-(n-1)} \int_{u_{min}}^{\bar{u}} G_j(u)^{n-1}(1 - u - c)\ \mathrm{d}G_i(u) = d_n m_n + \pi_r.$$

I now show that it is profitable for firm i to remove mass $\alpha\varepsilon$ from (the neighbourhood of) u_{max} and $(1-\alpha)\varepsilon$ from u_{min}, and to move it to some $u \geq \bar{u}$ if α is small.

First note that the profit from return customers π_r is unaffected if mass is moved from u_{min} and u_{max} to $u \geq \bar{u}$: offers $u_i^o = u_{max}$ and $u_i^o = \bar{u}$ are not offered to return customers, and offer $u_i^o = u_{min}$ always loses against other firms' offers. Thus, the change in profits from this reallocation of probability mass is

$$\Delta\pi = m_n \Delta d_n + d_n \Delta m_n$$

$$= \varepsilon \left\langle m_n\{-\alpha - (1-\alpha)G_j(\bar{u})^{n-1} + [G_j(\bar{u}) + G_j(u)(1-G_j(\bar{u}))]^{n-1}\} \right.$$

$$\left. + d_n[-\alpha(1-u_{max}-c) + 1 - u - c] \right\rangle .$$

The reallocation of mass must be unprofitable for all α, but note that

$$\frac{\Delta\pi}{\varepsilon} \to m_n \left\{[G_j(\bar{u}) + G_j(u)(1-G_j(\bar{u}))]^{n-1} - G_j(\bar{u})^{n-1}\right\} + d_n(1-u-c) > 0$$

as $\alpha \to 0$. In words, firm i always finds it profitable to reallocate probability mass away from u_{min} to some $u \geq \bar{u}$ if $u_{min} < \bar{u}$. Intuitively, the firm can do so because utilities above $\bar{u}$ generate disproportionately more demand that utilities below $\bar{u}$. □

Proof. (Proposition 3.) A fraction $\lambda > 0$ of the consumers are as before and fraction $1-\lambda$ are uninformed consumers who do not receive price information.

I solve for the equilibrium using the same method as when proving Proposition 2, but skip much of the detail because it is the same. In Step 0, I show that a single-u equilibrium never exists. Steps 1-8 follow closely the corresponding ones in the proof of Proposition 2.

Step 0: An equilibrium in degenerate distributions $G(u)$ does not exist.

Suppose that all firms set $u = \hat{u}$ in equilibrium with probability one. If all firms set $u = \hat{u}$ in equilibrium, then a consumer's expected value is $\mathbb{E}[u] - s = \hat{u} - s$ so he accepts any first offer if it is positive. The proposed equilibrium profits are $\hat{\pi} = \frac{1-\hat{u}-c}{n}$. For weakly positive profits in equilibrium, it must be that $\hat{u} \leq 1-c$. I show that firm i has an incentive to deviate to a dispersed distribution G_i' such that $P'(u = \hat{u} - \varepsilon) = \frac{1}{2}$ and $P'(u = \hat{u} + \frac{\varepsilon}{2}) = \frac{1}{2}$ for $\varepsilon > 0$ small.

Firm i's profit from this deviation is

$$\pi' = \left(\frac{\lambda}{2} + \frac{1-\lambda}{n}\right)\left[\frac{1}{2}\left(1 - \hat{u} - \frac{\varepsilon}{2} - c\right) + \frac{1}{2}(1 - \hat{u} + \varepsilon - c)\right]$$

because it attracts half of the consumers who partially direct search (those, who get the signal $u = \hat{u} + \frac{\varepsilon}{2}$ from it) and its fair share of uninformed consumers. This deviation is profitable since $\pi' > \hat{\pi}$.

Step 1: Necessary conditions on $D(u)$ and u_{max}.

Assume that $u_{min} \geq \bar{u}$. I need to derive the rest of $G(u)$. Recall that a firm's expected profit is

$$\pi = \left(\lambda \mathbb{E}[D(u)] + \frac{1-\lambda}{n}\right) \mathbb{E}[1-u-c],$$

and a consumer's cutoff solves $\bar{u} = \mathbb{E}[u] - s$.

Removing a small mass $\varepsilon > 0$ from u and putting mass $\alpha\varepsilon$ on u_{min} and mass $(1-\alpha)\varepsilon$ on u_{max} changes firm i's profit by

$$\Delta\pi = \lambda \Delta\mathbb{E}[D(u)]\mathbb{E}[1-u-c] + \left(\lambda \mathbb{E}[D(u)] + \frac{1-\lambda}{n}\right) \Delta\mathbb{E}[1-u-c]$$

$$= \varepsilon\{\lambda[\alpha D(u_{min}) + (1-\alpha)D(u_{max}) - D(u)](1-m-c)$$

$$+ \left(\lambda d + \frac{1-\lambda}{n}\right)[\alpha(1-\bar{u}-c) + (1-\alpha)(1-u_{max}-c) - (1-u-c)]\},$$

where $D := D_i$, $m := \mathbb{E}[u]$ and $d := \mathbb{E}[D(u)]$ for brevity. If $G(u)$ is optimal, $\Delta\pi \leq 0$ has to hold for all $\varepsilon > 0$. Since $\Delta\pi$ is proportional to ε, $\Delta\pi \leq 0$ if the term in the curly brackets, $\frac{\Delta\pi}{\varepsilon}$, is weakly negative. This holds for any small $\varepsilon > 0$ if $\frac{\Delta\pi}{\varepsilon} = 0$, i.e., if

$$\lambda(1-D(u))(1-m-c) - \left(\lambda d + \frac{1-\lambda}{n}\right)(u_{max} - u)$$

$$= \alpha[-(u_{max} - \bar{u})\left(\lambda d + \frac{1-\lambda}{n}\right) + \lambda(1-m-c)].$$

So the two necessary conditions that must be satisfied are that

$$D(u) = 1 - \lambda^{-1}(1-m-c)^{-1}\left(\lambda d + \frac{1-\lambda}{n}\right)(u_{max} - u),$$

and

$$u_{max} = \bar{u} + \left(\lambda d + \frac{1-\lambda}{n}\right)^{-1} \lambda(1-m-c).$$

Note that $D(u)$ is linear in u and the profit function depends on $G(u)$ only through $\mathbb{E}[D(u)]$ and $\mathbb{E}[u]$ so that Corollary 4 holds.

Step 2-5: Follow directly from the equivalent steps in the proof of Proposition 2. The only difference is that for the arguments to hold, $\lambda > 0$ must hold because the firms can affect their demand only from consumers who partially direct search.

Step 6: In this step, the only difference comes from the slightly different forms that $D(u)$ and u_{max} take. Altogether, these amount to

$$\bar{u} = 1 - c - (1 + \lambda^{-1})s,$$

and

$$u_{max} = 1 - c + \left(n - 1 - \lambda^{-1}\right) s,$$

Thus, a necessary condition for the equilibrium to exist is that $\bar{u} \geq 0$ or $s < \frac{\lambda(1-c)}{1+\lambda}$.

Step 7-8: These steps are almost exactly the same as in the proof of Proposition 2. An additional deviation that we need to rule out is the firm abandoning serving the partially informed consumers and only serving the uninformed consumers at the highest acceptable price. This deviation would yield profits equal to $\tilde{\pi} = \frac{1-\lambda}{n}(\bar{p} - c)$ to a firm. The equilibrium profits are $\pi = \frac{1}{n}(\bar{p} - s - c)$. Plugging in $\bar{p}$ shows that the deviation is not profitable because $\pi > \tilde{\pi}$. □

Proof. (Proposition 4.) I prove the statement without formally defining a metagame where the consumers' strategy would include the choice of a tie-breaking rule, should the pursuant equilibrium pricing strategies of firms be symmetric. I consider ordinal tie-breaking rules that can depend on the sample of free price signals that a consumer gets, but are anonymous. I argue informally that the following tie-breaking rule is consumer-optimal among all ordinal tie-breaking rules:

- if all price signals in the consumer's sample are equal, visit first any particular firm with probability $\frac{1}{n}$.

- if not all price signals in the consumer's sample are equal, visit first the cheapest firm in the sample with probability one. If firm i's price signal is below firm j's, visit firm i as the kth firm in sequence, for $k > 1$, with a strictly higher probability than firm j.

In general, let the probability with which firm i is visited as the kth firm if it is the mth-cheapest firm in a consumer's sample be denoted by μ_m^k. Note that $\sum_{m=1}^{n} \mu_m^k = 1$ for all k as long as the expected symmetric equilibrium price satisfies $\mathbb{E}[p|p \leq \bar{p}] + s \leq v = 1$. I again work with offered utilities instead of prices below.

Step i: Inducing an equilibrium where firms set singleton prices is not optimal for consumers.

We know that if all firms set singleton utility, then the best price for them is offer $u^* = s$. But we know from Proposition 2 that consumers can induce a symmetric equilibrium with a higher offered expected utility by using a tie-breaking rule that induces a dispersed pricing equilibrium.

Step ii: For all $\mu_m^k > 0$, if $u_i^s > u_j^s$, then a consumer-optimal ordinal tie breaking has $\mu_i^k \geq \mu_j^k$ for each k and the inequality is strict if $u_i^s > \bar{u}$.

Suppose otherwise: that firm i's utility signal is higher than firm j's, but firm i is visited as the kth firm with a strictly lower probability, i.e., that $u_i^s > u_j^s$, but $\mu_i^k < \mu_j^k$. Then firm i has a profitable deviation: it can reallocate mass within $G_i(u)$ from (the neighbourhood of) u_i^s to u_j^s. This mass reallocation increases i's chance of being visited as the kth firm and (weakly) decreases the expected utility that i offers to consumers. As long as $u_i^s > \bar{u}$, firm i can reallocate the appropriate amount of mass from u_i^s to u_j^s so that the expected utility that i pays to consumers, $\mathbb{E}_i[u|u \geq \bar{u}]$, decreases. In particular, i reallocates so much mass that after the mass reallocation, $\mathbb{E}_i[u] \geq \bar{u}$ still holds.

Note that if $\bar{u} \geq u_i^s > u_j^s$ and $\mu_i^k = \mu_j^k$, then firm i cannot reallocate mass profitably from u_i^s to u_j^s.

Thus, if consumers want to induce symmetric equilibrium utilities that are all acceptable (i.e., $u_{min} \geq \bar{u}$), then any consumer-optimal ordinal tie-breaking rule says that a firm with a higher utility signal is visited as the kth firm with a higher probability than a firm with a lower utility signal.

Step iii: A consumer-optimal ordinal tie-breaking rule involves $\mu_1^1 = 1$ and $\mu_2^1 = \mu_3^1 = ... = \mu_n^1 = 0$.

In this step I assume that all posted utilities are acceptable, i.e, $u_{min} \geq \bar{u}$ (I show that this must be the case in Step iv). I first show that in any induced symmetric equilibrium, a firm's demand must satisfy

$$D(u) = \frac{\mu_1^1(u - u_{min}) + \mu_n^1(u_{max} - u)}{u_{max} - u_{min}}, \tag{14}$$

and

$$u_{max} - u_{min} = n(1 - c - m)(\mu_1^1 - \mu_n^1). \tag{15}$$

I use the same procedure as in Step 2 of the proof of Proposition 2 to derive conditions on $D_i(u)$ such that firm i does not want to reallocate mass within the support of $G_i(u)$. First, only if $D_i(u)$ satisfies equation (14) is it unprofitable to remove mass $\varepsilon > 0$ from $u \in T_i$ and put mass $\alpha\varepsilon$, on u_{min} and the rest, mass $(1 - \alpha)\varepsilon$, on u_{max}. Such a mass reallocation changes firm i's profit by

$$\Delta\pi = \varepsilon\{[\alpha\mu_n^1 + (1 - \alpha)\mu_1^1 - D(u)](1 - c - m)$$

$$+d[\alpha(1 - c - u_{min}) + (1 - \alpha)(1 - c - u_{max}) - (1 - c - u)]\} \leq 0,$$

where I have used the fact that (in a symmetric equilibrium) u_{min} will be the lowest and u_{max} the highest utility signal in any consumer's sample of signals, and denoted $D := D_i$, $m := \mathbb{E}[u]$ and $d := \mathbb{E}[D(u)]$ for brevity. This condition is satisfied for any positive α only if $D(u)$ satisfies equation (14) and u_{max} satisfies (15) (where I have replaced a symmetric equilibrium outcome $d = 1/n$).

Now in a symmetric pricing equilibrium, firms choose the same $G(u)$. Then the probability that firm i's signal $u_i^s = u_{max}$ is the highest among a consumer's signals is equal to one if $G(u)$ has no mass point on $u = u_{max}$. A mass point on u_{max} can be ruled out in a similar manner as Step 4 in the proof of Proposition 2 (for all $\mu_1^1 > 0$). Thus, in addition to equation (14), an individual firm i's expected demand $D_i(u_{max})$ must satisfy

$$D_i(u_{max}) = Pr(u_i^s = u_{max} > u_j^s \text{ for all } j \neq i) = G(u_{max})^{n-1} = 1.$$

But this equation can hold together with equation (14) only for $\mu_1^1 = 1$. Altogether, the consumer-optimal tie-breaking rule satisfies $\mu_1^1 = 1$ and $\mu_2^1 = \mu_3^1 = ... = \mu_n^1 = 0$.

Note that $\mu_1^1 = 1$ means that all tie-breaking rules that differ in μ_m^k for $k > 1$ are equivalent to this one on the equilibrium path if $u_{min} \geq \bar{u}$ because all consumers stop searching at the first-visited firm.

Step iv: In any symmetric equilibrium induced by a tie-breaking rule that satisfies Step i, firms offer utilities that are all acceptable: $u_{min} \geq \bar{u}$.

Suppose otherwise: that a symmetric equilibrium $G(u)$ induced by a tie-breaking rule as described in Step i, assigns utilities below $\bar{u}$ positive probability or $G(\bar{u}) \leq 1$. I first show that $G(\bar{u}) = 1$ cannot hold in equilibrium and then that, if $G(\bar{u}) < 1$, firm i can profitably reallocate mass within $G_i(u)$.

If $G(\bar{u}) = 1$, each consumer looks through all firms before purchasing from the firm with the highest utility offer. When visiting any firm i, a consumer's continuation value is $\mathbb{E}_G[u] - s < \bar{u} - s$. But the cutoff utility $\bar{u}$ is defined as the utility that makes the consumer just indifferent between continuing and stopping, or $\bar{u} = \mathbb{E}_G[u] - s$. This equality cannot hold together with the previous inequality. Thus, in any symmetric equilibrium we must have $G(\bar{u}) < 1$.

I show now that if $G(\bar{u}) < 1$, an individual firm wants to deviate from the proposed equilibrium $G(u)$ if consumers use a tie-breaking rule that satisfies Step i. In particular, the firm can profitably reallocate mass from (the neighbourhood of) $u = \bar{u} - \eta$ for $\eta > 0$ small to $u = \bar{u}$. (The fact that $G(u)$ must be continuous in equilibrium can be shown in a similar way as in Step 4 of the proof of Proposition 2.)

Recall that the tie-breaking rules in Step i satisfy that for $u_i^s > \bar{u}$ and $u_i^s > u_j^s$, $\mu_i^k > \mu_j^k$: if firm i reallocates mass from lower to higher utilities, it increases the chance that it is visited as the kth firm in the sequence, k by k, so the mass reallocation increases its expected demand. The increase is strict if i moves mass to utilities that weakly exceed $\bar{u}$. So if firm i moves mass from $u = \bar{u} - \eta$ to $u = \bar{u}$, its signals get a bit better and it is visited earlier with a bit higher probability than without the mass reallocation.

Now suppose that firm i is visited by a consumer as the kth firm in his sequence and, instead of getting offer $u_i^o = \bar{u} - \eta$, he gets offer $u_i^o = \bar{u}$. The consumer buys from firm i, whereas previously he would have returned to firm i only if for all $j > k$ in the visiting sequence $u_j^o < \bar{u}$. (Note that u_i^o "wins" against almost all u_j^o because η is small.) The probability that the consumer gets such offers at all firms $j > k$ in the visiting sequence happens with a probability that is strictly below one for all $k < n$. Thus, firm i can strictly increase its demand from consumers at a negligible cost. In other words, this deviation is profitable. □

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