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Abstract

This paper studies measurement of welfare effects, transient and permanent, of stabilizing or deregulating prices in Cobweb-like settings. As in Cobweb-models, producers must commit inputs in face of uncertainty. Here, however, we consider producers who are concerned with adaptations of *inputs* rather than *price predictions*. This shift of emphasis reflects two things. First, since persistent randomness causes on-going price fluctuations, *point* predictions are of modest concern. Second, producers are likely to care more about profits than prices.

Explored below are economic issues related to transitions between regulated and unregulated equilibrium. Our focus is on convergence, stability and welfare. The main motivation and the running example centers on agricultural commodities.

The paper does, however, not review the case for or against market intervention. In stead it advocates and illustrates the use of tractable tools called stochastic approximation. These tools easily produce quantitative estimates to facilitate discussion of the pros and cons.

Keywords: price stabilization, cobwebs, convergence, stochastic approximation.

JEL Classification: C63, D60, L51, Q11.

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1 Introduction

Issues about price stabilization have generated extended and long-lasting debates, driven chiefly by concerns about random fluctuations in supply or demand of agricultural commodities. Many arrangements in subsistence economies aim expressly at smoothing food supply across time, contingencies, and members. Those arrangements have largely reflected prevailing institutions and ideologies. Indeed, the recurrence - and variability - of fat and lean years has left a legacy concerning distribution, fairness, and insurance.

While history often witness high income elasticity of basic food demand, modern times have seen such demand become fairly inelastic. In any case, farmers frequently prefer that some agency stabilize their product prices. Often such an institution is intended to insure or improve producers' incomes. Many theoretical studies stress however, that stabilization programs may affect various parties differently, and some even adversely, depending on the data and the source of uncertainty.

The received literature is large and since long burgeoning [6], but it still leaves many issues unanswered. For one, because repercussions and consequences of regulation/deregulation need time to become complete, welfare analysis ought not ignore transient effects. In addition, price stabilization may affect welfare in ambiguous ways, with unequal distribution of costs and benefits. Ever since the study of Newbery and Stiglitz (1981), intervention in primary markets has caused well-founded concerns about efficacy and fairness. In contrast, since conventional wisdom often regards *stability* as beneficial per se, the notion itself may have a positive, slightly seducing ring to it.

Therefore, discussions about the merits of price stabilization are difficult to settle. Additional difficulties stem from received models being fairly stringent or rather demanding. It is, for example, somewhat constraining to allow merely one good or linear market curves [17], [18], [22], [24], [27]. Further, to have the dynamics properly described by stochastic differential equations is quite a task in itself [8].¹

A more serious limitation of many approaches comes from their insisting on agents' competence, rationality and foresight being perfect. Behavioral assumptions of such sort ignore that human-like individuals need time to observe and learn. In response this study, like many others, contends with more realistic assumptions. Broadly, these reduce to specifying rather gen-

¹In response to such queries one might accommodate many outputs, with interrelated market curves, and several, possibly constrained inputs. Also, one would rather use statistical data in original, representative form. The method developed here then suits well.

eral forms of adaptive behavior. The forms fit the frames of *stochastic approximation*, and they require neither analytic process descriptions nor divine agents. A similar approach is already prominent in recent studies of economic dynamics [20], technological evolution [1], [9], Walrasian tatonnement [10], macroeconomic expectations and learning [11]. To the best of our knowledge it has not previously been used to study price controls. In that particular context stochastic approximation, provided it be applicable, offers several advantages. Notably it can:

• accommodate agents who are somewhat short on foresight, competence, or knowledge;

- trace the welfare effects of replacing one regulatory regime with another;
- indicate transition paths and record associated costs;
- avoid restrictions on functional forms or random mechanisms; and finally,
 use statistical time series in raw form.

Most important, the said methods permit inquiries into how and where uncertainty affects behavior and hence outcomes. In particular, they expressly direct attention to the presence and impact of non-linearities. Exploring these features is our chief errand. Other, more minor topics include • identification of stable equilibria;

- inquiries as to whether, which and how equilibrium might be reached;
- accommodation of several interdependent inputs and outputs.

Throughout the paper, our main interest is with welfare effects whether transient or permanent. Rational expectations are never presumed. Instead iterated adaptations drive the dynamics. Each agent's choice is always based on his local information. However, even if he remains myopic and imperfectly informed throughout, one hopes that rational expectations obtain in the long run. Plainly, such a hope isn't warranted without justification. So, the major question reads: *Can equilibrium be reached by a plausible, decentralized, informationally parsimonious processes which, at best, should steer away from unstable equilibria?*

While that question is addressed below, some short cuts are made. For one, we assume that output is perishable or not stored. Accordingly, all inputs are construed as variable, meaning that there is a rental market for durable production factors and no start-up/closure cost [8]. For another simplification, no account is given for the increased capacity of modern financial markets and various derivatives or insurance instruments to smooth producers' profits. Thus, in dealing with risk, we tend to underestimate the rationality or competence of private agents relative to that of regulators. Likewise, we do not inquire whether suitable design of taxes, say by averaging or deferring liabilities, might serve welfare and efficacy better than price policies. A running example, coming from agriculture, provides illustration. It captures that major inputs must be committed before uncertainty is unveiled. The setting thus resembles the one that dominates the large literature on cobwebs [16]. The standard cobweb story, couched in terms of *adaptive price expectations*, is framed here, in the form of *adaptive inputs*. Also the received cobweb story will, at places, be turned on its head. Specifically, suppose some agency *sets* prices. In doing so its challenge is to learn a stable and financially viable stipulation.

While agriculture always lurks in the background as a leading example, Section 2 spells out, rather generally, how an unregulated equilibrium may materialize in a decentralized scenario featuring scantly informed agents. Section 3 illustrates welfare impacts of deregulating the Norwegian grain market. Section 4 depicts the opposite transitions, from laissez-faire to a regulated market. In this case, there is a buffer agency concerned with fixing a stable price level. Such a body can hardly identify the appropriate level right away. To illustrate how it might eventually learn, a numerical example again revolves around the Norwegian grain sector.

Data that pertain to the running example are listed in Appendix 1. Since stochastic approximation theory is a main vehicle throughout, some chief results are briefly reviewed in Appendix 2.

2 Theoretical underpinnings

Consider henceforth a finite, fixed set I of producers, all supplying the same homogeneous, perfectly divisible goods to common markets. Everybody faces uncertainty about the state ω of the world. That state belongs to a finite list Ω of mutually exclusive, relevant outcomes. An exogenous, possibly unknown probability distribution governs the likelihood of diverse outcomes in Ω .

Uncertainty has, in each period, the temporal aspect that producers must commit inputs before ω is unveiled. In contrast, consumers make their purchases under perfect information, after uncertainty has resolved. Across periods, labelled t = 0, 1, ..., the state ω^t is sampled time and again. We shall, in the main, assume that $\omega^0, \omega^1, ...,$ be independent and identically distributed. Only in passing do we allude to generalizations, of which there are many.

2.1 Spot market equilibrium

Consider a representative period in which some state $\omega \in \Omega$ already came about. Agent *i* then enjoys state-dependent utility $u_i[\pi_i, \omega]$ of his realized profit

$$\pi_i(x_i,\omega) := p(\omega) \cdot q_i(x_i,\omega) - c_i(x_i,\omega).$$
(1)

 $p(\omega)$ denotes the contingent price vector that clears markets in state ω . If G is a finite set of produced goods, then plainly, $p(\omega) = [p_g(\omega)] \in \mathbb{R}^G_+$. The quantity vector $q_i(x_i, \omega) = [q_{ig}(x_i, \omega)] \in \mathbb{R}^G_+$ records the output bundle supplied by agent i who *ex ante*, in front of uncertainty, used input x_i , and *ex post* faces state ω . Decisions made after uncertainty resolves, if any, have already been incorporated. $c_i(x_i, \omega)$ is production cost. All inputs are variable. Therefore, in each period, before ω is unveiled, a well informed, competent, foresighted decision maker would

maximize $Eu_i[\pi_i(x_i, \omega), \omega]$ with respect to own factor input $x_i \in X_i$.

E denotes the expectation operator with respect to ω . The decision variable x_i must reside in prescribed feasible set X_i , assumed nonempty compact convex.² Note that costs, preferences, technologies, and risk exposure may differ across agents. Uncertainty is, however, generated by the same probabilistic and exogenous mechanism.

To close the model we must specify how prices are formed. On this account posit that a "demand curve"

$$Q \in \mathbb{R}^G_+ \mapsto P(Q, \omega) \in \mathbb{R}^G_+$$

tells the price vector at which consumers are willing to purchase the aggregate commodity bundle

$$Q := \sum_{i \in I} q_i$$

in state ω . As customary, $P(\cdot, \omega)$ is the inverse of the state-contingent demand $D(\cdot, \omega)$.

Note that the economy, just depicted, is stationary in structure. Accordingly, if ω has a time-invariant distribution, x_i should not vary. That is, in steady state only outcomes, not inputs would fluctuate together with ω .

Also note that the economy is very much reduced, totally focused on production, and organized around interrelated spot markets. Indeed, consumers enter, in various markets, simply and merely via their aggregate demand schedules. Intuitively, the wait-and-see aspect of buyers' behavior gives them an informational advantage over suppliers. Buyers' opportunism allows them full adaptation to ups and downs in supply.

 $^{{}^{2}}X_{i}$ accounts for all constraints imposed on manifold decision variables, including nonnegativity or upper bounds. Various restrictions could reflect limits to acreage and capacity. These are not spelled out here.

Definition 1. (Unregulated, spot market equilibrium) An input profile $x = (x_i)$ is said to support a spot market equilibrium if for all i

$$x_i \in \arg \max Eu_i [\pi_i(\cdot, \omega), \omega],$$
 (2)

and, at the same time, rational price expectations prevail ex ante in that

$$p(\omega) = P(\sum_{i \in I} q_i(x_i, \omega), \omega) \text{ for all } \omega \in \Omega.$$
(3)

Stochastic elements feature prominently in (2) and (3). Their importance depends on whether and where they enter non-linearly.³ Plainly, cost $c_i(\cdot, \omega)$ and production $q_i(\cdot, \omega)$ might vary at the margin. But even if they don't, given elastic demand, revenue $P(Q(x, \omega), \omega) \cdot q_i(x_i, \omega)$ becomes non-linear. Further, even if u_i lacks significant curvature, a progressive tax would make producer's net income concave. So, following Colman [6], we remark, in passing, that if risk aversion impedes efficiency, then smoothing of taxes might compete well against price stabilization.

At any rate, one naturally wonders how spot market equilibrium might come about. We explore that issue next.

2.2 How can an unregulated equilibrium be reached?

To deal with this question assume henceforth that each function $u_i [\pi_i(x_i, \omega), \omega]$ be concave and smooth with respect to x_i . Further let

$$M_i(x,\omega) := \frac{\partial}{\partial x_i} u_i \left[\pi_i(x_i,\omega), \omega \right]$$

denote the corresponding marginal utility of producer i, as realized and perceived in state ω . Clearly, when he uses several inputs, the gradient $M_i(x, \omega)$ has just as many components. If agent i comes forward as a price-taker,⁴ and if all functions

$$u_i[\cdot,\omega], \quad q_i(\cdot,\omega), \quad c_i(\cdot,\omega)$$

$$\sum_{g \in G} \left[p_g(\omega) \frac{\partial}{\partial x_i} q_{ig}(x_i, \omega) + \left\{ \frac{\partial}{\partial x_i} P_g(Q, \omega) \right\} q_{ig}(x_i, \omega) \right].$$

³Absent non-linearities in ω , uncertainty wouldn't affect the optimal x_i .

⁴Nothing in our set-up precludes though that i be a strategist, enjoying some market power. If indeed he does, the sum in (4) should be replaced by

are smooth for each $\omega\in\Omega$, then

$$M_i(x,\omega) = \frac{\partial}{\partial \pi_i} u_i \left[\pi_i(x_i,\omega), \omega \right] \left\{ \sum_{g \in G} p_g(\omega) \frac{\partial}{\partial x_i} q_{ig}(x_i,\omega) - \frac{\partial}{\partial x_i} c_i(x_i,\omega) \right\}.$$
(4)

An unregulated, spot market equilibrium is characterized by satisfaction of (2) and (3). Equivalently, when (3) is in force, it should hold for each agent i that

$$x_i = Proj_i \{x_i + sEM_i(x, \omega)\} \quad \text{for all } s > 0.$$
(5)

 $Proj_i$ denotes the orthogonal projection onto the closest approximation in the constraint set X_i . This operation, enforces feasibility. Optimality condition (5) is necessary and sufficient for x_i satisfying (2).

Our hope is that agents reach equilibrium via some adaptive and plausible procedure. To that end system (5), which depicts x_i as a stationary point, immediately invites a corresponding dynamic version. Specifically, consider iterative updating as follows:

$$x_i^{t+1} := \operatorname{Proj}_i \left\{ x_i^t + s_t M_i(x^t, \omega^t) \right\} \quad \text{for all} \quad i.$$
(6)

The parameter s in (5) could be any positive number just serving to scale $EM_i(x, \omega)$. By contrast, its time dependent counterpart s_t in (6) plays the role of a step size. Having decided on that size, the updating merely requires that each agent knows his last choice x_i and the corresponding marginal utility $M_i(x^t, \omega^t)$. More precisely, he is only presumed to make a step of length s_t along $M_i(x^t, \omega^t)$. If necessary, his move in that direction must be bent (projected) so as to preserve feasibility.

Diverse sorts of step sizes are applicable. It suffices that

$$\sum_{t=0}^{\infty} s_t = +\infty \quad \text{and} \quad \sum_{t=0}^{\infty} s_t^2 < \infty, \tag{7}$$

a possible choice being $s_t = \alpha/(1 + \beta t)$ with $\alpha, \beta > 0$. Procedure (6) might look complex but is indeed both simple and natural. It captures that marginal profit, regarded as direction of adaptation, points towards higher profit. This reasoning points to how agents might learn and adapt step by step. We restate the process next as a formal **algorithm** suppressing mention of time:

Start at time t := 0 with step-size $s := s_0$ and individual input choices $x_i \in X_i, i \in I$, determined by (historical or accidental) factors not discussed here.

Sample anew the state $\omega \in \Omega$ according to the fixed, exogenously given

distribution.⁵

Update individual supply (or observe that entity) for each *i* as a realization $\hat{q}_i = q_i(x_i, \omega)$.

Record aggregate supply $\hat{Q} := \sum_{i \in I} \hat{q}_i$ and corresponding realized price $\hat{p} := P(\hat{Q}, \omega)$.

Update individual inputs by the rule

$$x_i \longleftarrow Proj_i \{x_i + sM_i(x,\omega)\}$$
 for all i (8)

in which M_i has been recalculated to account for the newly observed price. Increase time: $t \leftarrow t + 1$ and modify the step-size $s \leftarrow s_t$. Continue to Sample until convergence.

Note that the process is driven by the producers, by private information and by the quest for profit improvement. Also note that price prediction is not an issue. Instead the chief question is whether iterated adjustments of inputs will bring about stability or not? Does the resulting trajectory (x^t) converge? If so, will the limit entail spot market equilibrium?

One hopes of course that (x^t) eventually clusters to a *deterministic limit*. To explore that issue let $T_i x_i := cl \{r(x'_i - x_i) : r \ge 0, x'_i \in X_i\}$ denote the *tangent cone* of X_i at a member point x_i ; see [23]. It is known from [25] that

$$\lim_{s \to 0^+} \frac{Proj_i \{x_i + sd_i\} - x_i}{s} = Proj_{T_i x_i} \{d_i\}$$

for any direction d_i . Therefore (8), provided s_t is small, can be seen as a discrete-time, stochastic Euler step of a corresponding continuous-time, deterministic, differential system, namely:

$$\dot{x}_i = Proj_{T_ixi} EM_i(x,\omega) \text{ for all } i.$$
(9)

It is a major insight of stochastic approximation theory [2], [3] that (8) and (9) tend to have the same limits.

Proposition 1. (Attaining equilibrium with spot markets) Suppose the limit set \mathcal{L} of the differential equation (9) is finite. Then process (8) converges with probability one to equilibrium. \Box

 $^{^5 \}rm More$ generally, a stationary Markov chain on Ω might be used. In that case probabilities are conditioned on the current state.

2.3 Measuring the welfare effects

Transition costs being important, we need to record welfare along any trajectory. Therefore this section ends with a procedure for computing consumers' and producers' surplus.

Start at time t := 0 with step-size $s := s_0$, initial consumer/producer surplus CS := 0, PS := 0, and discount factor d := 1. Update x_i by (8) and surpluses by the rule

consumer surplus
$$CS \leftarrow CS + d \int_{p(\omega)}^{+\infty} D(p,\omega) dp$$

producer surplus $PS \leftarrow PS + d \sum_{i \in I} \pi_i(x_i,\omega)$ $\left. \right\}$ (10)

The items q_i , Q and $p(\omega)$ are computed as in (8). Discount factors are updated using $d \leftarrow 1/(1+r)$. \Box

CS and PS will, even in the limit, depend on the trajectory $\omega^0, \omega^1, \omega^2, ...,$ with its early part playing a dominant role. In short, welfare will exhibit path dependence, and the transient component become important.

3 Deregulating a Grain Market

The preceding section described how producers might adapt when prices are deregulated. This section provides an illustration, using a simple model of the Norwegian grain market where the price is fixed at a high level. Foreign competition is fenced off by import tariffs. Market balance is ensured via import and export.

Our interest revolves around welfare effects when domestic farmers need time to approach a new equilibrium. Clearly, total welfare would be best served by an immediate transition to a new equilibrium. Belated or slow adaptations cause a loss.

In a hypothetical experiment we let the domestic grain price move freely but maintain prohibitive import tariffs. We first locate *all* spot market equilibria by global search. These may be several, and some are unstable. Then, we let process (8) loose, starting out from a fixed-price, steady-state regime. Convergence always obtains and transition costs are easily recorded. Details are given next.

3.1 A stylized model of the Norwegian grain market

Suppose farmers repeatedly allocate land either to grain production or to alternative purposes. While grain production is risky, the alternative is not.

For simplicity, posit that production planning concentrates on merely one input, namely land use. Thus, in each period the profit π_i of farmer *i* in (1) depends on his choice *ex ante* of land use in grain production x_i and the *ex post* realized state ω :

$$\pi_i(x_i,\omega) := p(\omega)y_i(\omega)x_i + p^A(\bar{x}_i - x_i) - c_i x_i.$$

Here \bar{x}_i is total acreage and $x_i \in X_i := [0, \bar{x}_i]$. The grain price $p(\omega)$ depends on ω whereas the price of alternative production p^A is assumed state independent. The yield per unit acreage in grain production is $y_i(\omega)$ while in alternative production it is normalized to 1. Finally, marginal cost c_i in grain production is assumed constant. The marginal costs in the best alternative have already been subtracted from the price, so p^A expresses the net contribution. For simplicity, let farmers be identical. Therefore subscript i is omitted from here on.

3.1.1 Spot markets equilibria: existence and stability

Consumers are assumed risk neutral when it comes to food prices, but farmers are risk averse. The farmer's utility of profit is taken to be either quadratic or exponential, featuring parameters calibrated so as to replicate the fixed-price equilibrium shown in the first column of Table 2. All parameters are spelled out in Appendix 1.

All spot market equilibria are first found by global search as follows: Any tentative x generates a probability distribution Pr(x) over contingent prices.⁶ Farmers next base their best input response $\hat{x} := B(Pr(x))$ on the resulting distribution. Equilibrium prevails when $\hat{x} = x$.

Figure 1 illustrates this procedure. The vertical axis measures the area x put into grain production, the total area being normalized to 1. Farmers' best response \hat{x} is reported along the horizontal axis. Table 1 brings out the three cases, depicted in Figure 1. *Case a* assumes that demand is linear and that farmers utility are quadratic. *Case b* invokes an exponential demand but maintains quadratic utility. Finally, *case c* assumes both demand and utility exponential.

⁶A proposal x generates contingent supply $q_i(x,\omega) = y(\omega)x$ from farmer *i* and state contingent market clearing prices $p(\omega) = P(\sum_i q_i(x,\omega),\omega)$.

Case	Utility	Demand	Revenue
a	quadratic	linear	concave
b	quadratic	exponential	concave if $\varepsilon \geq 1$
с	exponential	exponential	concave if $\varepsilon \geq 1$

Table 1: Basic assumptions

In Figure 1, case a, x = 0.10 generates best response $\hat{x} = 0.18$ (point A₁). Increasing x to 0.11 yields a response slightly above 0.18. Continuing so on until $x = \hat{x}$, equilibrium prevails in points A₂, A₃ and A₄. Among these A₂ and A₄ are stable; A₃ is not.⁷ In case b, the only equilibrium stable is B₂. Case c features a unique and stable equilibrium C.

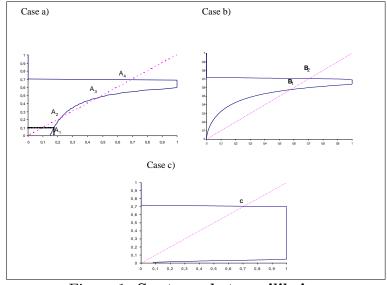


Figure 1: Spot market equilibria

The last three columns of Table 2 report quantitative results for selected spot market equilibria (A_4 , B_2 and C, respectively). Interestingly, farmers

⁷Stability obtains if and only if the response curve crosses the line $\hat{x} = x$ from below.

	Fixed price	Spot m equilibri		
		Case a	Case b	Case c
Land use grain (share)	0.6900	0.6940	0.7054	0.7081
Expected price (NOK per kg.)	2.000	1.979	2.020	2.007
Expected output (10^6 kg.)	1295.0	1303.0	1324.5	1329.5

plant more under the spot market regime. This phenomenon will be explained in Section 3.2.2.

Table 2: Stable rational expectations equilibria

3.1.2 Paths from fixed price to spot market equilibria

Having identified spot market equilibria, we next inquire how they can be reached. Figure 2 depicts paths to equilibrium, simulated over 100 periods, in each of which a new ω is sampled uniformly and independently among the 10 yields listed in Appendix 1. The experiment is repeated 100 times and the paths in Figure 2 are computed as averages. In all instances convergence obtains to limits that comply with the first line of Table 2. Noteworthy is the speed of convergence when demand is exponential. Approximate equilibria are then reached after about 3-4 periods. Also noteworthy, albeit expected, is that unstable equilibria are never reached. On the other hand, there is path dependence: final equilibrium depends on the initial point. In our experiments that point of departure is the fixed-price equilibrium.

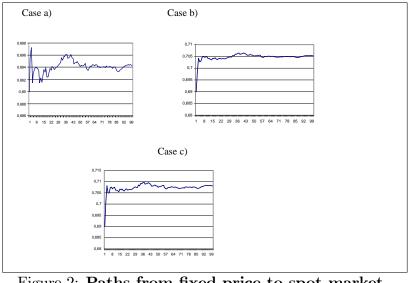


Figure 2: Paths from fixed price to spot market equilibrium

3.2 Welfare effects of deregulation

What are the welfare effects of allowing prices to float freely? To elucidate that question we experiment with three different assumptions as to how farmers respond to deregulation. First, and in extremis, we follow the traditional approach by Waugh (1944), Oi (1961) and Massell (1970) by assuming that deregulation doesn't induce farmers to change their factor inputs. Next, we suppose that farmers, after deregulation, switch instantaneously to the attended spot market equilibrium. Finally, and for more realism, we allow them time to reach equilibrium as in Figure 2.

We invoke the same draws of yields as in the preceding subsection. For each of the 100 years we compute the change in various measures, e.g. consumer and producer surplus, compared to the fixed price equilibrium. Also recorded are estimates of expected profit and price - together with standard deviations. The change in agents' welfare is discounted, using an interest rate of 4 %.

3.2.1 No input response

In the Waugh-Oi-Massell studies demand curves are linear. Therefore, it comes as no surprise that our results in case a fit nicely with theirs. Upon

moving from fixed to floating price, consumers' (producers') surplus increases (decreases). Furthermore, producers' loss dominates consumers' gain, hence a fixed price yields the highest total economic surplus.

The above conclusions are informative for risk neutral farmers. For risk averters some caution is appropriate. While the expected profit decreases in case a (-4.7 %), so does the standard deviation of profit (-51.3 %); see Table 3. Therefore, to assess producers' gain from reduced uncertainty, we calculate the corresponding certainty equivalent. Welfare considerations now favor the spot market regime, thus reversing the conclusion of Waugh-Oi-Massell.

Change in consumer surplus Change in producer surplus Change in certainty equivalent	Case <i>a</i> 1042.3 -1972.3 -657.5	Case b -1692.5 609.7 1300.0	Case c -1692.5 609.7 1527.5
Change in profit (%) Change in st. dev. of profit (%)	-4.7 -51.3	1.7 -22.8	1.7 -22.8
Average spot price St. dev. spot price Average production spot market	$2.000 \\ 0.437 \\ 1295.0$	$2.095 \\ 0.563 \\ 1295.0$	$2.095 \\ 0.563 \\ 1295.0$

Table 3: No input response

With constant-elasticity demand (cases b and c), the effects on consumers and producers are reversed as compared to the linear case. Also, the sum of producer and consumer surplus is negative, thus favoring the fixed price regime. This result applies even if the certainty equivalent is used as welfare measure for the producers.⁸ The explanation for these results goes as follows: According to Newbery and Stiglitz (1981), producers lose from floating prices if the revenue function is concave in quantity.⁹ Reflecting on this, the properties of the revenue function in the various cases are reviewed in the last column of Table 1. With linear demand, the revenue function is indeed concave in quantity, this explaining producers' loss in case a. With constantelasticity demand, the curvature of revenues depends on the price elasticity,

⁸Cases b and c give identical results with the exception of the certainty equivalent. The reason is that the two cases have identical demand but different utility functions.

⁹Effects on costs are ignored.

the threshold 1 giving a shift from convexity to concavity. An elasticity of $\varepsilon = 0.6$, as is assumed in our cases, means a convex revenue function. Thus, producers gain from floating prices.

3.2.2 Immediate transition

Newbery and Stiglitz (1981) earlier stressed that one can hardly defend the Waugh-Oi-Massell assumption that producers won't modify factor inputs. In response, this section examine a simulation, allowing farmers to adapt not only fully but also immediately.

The results of this simulation show that farmers plant more under the spot market regime. The main explanation, opposing widespread beliefs, is that profit stabilizes because price and yield fluctuate inversely (see Table 3). Thus, other things equal, risk averse farmers who are concerned not with price uncertainty per se, but more prudently with *profit uncertainty*, will plant more. This result hinges critically on the price elasticity, which in all our cases equals 0.6. Generally, an elasticity lower than 0.5 generates larger profit variation.¹⁰

For case a, it would be intuitive to expect grain to become less attractive after deregulation since the certainty equivalent (which balances the preferences for risk versus return) declines. Less planting is then expected, not more as Table 2 tells. However, the quadratic utility function, employed in cases a and b, displays increasing absolute risk aversion. Consequently, lower return means higher risk tolerance, making for a higher share in the risky alternative.

Generally, planting more grain means higher average production and lower prices. This benefits consumers and hurts producer. Table 4 reports the results for case c.

¹⁰See Newbery and Stiglitz (1981), pp.26-27.

	Wh	When input response is			
	None	Immediate	Delayed		
Change in consumer surplus	-1692.5	1160.6	837.4		
Change in producer surplus	609.7	-2171.3	-1869.2		
Change in certainty equivalent	1527.5	-1177.5	-1090.0		
Change in profit $(\%)$	1.7	-5.0	-4.7		
Change in st. dev. of profit $(\%)$	-22.8	-24.2	22.1		
Average spot price	2.095	2.007	2.011		
Standard dev. spot price	0.563	0.540	0.544		
Average prod. spot market	1295.0	1329.5	1328.0		

Table 4: From fixed to floating price. Case c

For consumers the beneficial effect of lower prices outweighs the otherwise negative impact of price variation observed under the no response assumption. For the producers it's the other way around; they lose from spot prices. Welfare effects are qualitatively the same as under the no response assumption. That is, the sum of consumers' and producers' surplus (measured by the certainty equivalent) is highest under fixed prices.

3.2.3 Delayed transition and transition costs

Next we consider welfare effects when, more realistically, it takes time to reach a new equilibrium. Using the same 100 draws described above, we now compute results when the trajectory for land use is identical to the one depicted in Figure 2.¹¹ The last column of Table 4 reports the effects.

The average price is higher during the transition period than in the equilibrium; see Table 4. The reason is of course that it takes time to raise the production to the new equilibrium level. Consequently, the transition period affects the consumers negatively with a loss of 323.2 (837.4-1160.6). The producers are positively affected with a gain of 87.5 (-1090.0+1177.5). The economic loss caused by the transition period is then 235.7. This figure is minor though, being merely 0.3 % of the production value.¹² The losses can be interpreted as the cost of learning.

¹¹Naturally, we use the individual trajectories behind the curves in Figure 2.

 $^{^{12}{\}rm The}$ economic loss for the cases a and b (not reported) is 0.15 % and 0.25 %, respectively.

If the transition is immediate the sum of consumers' and producers' surplus (measured by the certainty equivalent) is highest when the price is fixed. The cost of learning strengthens the case for maintaining the fixed price.

4 Price Stabilization

Stability in primary commodity markets are of utmost topicality among policy makers and economists.¹³ We simply suppose that some agency hereafter be assigned the task to stabilize prices. Its instruments include inventory, market intervention and export/import. Among these, for simplicity, we exclude the first, implying absence of storage.

The agency's objective, and its legitimacy, is then to smoothen allocations across contingencies, time, parties, production lines, and regions. Whatever be the precise measures taken, it faces the financial constraint of maintaining a balance over the long-run. Assuming ergodicity we posit that time averages coincide with sample means. Therefore the *excess demand*

$$\mathcal{E}(p,\omega) := D(p,\omega) - S(p,\omega) \tag{11}$$

must satisfy $E\mathcal{E}(p,\omega) = 0$. To achieve such balance the agency must, of course, learn a suitable mode of operation. We shall briefly return to that issue. For now, and for the definition that follows, simply suppose the body at hand has operated long enough to already have learned its best business. Specifically, suppose it has identified a constant price that makes expected excess demand equal zero.

Definition 2. (Fixed price equilibrium) A price p is said to constitute a *fixed price equilibrium* if the regime

 $x_i \in \arg \max Eu_i[\pi_i(\cdot, \omega), \omega] \quad s.t. \ p(\omega) = p \ for \ all \ \omega \in \Omega,$

yields $ED(p, \omega) = ES(p, \omega)$; that is, expected excess demand is nil.

¹³Colman [6] emphasizes that a "clear distinction must be drawn between stabilization as a by-product of income support, and stabilization with no intended transfers over the long-run." Price stabilization may also affect competitiveness [15]. We shall elaborate none of these issues.

4.1 Stabilized price equilibrium

Instead of "just" solving the equation $E\mathcal{E}(p,\omega) = 0$, let, more generally, $h(\omega, \cdot) : \mathbb{R}^G \to \mathbb{R}^G$ be a convex function that vanishes only at the origin. Suppose moreover that the operative cost of the agency is given by $h(\omega, \mathcal{E}(p, \omega))$. The function h may well reflect cost when acquisition of an extra amount (import) is compared to release of surplus (export). The agency presumably seeks to

minimize
$$Eh(\omega, \mathcal{E}(p, \omega)),$$
 (12)

the instance $h(\omega, \cdot) := \|\cdot\|^2$ being one possible choice.

We find it tempting, in this section, to assign a more general meaning to p. This item is now construed as a *parameter vector*, affecting the realized prices of the commodities at hand. For instance, p might embody acquisition and release prices (also called price floors and ceilings) for any item. We shall write

$$\pi_i(x_i, p, \omega) := P(\omega, p) \cdot q_i(x_i, \omega) - c_i(x_i, \omega).$$

to stress that individual profit, and notably the realized price $P(\omega, p)$, depends on the regulated parameter vector p, the simplest instance being $P(\omega, p) = p$.

Definition 3. (Stabilized equilibrium). A vector $p \in R^G_+$ is now said to be a stabilized equilibrium price iff

$$x_i \in \arg \max Eu_i[\pi_i(\cdot, p, \omega), \omega]$$
 for all $i, p \in \arg \min Eh(\omega, \mathcal{E}(\omega, \cdot))$

where $S(p,\omega) := \sum_{i \in I} q_i(x_i,\omega)$ and expected excess demand is nil.

4.2 How can stabilized equilibrium be reached?

To hit this sort of equilibrium in one shot would demand considerable information - and much competence. Indeed, as formulated, it presumes knowledge of functions $S = \sum_{i \in I} q_i$, D and the underlying probability distribution. One can hardly suppose all such information be readily available. So, we shall pursue an extreme, opposite tack, namely: suppose the agency knows neither. Indeed, suppose it must contend with sequential observation of *realized* supply and demand. But then, how can it eventually learn to stabilize p? Classical micro-economics immediately suggests a procedure, kin to Walrasian tatônnement. Let now

$$M_i(x, p, \omega) := \frac{\partial}{\partial x_i} u_i \left[P(\omega, p) \cdot q_i(x_i, \omega) - c_i(x_i, \omega) \right]$$

denote the marginal utility of agent i when promised "price" p, and consider the following process:

Price stabilization undertaken iteratively by a buffer agency:

Start at time t := 0 with $s := s_0$ and a reasonably informed guess p. Suppose the producers have already committed inputs $x_i, i \in I$. Sample the state ω . Initiate consumer and producer surplus at CS := 0 and PS := 0, respectively. Posit the discount factor d := 1.

Update surpluses, parameters and inputs as follows:

Increase time: $t \leftarrow t+1$, modify the step-size: $s \leftarrow s_t$, and update the discount factor $d \leftarrow d/(1+r)$.

Sample a new state ω independently from the given distribution. **Continue** to **Update** until convergence. \Box

In (13) $Proj_+$ denotes projection onto the non-negative price orthant \mathbb{R}^G_+ - or onto some bounded closed convex set \mathbb{P} to which p must belong. Note that (13) presumes no knowledge neither about supply/demand functions nor about the underlying probability distribution. It more modestly contends with agents being able to observe or calculate derivatives. Then, the process merely requires the possibility to observe the evolution of realized aggregate supply $\sum_{i \in I} q_i(x_i, \omega)$ and demand.

For the next assertion let $Proj_{T_+p}$ denote projection onto the corresponding tangent cone at $p \in \mathbb{R}^G_+$.

Proposition 2. (Convergence under no handling cost) Suppose the limit set \mathcal{L} of the coupled set of differential equations

$$\dot{p} = Proj_{T_{+}p} \left[-\frac{\partial}{\partial p} Eh(\omega, \mathcal{E}(\omega, p)) \right]$$

$$\dot{x}_{i} = Proj_{T_{i}x_{i}} EM_{i}(x, p, \omega) \text{ for all } i.$$

is finite. Then, provided price parameter p remains bounded, under (7) process (13) converges with probability one to a stabilized equilibrium.

4.3 Stabilizing a grain market

The stabilizing procedure just described is demonstrated next by examples. For the argument, let the market initially reside in unregulated spot equilibrium as explained in Subsection 3.1.1. Further, the authorities opt to stabilize the price, giving some agency this task. The agency observes excess demand, and it keeps on adjusting the price until a stationary equilibrium comes up.

4.3.1 Stabilized price equilibria

As in Subsection 3.2 we first identify all fixed-price, rational-expectations equilibria.

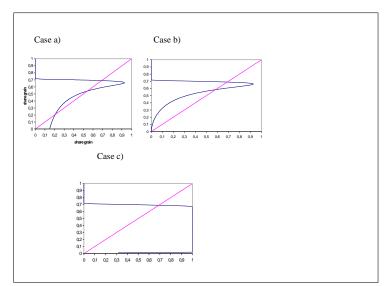


Figure 3: Fixed price equilibria

Figure 3 displays the same features as seen above in the spot markets. The number of equilibria remains the same, but deviations from the 45^0 line are smaller. Observe that the fixed-price, rational-expectation equilibrium reported in Table 2 (share 0.69) remains stable in all cases.

4.3.2 Paths from spot market to stabilized price equilibrium

Suppose we initially are in a spot market equilibrium, given as one of the last three columns of Table 2. From there on stabilization is not immediate; see the last two equations of (13). Figure 4 illustrates some evolutions. In each case convergence obtains, but compared to Figure 2, it takes longer time. A most plausible reason is that learning is more complex now, involving two sorts of scantly informed agents, namely farmers and the agency. Observe in particular that the price path approaches equilibrium from above.

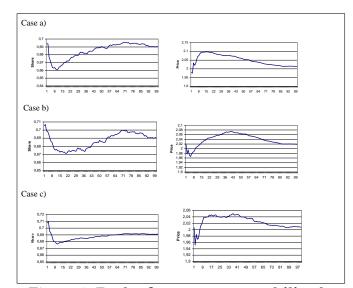


Figure 4: Paths from spot to stabilized price equilibrium - fast learning

The intuition behind this phenomenon is that producers are fast learners, employing large step sizes. This means that they quickly lower the share of land used for grain production. Doing so generates excess demand and a price hike. Consequently, the price path approaches equilibrium from above. In contrast, if producers are slow adapters/learners, the path converges from below, as shown in Figure 5.

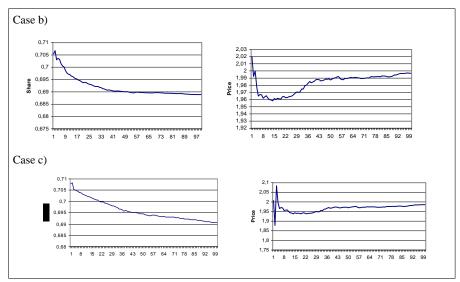


Figure 5: Paths from spot to stabilized price equilibrium - slow learning

4.3.3 Welfare effects of stabilization - transition costs

Welfare effects for case c is given in Table 5. The various measures are now given as the difference between the results in the fixed and the floating price regime. The first column reiterates the results for immediate transition (with the opposite sign) found in Table 4. The last two columns report the more realistic, time consuming instances, as given by the trajectories for case c in the Figures 4-5.

	Immediate transition	Delayed transition		
		Fast learning	Slow	
Change in consumer surplus	-1160.6	-1790.9	315.4	
Change in producer surplus	2171.3	2801.0	540.9	
Change in certainty equivalent	1177.5	1527.5	-410.6	
Change in profit $(\%)$	5.0	7.1	2.6	
Change in st. dev. of profit %)	24.2	47.8	45.4	
Average price	2.000	2.025	1.968	
St. dev. price	0.000	0.117	0.112	
St. dev. spot price	0.540	0.540	0.540	
Average production stab. price	1295	1291.7	1306.0	
Average demand stab. price	1295	1288.2	1309.5	

Table 5: From floating to a fixed price. Case c

On average, when learning is fast, prices stay high. The transition period therefore affects the consumers negatively, with a loss in surplus of 630.3 (1790.39-1160.6). In contrast, the producers are positively affected with a gain of 350.0 (1527.5-1177.5). The economic loss caused by the transition period is 280.3. As in Section 3.2.3 this loss is minor, amounting to about 0.32% of the production value. If learning is slow, the distribution effects go in the opposite direction: consumers gain (1476.0) while producers loose (1588.1), summing up to 112.1.

Because the price is markedly off equilibrium during the transition period, there are sizable distributional effects. For example, consumers will lose considerably under fast learning as compared to the slow learning case. Naturally, for producers it is the other way around.

When computing the economic loss we have abstracted from the cost of operating the buffer agency. We are therefore underestimating the economic loss.

Comparing long run equilibria, ignoring transition costs, the fixed price case comes out best. This finding provides some partial justification of the regime in place. It does however, not tell that a deregulated market, if already operative, had better be abandoned. Indeed, our numerical exercise indicates that, when transition costs are incorporated, a deregulated market merits to be maintained.

5 Conclusion

We have considered some recurrent issues in dynamic adjustment to market equilibrium, as a tool for evaluating the merits of stabilized versus floating prices. The facts that adaptation is stepwise, learning comes by degrees, and randomness persists, makes analysis rather intricate. In particular, closed from solutions are hard to come by. The numerous difficulties not withstanding, we think that stochastic approximation offers a particularly useful and tractable toolbox, able to handle quite intricate situations.

That box frees one from the straight-jacket of equilibrium analysis. Also, it relieves agents from having rational expectations, divine competence, and perfect information. Instead it accommodates stochastic dynamics, agents who steadily grope for improvement, and planners considerate (or decent) enough to be concerned with transition costs.

For the purpose of illustration we have singled out a domestic grain market. That instance, albeit highly stylized and special, serves well to emphasize the importance of caution and detail. It stresses that one should identify precisely where and how the setting at hand is affected by non-linearity and randomness. It also brings out that this is a field in which general or sweeping results might be few and far between.

Most applications of stochastic approximation, say in game theory [12], [13] or macroeconomics [11], deal with *predictions* of crucial parameters. Rather few studies have considered stepwise *adaptation* of chief factor inputs. This paper shows that relatives of Smith's invisible hand may work well not merely in the market place, but in production planning as well. By doing so it invites studies of industries where capacity choice remains an intricate and recurrent issue. And finally, in case the environment is non-stationary, it offers a perspective on how agents might move in short or long term.

Appendix 1: Data

Demand functions: Two forms are used, either linear P(Q) := a - bQ, featuring constants a and b, or constant elasticity: $P(Q) := KQ^{-\varepsilon}$, where K is a constant, and ε denotes the price elasticity. We posit

$$a = 5.333, b = 0.333, K = 1962.9, \varepsilon = -0.6.$$

Utility functions are state independent. Further posit that they be either quadratic: $u_i(\pi_i) := \pi_i - C\pi_i^2$ with C reflecting risk aversion, or exponential:

 $u_i(\pi_i) := -e^{-A\pi_i}$, A being then the absolute risk aversion. We posit

 $A = 0.001, \quad C = 0.0002.$

Marginal costs (in $10^3 NOK$ /hect.) are for:

grain production: 41.1, land rent: 30.

Total acreage for grain, 10^6 hect.	\bar{x}	.345
Producer price of grain, NOK/Kilo	p^{Base}	2.
Consumption (average) 10^6 Kilo		1295.

Table A.1: Data for the Norwegian grain sector as of year (2000) [4].

Crop yields estimated from data during $[1990 - 1999]$:										
y	4396	4069	2815	3927	2840	3626	4026	3876	4084	3875
Table A.2: Yield kg./hectare,uniform probabilities [?], [?].										

Appendix 2: Stochastic Approximation

This section collects some technicalities that have served us time and again. For full exposition see [2], [3]. We repeatedly consider some stochastic process z^t constrained to evolve within a non-empty closed convex subset Z of some ambient finite-dimensional (Euclidean) vector space Z.

The key issue is always whether z^t converges in discrete time t = 0, 1, ... to a stationary point. The dynamic is stochastic, at time affected by the upcoming state ω^t . The latter belongs to a list Ω , comprising all relevant, mutually exclusive outcomes. For simplicity assume Ω finite. Also for simplicity, posit that $\omega^0, \omega^1, ...$ be independent and all distributed according to a (possibly unknown) probability measure μ over Ω . That measure generates an expectation operator denoted E. The state evolves as follows:

$$z^{t+1} := \operatorname{Proj}_{Z} \left\{ z^{t} + s_{t} F(z^{t}, \omega^{t}) \right\}$$

$$(14)$$

Here the operator $Proj_{\mathbb{Z}} \{\cdot\}$ denotes the *orthogonal projection* of any point in \mathbb{Z} onto its *closest approximation* in \mathbb{Z} . Thus, by construction, z^t always belongs to $\mathbb{Z} \subseteq \mathbb{Z}$. The function $F : \mathbb{Z} \times \Omega \to \mathbb{Z}$ is given a priori. The sequence $\{s_t\}$ of *step-sizes* $s_t > 0$ is specified at the outset subject to (7). Declare a point $z \in \mathbb{Z}$ stationary iff

$$z = Proj_Z \{ z + sEF(z, \omega) \} \text{ for all } s > 0.$$

We deem it highly desirable that $\{z^t\}$ converges to such a point. To inquire about such behavior, introduce the auxiliary function

$$f(z) := Proj_{Tz}EF(z,\omega)$$

Here $Proj_{Tz}$ denotes the orthogonal projection onto the *tangent cone* $Tz := Cl\mathbb{R}_+(Z-z)$ of Z at z. Assume that from any initial point $z^0 \in Z$ there emanates a unique, infinitely-extendable solution trajectory of differential equation

$$\frac{dz(\tau)}{d\tau} = f(z). \tag{15}$$

The crucial effect of the projection $Proj_{Tz}$ is to maintain $z(t) \in Z$ for all times $t \ge 0$. Now, invoking the results in [2] we have:

Theorem 1. (Asymptotic stability and convergence) Suppose that the limit set

 \mathcal{L} of all accumulation points of solution trajectories to (15), is finite.

Also suppose that process $\{z^t\}$ defined by (14) is bounded. Then, if

$$f(z) \cdot (z - \bar{z}) > 0$$
 for all zsufficiently close to $\bar{z} \in \mathcal{L}$, (16)

 $\{z^t\}$ converges with probability one to a stationary point. \Box

For spot market equilibrium $[z, Z, F] := [x := (x_i), X := \prod_i X_i, M := (M_i)]$, whereas for price stabilization [z, F] := [(x, p), (M, h)]. Arguing whether (16) is satisfied would expand the paper too much. Suffice it to say that linear demand causes no worries.

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