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OUTSOURCING IN CONTESTS

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Outsourcing in Contests*

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Abstract

We study ex post outsourcing of production in an imperfectly discriminating contest, interpreted here as a research tournament or a procurement contest for being awarded some production contract. We find that the possibility of outsourcing increases competition between the contestants, leading to higher total contest effort, unless there are very few contestants and/or the ex-post bargaining strength of the contest winner is sufficiently low. However, even in the case of two contestants, outsourcing reduces the procurement costs of inducing a given level of effort if the contest organizer can collect entry fees. With respect to contest design, this suggests that outsourcing should generally be allowed if the objective is to induce stronger competition.

Keywords: Contests; Outsourcing; Bargaining; Contest design

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1 Introduction

Horizontal outsourcing, where inputs or parts of the final production of a good are subcontracted to rival firms within the same industry, is a common phenomenon in many industries, such as automobile, electronics, computer and aircraft manufacturing, as well as marine insurance and architectural design.[1] For example, Alexander (1997) finds a large extent of post-award subcontracting between firms who were rivals in competition for 'prime contracts' from the U.S. defense department.

In the present paper, we ask how the possibility of such horizontal outsourcing is likely to affect competition between firms in markets where *non-price competition* plays an important role. Competition for big projects or large-scale production contracts are examples where the final allocation of the contract to a supplier not only depends on prices, but may crucially depend on different types of sunk effort undertaken by competing suppliers in order to influence the buyer's decision. For example, a potential supplier might spend effort on R&D in order to improve the quality of his product and tailoring it to match the buyer's needs and requirements, thereby increasing the probability of being awarded the contract. Firms may also spend considerable resources on lobbying – even direct bribes – in order to secure lucrative licences or contracts.[2]

We analyze a situation where the allocation of a fixed prize, interpreted as the gross value of a licence or contract for the supply of a certain good, is determined in an *imperfectly discriminating contest*, where a given number of potential suppliers exert sunk effort in order to increase the probability of being awarded the prize. In addition to the above mentioned examples, this model can also be interpreted as an R&D tournament, where firms spend resources on R&D in order to obtain a profitable patent, licence or production contract.[3] For example, Rogerson (1989) argues that this type of rent-seeking framework applies well to regulatory structures in U.S. defense procurement, where the 'prizes' of research contests held by the Department of Defense are

[1] See, e.g., Kamien et al. (1989), Spiegel (1993) and Chen et al. (2004).

[2] See, e.g., Konrad (2000) for a further discussion.

[3] See, e.g., Taylor (1995) and Fullerton and McAfee (1999).

profitable production contracts. By imposing cost asymmetries between the potential suppliers of the good we create incentives for ex post outsourcing of production, in case the contested prize is not awarded to the most cost-efficient firm.[4]

How is the possibility of ex post outsourcing likely to affect firms' choice of effort in the contest? In general, outsourcing tends to increase effort incentives for high-cost firms, due to reduced effective production costs, while the most efficient firm has reduced incentives, since this firm will expect to appropriate part of the contested prize through ex post subcontracts in any case. More specifically, we find that the possibility of outsourcing will increase total contest effort unless there are very few contestants and the ex post bargaining strength of the contest winner is sufficiently low. If there is free entry to the contest, outsourcing tends also to increase the number of active contestants. This result sheds some new light on the common view of horizontal subcontracting as a collusive device.[5] Collusion is not an issue in the present paper, but our results show that competition may actually increase from subcontracting between potential suppliers.

The effect of outsourcing on aggregate contest effort naturally direct the attention towards optimal contest design.[6] We argue that the decision to allow or disallow outsourcing or subcontracting is not only a potentially effective tool in contest design, it is also a tool that seems easier to support in practice than some of the other measures proposed in the literature.[7] In-

[4]A contest may be set up so as to avoid this mismatch problem, but it raises moral hazard concerns: Launching a design contest and then auctioning out the production licence can clearly lead to the winning design being produced by the lowest-cost firm. However, a firm may then have an incentive to design a project in such a way that it is very hard for the competitors to produce it. Furthermore, it may be of great importance to the producers to have unlimited access to the designers during the production phase, something which is not likely to be the case if a rival of the designer firm gets the production contract.

[5]In a standard oligopoly model of international trade, Chen et al. (2004) find that horizontal outsourcing has a collusive effect that could raise prices, while Alexander (1997) argues that subcontracts may help facilitate collusive bidding in prime contract auctions.

[6]Several suggestions for designing a contest to achieve a specific objective have been formulated in the contest literature, see, e.g., Baye et al. (1993), Fullerton and McAfee (1999), Gradstein and Konrad (1999), Amegashie (1999,2000), Clark and Riis (2000) and Szymanski and Valletti (2004).

[7]For instance, if the aim is to increase contest effort, there may be a case for discrimi-

deed, practices on these matters differ, and while allowing outsourcing would probably raise no eyebrows, neither would *not* accepting such schemes, since it can easily be argued (as it frequently is) that this is needed to prevent the diffusion of control and responsibility for the project in question.

If the aim of the contest designer is to maximize total effort, our results suggest – as indicated above – that outsourcing should be allowed in a majority of cases. A case to the contrary, though, is a situation with only two contestants, where outsourcing will *reduce* total effort if the contest winner has sufficiently low bargaining power in setting the terms of the subcontract. However, we show that this latter result is overturned if the contest administrator can also collect *entry fees*. For the case of two contestants, we find that outsourcing will always reduce procurement costs (i.e., the costs of inducing a given level of effort in the contest), which reinforces the policy recommendation of allowing outsourcing.[8]

From a viewpoint of social welfare, increased competition for winning a contested prize might not always be desirable, though. In situations where a considerable amount of contest effort is resources spent on lobbying or bribes, a social planner might want to introduce measures to reduce such competition. In order to capture this possibility, we also study the case where all contest effort is considered socially wasteful. In this case, social welfare is equivalent to aggregate profits. For the case of two contestants, we find that outsourcing is always socially desirable if the contest winner has sufficiently low bargaining power. Otherwise, outsourcing might reduce welfare if cost asymmetries are relatively large. Since our previously discussed results suggested that outsourcing generally leads to increased effort, one might expect that outsourcing should not be allowed in these cases if effort

nating against the high-ability/low-cost contestants, by limiting their possibilities to win the contest, in some cases excluding them from the contest altogether (see, e.g., Baye et al., 1993, and Che and Gale, 2003). The aim is to level the playing field and thus induce higher effort, but the approach is questionable when it comes to fair treatment in the more common sense of the term.

[8] Fullerton and McAfee (1999) consider a situation where the objective of the contest designer is to achieve a given level of effort at minimum cost, defined as prize money net of entry fees. They show that, for a large class of contests, the optimum number of contestants is two. Similar results are found by Taylor (1995) and Che and Gale (2003).

is wasteful. However, since high-cost firms may be awarded the project, outsourcing may increase aggregate firm profits even though rent-seeking expenditures increase.

To the best of our knowledge, ours is the only paper dealing with horizontal outsourcing in a contest framework. However, the present paper relates to several earlier contributions on horizontal outsourcing focusing on other modes of competition. Kamien et al. (1989) analyze how the possibility of ex post subcontracting affects the initial competition for a contract in a duopoly under price competition, where the incentive for outsourcing stems from strictly convex production costs. Particular attention is directed towards two polar cases, where either the winner or the loser of the initial contract dictates the terms of the subcontract, and the authors find that competition is higher in the former case. An equivalent result is derived in the present paper, although the framework is quite different.

Spiegel (1993) analyses a duopoly situation which is quite similar to Kamien et al. (1989), the important difference being that firms are assumed to compete in quantities rather than prices.[9] As in our model, but in contrast to Kamien et al. (1989), incentives for subcontracting arise from cost asymmetries. Spiegel (1993) finds that ex post outsourcing is more likely to increase social welfare if the subcontractor's share of the outsourcing surplus is relatively small. Unless contest effort is socially wasteful, this result is also reflected in the present analysis since, in our model, low bargaining strength for the subcontractor tends to increase competition.

Another related paper is Gale et al. (2000), who consider a sequential auction for multiple contracts with ex post subcontracting possibilities between the initial bidders. Once more, outsourcing incentives arise because of cost asymmetries. The authors find that the possibility of ex post outsourcing might make the sellers worse off ex ante. Although the framework is quite different from ours, this result reflects the situation with low bargaining strength for the subcontractor in our model, where we show that outsourcing might reduce ex ante aggregate profits.

[9] Chen et al. (2004) also study horizontal outsourcing under Cournot competition, but in a specific context of international trade.

The rest of the paper is organized as follows. In the next Section we present the benchmark model without outsourcing – a standard fixed-prize Tullock contest with asymmetric valuations. In Section 3 we introduce the possibility of ex post outsourcing under the assumption of price competition for the subcontract. In Section 4 we relax the assumption of price competition to consider alternative mechanisms for determining the price of the subcontract, including bilateral Nash bargaining and cooperative bargaining. In Section 5 we discuss contest design, while Section 6 concludes the paper.

2 A benchmark model

There are n firms participating in a contest for being awarded a contract for the supply of a good with a gross value of V. Alternatively, the set-up can be thought of as an R&D tournament, with n firms competing to obtain a patent, license, production contract or simply a technological lead, which generates a revenue of V. We assume initially that the firm that wins the contest must supply the good by producing it in-house. The firms are different with respect to cost efficiency in production, implying that their valuation of the contested prize also differ. The net valuation for firm i is given by

$$V_i = V - c_i, \quad i = 1, ..., n. \tag{1}$$

where c_i is firm i's cost of producing the good. We rank the contestants according to their net valuations, so that $c_i \leq c_{i+1}$, or $V_i \geq V_{i+1}$.[10]

The probability of being awarded the prize depends on the relative up-front efforts of the contestants. Applying a standard Tullock framework[11] the probability that firm i wins the contest is given by

$$P_i = \frac{x_i}{S_n}, \tag{2}$$

where x_i is firm i's effort (in monetary terms), while $S_n := \sum_{j=1}^{n} x_j$ is total

[10] Net valuations are assumed to be common knowledge.
[11] See Tullock (1980).

effort exerted by all active contestants.[12] Expected profits for firm i are thus given by

$$\pi_i = P_i V_i - x_i. \tag{3}$$

The first-order conditions for profit maximization define individual efforts as

$$x_i = S_n \left(1 - \frac{S_n}{V_i}\right). \tag{4}$$

Contest effort is monotonically increasing in net valuation for each player. In our setting, this means that low-cost firms exert more effort than high-cost firms in the contest. By summing over n and re-arranging (4), assuming that all n firms actively participate in the contest, we can derive total effort in equilibrium:

$$S_n = \left(\frac{n-1}{n}\right) \overline{V}_n, \tag{5}$$

where $\overline{V}_n := n \left(\sum_{j=1}^{n} \frac{1}{V_j}\right)^{-1}$ is the harmonic mean of the n firms' valuations of the contested prize.

It remains to ensure that all n agents will actually choose to participate in the contest.[13] Following Hillman and Riley (1989), we check whether firm $n+1$ has an incentive to contribute a positive amount of effort, given that the other n contestants expect that firm $n+1$ will not contribute. Since π_i is concave in x_i it suffices to evaluate firm $(n+1)$'s contribution incentives at $x_{n+1} = 0$:

$$\left.\frac{\partial \pi_{n+1}}{\partial x_{n+1}}\right|_{x_{n+1}=0} = \frac{V_{n+1}}{S_n} - 1. \tag{6}$$

We see that firm $n+1$ will only contribute if $V_{n+1} > S_n$. In the case of free entry of contestants, this also provides the condition for the maximum number of firms that will enter the contest. Using (6), the number of contestants

[12] This success function also arises from the more elaborate probability structure presented in Fullerton and McAfee (1999). Baye and Hoppe (2003) establish the strategic equivalence between the Tullock model and a variety of research tournaments and patent race games.

[13] In a perfectly symmetric contest, it is easily shown that all n firms will participate, and that free entry implies $n \to \infty$. Not so when the players' valuations of the contested prize differ.

in a free entry equilibrium is the n lowest-cost firms, where n is the lowest integer that satisfies the following condition:

$$V_{n+1} \leq S_n = \left(\frac{n-1}{n}\right) \overline{V}_n. \tag{7}$$

From (6) it also follows, as noted by Stein (2002), that total effort is always lower than the valuation of the active player with the lowest valuation of the prize.[14]

A parametric example

For later comparison, consider the following example. Suppose, like Hillman and Riley (1989) do, that net valuations are geometrically decreasing, such that $V_{i+1} = \alpha V_i$, $\alpha \leq 1$. The net valuation of firm i can then be characterized as

$$V_i = \alpha^{i-1} v, \quad \alpha \in (0,1), \quad v > 0. \tag{8}$$

Using this specification[15] in (8), total effort in the contest is given by

$$S_n = \frac{v(n-1)(1-\alpha)}{\alpha(\alpha^{-n}-1)}. \tag{9}$$

It is straightforward to verify that $\partial S_n / \partial n < 0$, which complies with the above-stated result that total contest effort is lower than the lowest valuation among the active players in the contest.

3 Outsourcing

Now we depart from the standard set-up of the previous section to allow for ex-post outsourcing of the awarded prize. More specifically, the winner of the contest can subcontract, or outsource, some or all of the post-contest production activities to one of its competitors. A realistic scenario would

[14] For a further discussion of equilibrium existence, see e.g., Fullerton and McAfee (1999).

[15] Equivalently, the production costs of firm i are given by $c_i = V - \alpha^{i-1} v$.

be that only parts of the total production is outsourced (e.g., production of some parts and components). Here, we assume – like Kamien et al. (1989) do – that the winner can outsource the entire production of the good. This assumption is only made for analytical clarity and does not qualitatively affect the main workings of the model.

In this model, incentives for ex post outsourcing arise from cost differences in production. Thus, unless the lowest-cost firm (i.e., firm 1) wins the contest, there will always be an incentive for ex-post outsourcing. The crucial questions are to which firm the subcontract is allocated and how the price of the subcontract is determined. As a starting point, we make the simple assumption that the losers in the contest engage in a price competition for being allotted the subcontract. In this case, the equilibrium price for the subcontract is (marginally below) c_2, and firm 1 produces the good in all cases.[16]

With this assumption, all except the lowest-cost firm maximize

$$\pi_i = P_i\left(V - c_2\right) - x_i, \quad i = 2, .., n. \tag{10}$$

Thus, the possibility of ex post outsourcing increases the incentives for high-cost firms to exert effort in the contest. This applies even to firms with very high costs, which would otherwise not have participated in the contest.

Firm 1 is different since it can produce with costs c_1, if winning the contest. However, this firm also receives a positive payoff if other firms win, since it then gets paid c_2 to produce the good for the winner. Expected payoffs for firm 1 are thus

$$\pi_1 = P_1\left(V - c_1\right) + \left(1 - P_1\right)\left(c_2 - c_1\right) - x_1,$$

[16] The typical contest discussed in this paper is characterised by 'design' and 'production' phases. At the outset, therefore, the contest designer (if any) is not able to use this type of auction mechanism for the ex post allocation of production. Furthermore, for the reasons discussed in Footnote 4, it may not be desirable (or feasible) to divide the contest into two separate design and production stages.

which can be re-arranged to

$$\pi_1 = P_1 (V - c_2) + (c_2 - c_1) - x_1. \tag{11}$$

Because of the positive payoff from others winning the contest, we see that firm 1 also behaves in the contest *as if* it had costs c_2. Accordingly, firm 1 has a lower incentive to exert effort to win the contest, compared with the benchmark case without the possibility of outsourcing.

Since all firms perceive their costs to be c_2, a *symmetric* equilibrium exists and is given by[17]

$$x_i = (\frac{n-1}{n^2})(V - c_2), \tag{12}$$

which yields equilibrium total contest effort

$$S_n = \left(\frac{n-1}{n}\right)(V - c_2). \tag{13}$$

We summarize the effect of outsourcing on contest effort incentives as follows:

Proposition 1 *The possibility of ex post outsourcing, with price competition for the subcontract, implies that all contestants exert the same level of effort in the contest.*

Thus, ex post outsourcing with price competition levels the playing field completely with respect to the contest, and cost differences between firms do not affect the probability of winning. Furthermore, with respect to expected profits, only the production costs of the two most cost-efficient firms matter. For firms $i \geq 3$, relative cost efficiencies are effectually irrelevant. This follows from the assumption that the price of a subcontract is determined by price competition.

How does the possibility of outsourcing affect total contest outlays? Let us first consider the case where the number of active firms, n, is constant. In

[17] The result that ex post outsourcing with price competition leads to a perfectly symmetric contest holds also for a more general contest success function of the form $P_i = \frac{x_i^r}{\sum_{j=1}^{n} x_j^r}$. Contrary to the benchmark case, closed form solutions for the equilibrium may then be obtained also for $r \neq 1$.

this case, the effect of outsourcing on total effort is determined by how much firm 1 reduces its effort relative to how much the other firms may increase their effort. This, in turn, depends on the number of contestants and the distribution of net valuations.

Proposition 2 *With n contestants, ex post outsourcing with price competition increases total contest effort if $V_2 > \overline{V}_n$.*

Proof. Follows from a trivial comparison of (5) and (13). ■

For $n = 2$, the net valuation of firm 2 must necessarily be lower than the harmonic mean of the two players' valuations. In this case, the net effect is simply that firm 1 has a lower incentive to win the contest (i.e., firm 1 behaves *as if* it has a lower net valuation), while the objective function of firm 2 remains constant.[18] Accordingly, total effort drops.[19] For $n > 2$, however, the effect on total outlays is a priori ambiguous, and determined by the condition given in Proposition 2. It is possible, though, to say something general about the effect of the number of contestants, n. By applying the entry condition in the benchmark contest without outsourcing, we see that $\overline{V}_n$ is decreasing in n. It follows that a larger number of contestants will increase the probability that outsourcing leads to higher total effort in the contest. This also makes intuitive sense, since a higher number of contestants implies that there are more high-cost firms that have increased incentives to exert effort in the contest due to the possibility of ex post outsourcing.

In the parametric example presented in the previous Section, total contest effort is given by

$$S_n = \left(\frac{n-1}{n}\right)\alpha v. \tag{14}$$

[18] With only two players, price competition for the subcontract is equivalent to one of the games considered by Kamien et al. (1989), where the terms of the subcontract is determined by the loser of the initial contract.

[19] From (5) we can easily derive the effect of a change in one player's valuation on total effort in the contest. This is given by

$$\frac{\partial S_n}{\partial V_i} = \frac{S_n^2}{(n-1)\,V_i^2} > 0.$$

A comparison of (9) and (14) confirms that the possibility of outsourcing leads to higher total contest outlays if

$$\frac{n(1-\alpha)}{\alpha^2(\alpha^{-n}-1)} < 1. \tag{15}$$

The left-hand side of (15) is increasing in n, implying that outsourcing leading to higher contest effort is more likely the larger the number of firms participating in the contest, as we would expect.[20] In fact, a closer scrutiny of (15) reveals that total effort is always higher in the outsourcing regime if $n \geq 3$. This also tallies well with the results of Stein (2002), who find that more similar valuations generally increase total effort in a Tullock contest.

Entry

The possibility of ex post outsourcing may not only affect contest incentives for a given number of firms, it may also greatly affect entry of new firms into the contest. From Proposition 1 we know that the effect of outsourcing with price competition is to transform an asymmetric contest into a symmetric one, which may trigger entry. With the assumption of free entry and that there are infinitely many firms able to enter the contest, we can establish the following results:

Proposition 3 *Under free entry and an infinite number of potential entrants, ex post outsourcing with price competition leads to*
(i) increased entry of firms,
(ii) increased total contest effort,
(iii) under-dissipation of the contested prize.

Proof. (i) With symmetric valuations, $V_i = \overline{V}_i = V$. From (7) it follows that firm $n+1$ has an incentive to participate in the contest as long as

[20] An imbedded assumption is then that all n firms would actually participate in the contest under free entry. However, since costs are symmetric for $\alpha = 1$ (where all firms want to participate in the contest), there always exists a range for α where a given number of firms will want to participate in the non-outsourcing contest. In the contest with outsourcing, all firms would like to participate.

$\frac{n-1}{n} < 1$, which holds trivially for all n. (ii) Since free entry implies $n \to \infty$ in the contest with outsourcing, it follows from a comparison of (5) and (13) that outsourcing leads to an increase in total effort if

$$V_2 > \left(\frac{n-1}{n}\right) \overline{V}_n. \tag{16}$$

From the entry condition in the asymmetric contest we know that, *in equilibrium*, the right-hand side of (16) is decreasing in n. Thus, it suffices to check for $n = 2$. In this case, the condition reduces to $\frac{V_2^2}{V_1+V_2} > 0$, which is trivially true. (iii) In the free entry equilibrium, total contest outlays are given by $V - c_2$, while production costs are c_1. Thus, rent dissipation is given by $\frac{V-c_2+c_1}{V} < 1$. ■

When outsourcing is possible, firms that would otherwise not find it profitable, will enter the contest if entry is free. Price competition for the subcontract implies that the effective cost of all firms except the most cost-efficient, is c_2, which triggers entry of new firms. In a free entry equilibrium, this means that total contest outlays increase. Furthermore, since own production costs do not matter for expected profits, an interesting implication of the equilibrium derived in this Section is that firms that are not able profitably to produce the good themselves have incentives to participate in the contest if they can outsource production to a lower-cost firm ex post.

Once more, we can illustrate our results by applying the parametric example from Section 2. In the asymmetric contest *without outsourcing*, the number of firms, n, participating in the contest under free entry is given by the general condition $V_{n+1} \leq S_n$. In our specific example, n is given by the smallest integer satisfying

$$\alpha^n \leq \frac{(n-1)(1-\alpha)}{\alpha(\alpha^{-n}-1)}. \tag{17}$$

Assume that $\alpha = 0.9$. This yields $n = 5$, and total contest outlays in equilibrium are approximately $0.64v$. On the other hand, if these 5 firms enter the contest with the possibility of ex post outsourcing, total contest effort is given by (14), which in this example amounts to $0.72v$. Finally, if there are infinitely many potential entrants to the contest, the possibility of outsourcing produces a total contest outlay of $0.9v$. Thus, outsourcing leads to more rent dissipation, and the possibility of additional entry reinforces this effect.

Summing up, the results of this Section suggest that rent-seeking will generally be higher with outsourcing than without. The only clear-cut case to the contrary, is when there are only two potential contestants, in which case contest effort is certain to fall.

4 Bargaining

In the analysis thus far we have assumed that, in case of outsourcing, the terms of the subcontract are determined by price competition among the potential subcontractors. Although this is, in some sense, a natural assumption, it also yields implications that might seem somewhat unrealistic. In particular, it seems reasonable to argue that a firm's own production costs should somehow affect its expected profits in the contest. Furthermore, pure price competition might be a particularly strong assumption in the case of few contestants. For example, with only two contestants, price competition for the subcontract implies that the losing firm unilaterally determines the price of the subcontract, which is a somewhat extreme assumption. In order to deal with these concerns, we relax the assumption of price competition and assume that, in case of outsourcing, the terms of the subcontract is determined in bargaining. We consider two different cases; bilateral Nash bargaining between the contest winner and the lowest-cost firm, and cooperative bargaining among the winner and all lower-cost firms.

4.1 Bilateral Nash bargaining

Assume that, upon winning the contest, firm i (≥ 2) can go to only one firm for negotiating a possible subcontract. This would be a reasonable scenario if bargaining costs are high. We assume that the winner always maximizes the total surplus of outsourcing by approaching firm 1 to negotiate the terms of a subcontract, and that the payoff to each party is given by the asymmetric Nash bargaining solution. We let the bargaining power of the contest winner be given by $\beta \in (0,1)$. This implies that outsourcing transforms the effective costs of firm i from c_i to $(1-\beta)c_i + \beta c_1$.[21]

Under these assumptions, the maximand of firm $i \geq 2$ is

$$\pi_i = P_i \left(V - (1-\beta)c_i - \beta c_1\right) - x_i, \quad i = 2, .., n, \tag{18}$$

while firm 1 maximizes

$$\pi_1 = P_1 \left(V - c_1\right) + (1-\beta) \sum_{i=2}^{n} \left[P_i \left(c_i - c_1\right)\right] - x_1. \tag{19}$$

The first observation worth making is that, unlike for the case of pure price competition for the subcontract, the contest is generally asymmetric. This means that the participation condition given in (7) applies.

Maximizing (18) and (19), the first order conditions for firm 1 and for any firm $i \geq 2$ are given by

$$x_1 = S_n \left[1 - \left(\frac{S_n}{V - c_1}\right)\right] - \left(\frac{1-\beta}{V - c_1}\right) \sum_{i=2}^{n} \left[x_i (c_i - c_1)\right] \tag{20}$$

and

$$x_i = S_n \left[1 - \left(\frac{S_n}{V - (1-\beta)c_i - \beta c_1}\right)\right]. \tag{21}$$

Inserting (21) into (20) and aggregating, we arrive – after some manipulations – to the following expression for equilibrium total contest effort

$$S_n = \left(\frac{n-1}{n}\right)(V - c_1) - \left(\frac{1-\beta}{n}\right) \sum_{i=2}^{n} (c_i - c_1) \tag{22}$$

We can summarize the characteristics of the contest equilibrium as follows:

Proposition 4 *With ex post outsourcing and bilateral Nash bargaining for*

[21] This cost could also arise if, for instance, the contest winner were able to make a credible take-it-or-leave-it offer to firm 1, but had to do a portion $(1-\beta)$ of the production himself.

the price of the subcontract, then

(i) the contest is asymmetric if $\beta < 1$ and $n > 2$,

(ii) total contest effort increases in β,

(iii) the contested prize is fully dissipated under free entry if $\beta \to 1$.

Proof. Follows straightforwardly from (18)-(22). ■

The bargaining parameter β plays a crucial role for the firms' behavior in the contest. From the payoff functions and the first-order conditions, we see that a higher value of β effectually contributes to making the contest more symmetric, which increases total effort in the contest equilibrium. Intuitively, a higher value of β means that the lowest-cost firm gets stronger incentives to win the contest, since the expected terms of the subcontract will be worse, from firm 1's viewpoint, if another firm wins the contest. On the other hand, since a higher value of β implies that all other firms $i \geq 2$ will have lower effective costs if winning the contests, these firms are also spurred to exert more contest effort.[22] In the limit case, $\beta \to 1$, where the winner of the contest has all bargaining power in determining the price of the subcontract, total contest effort approaches $\left(\frac{n-1}{n}\right)(V - c_1)$. Thus, free entry will contribute to full rent-dissipation if $\beta \to 1$, something that was not possible with price competition for the subcontract.[23]

It is also clear that if n and β are sufficiently low, outsourcing with Nash bargaining can reduce total contest effort compared with the case of no outsourcing.[24] For the special case of $\beta = 0$, rent-seeking is actually reduced regardless of the number of firms. One might view this result with some scepticism, though. If firm i wins the contest and has low bargaining strength, the agreed price (and hence, the effective cost of firm i) will be close to firm i's own costs, even though these may far exceed the production

[22] For the case of $n = 2$, the effect of an increase in β on the effort incentives of both firms are equally strong, implying that the contest remains symmetric for all $\beta \in (0, 1)$.

[23] The extreme case of $\beta \to 1$ corresponds to the other of the (two-player) games considered by Kamien et al. (1989), where the terms of the subcontract is determined by the winner of the initial contract.

[24] For example, if $n = 2$ and $\beta = 0$, the price for the subcontract if firm 2 wins will be identical under Nash bargaining and price competition. In this case, the terms of the subcontract are effectually determined by the loser of the contest, and we have already seen that this reduces total effort.

costs of other low-cost firms. Incentives to bargain with other firms thus naturally emerge. Let us therefore restrict attention to situations where the agreed price is actually lower than or equal to c_2, i.e., $(1-\beta)c_i + \beta c_1 \leq c_2$. This places a lower bound on β, given by $\underline{\beta} := (c_i - c_2) / (c_i - c_1)$. If we let superscripts PC and NB denote price competition and Nash bargaining, respectively, this means that

$$\begin{aligned} S_n^{NB} &= \left(\frac{n-1}{n}\right)(V-c_1) - \left(\frac{1-\beta}{n}\right)\sum_{i=2}^{n}(c_i - c_1) \\ &\geq \left(\frac{n-1}{n}\right)(V-c_2) = S_n^{PC}. \end{aligned} \tag{23}$$

Thus, with the assumption that the bargained price should be better than any feasible alternative, total contest effort under Nash bargaining is never less extensive than in the price-competition case.[25]

4.2 Cooperative bargaining

As we have just indicated, a potentially crucial feature of the above outlined model with bilateral Nash bargaining is that the existence of outside opportunities does not affect the bargaining position. In other words, if the contest is won by a firm $i \geq 2$, the bargaining parameter β is independent of the number of other firms with lower costs than the winner. We now proceed to relax this assumption by applying a cooperative bargaining framework and use the Shapley value as an approximation to the expected earnings of a firm.

The Shapley value of firm i is given by the average contribution i gives to any coalition of the other firms. Let M be the set of firms in such a coalition and N be the total set of firms. $|N| = n$ and $|M| = m$ denote the number of firms in N and M, respectively. Finally, let $\Pi(K)$ denote the profit that a coalition K can get on their own. The Shapley value for firm i when firm

[25] The somewhat ad-hoc assumption that the bargained outsourcing price should not exceed any feasible alternative can be conceptualised by considering the following joint bargaining/price competition framework. If firm 2 wins the contest, it enters into bargaining with firm 1, since no other firm can offer a more beneficial outsourcing contract. However, if any firm $i \geq 3$ wins the contest, this firm may play the lower cost firms against one another to obtain an outsourcing price of c_2.

j is the winner of the contest is then defined as

$$\phi_{ij} = \sum_{M \subseteq N-i} \left\{ m! \frac{(n-m-1)!}{n!} \left[\Pi(M \cup \{i\}) - \Pi(M) \right] \right\}. \tag{24}$$

The Shapley value of firm i is determined by which firm is actually the winner of the contest (firm j), since the value added to a random coalition by any one firm is strongly determined by which firm is the winner. Since the Shapley value averages the contributions a firm provides to all coalitions, it follows that the winner of the contest always gets a relatively large share of the pie – since without this firm, no coalition gets anything.

Expected profits for firm i is then given by

$$\pi_i = \sum_{j=1}^{n} P_j \phi_{ij} - x_i, \tag{25}$$

from which we can derive the first-order condition for optimal effort in the contest:

$$x_i = S_n - \frac{S_n^2}{\phi_{ii}} - \frac{1}{\phi_{ii}} \sum_{j \neq i} x_j \phi_{ij}. \tag{26}$$

From (26) we can see that, in general, participation in the contest (i.e., $x_i > 0$) requires that ϕ_{ii} is not too low. This reflects the familiar result that, in an asymmetric contest, firms that expect to have high effective costs if they win (or equivalently, a low valuation of the contested prize), do not participate in the contest.

Unfortunately, it proves impossible to carry the analysis much further without narrowing the focus. Thus, for the remainder of this Section, we concentrate on the case of $n = 3$. Using the definition of the Shapley value, the three-firm case produces the following results for ex post payoffs:

$$\phi_{21} = \phi_{31} = \phi_{32} = 0, \tag{27}$$

$$\phi_{11} = V - c_1, \tag{28}$$

$$\phi_{22} = V - \frac{c_1 + c_2}{2}, \quad \phi_{12} = \frac{c_2 - c_1}{2}, \tag{29}$$

$$\phi_{33} = V - \frac{3c_1 + c_2 + 2c_3}{6}, \quad \phi_{23} = \frac{c_3 - c_2}{6}, \quad \phi_{13} = \frac{c_3 + 2c_2 - 3c_1}{6}. \tag{30}$$

(27) states that the firms with higher costs than the winner get zero payoffs, while (28) shows that there is no outsourcing if firm 1 wins, as expected. On the other hand, if firm 2 wins the contest, (29) shows that firms 1 and 2 split the surplus in the same manner as in a 50/50 Nash bargaining solution. If firm 3 is the winner, all firms get a positive share of the total available surplus, as demonstrated by (30), although firm 1 gets a larger share of the pie than firm 2. Furthermore, as long as firm 3 gets nothing unless it wins the contest, a necessary condition for participation is that $V > \frac{3c_1+c_2+2c_3}{6}$.

Inserting the Shapley values given in (27)-(30) into (26), and solving for individual efforts, yield

$$x_1 = x_2 = \frac{1}{27} \frac{(6V - 3c_1 - 2c_2 - c_3)^2}{6V - 3c_1 - c_2 - 2c_3}, \tag{31}$$

$$x_3 = \frac{(6V - c_3 - 2c_2 - 3c_1)}{27} \left(\frac{2(V - c_1) + 4(V - c_3) + c_2 - c_1}{6V - 3c_1 - c_2 - 2c_3} \right). \tag{32}$$

Total contest effort is then given by

$$S_3 = \frac{6V - 3c_1 - 2c_2 - c_3}{9}. \tag{33}$$

Like in the case of bilateral Nash bargaining, the production costs of all firms matter for total contest effort. We also see that the two lowest-cost firms exert the same amount of effort, and thus have the same probability of winning the contest, even though firm 1 has higher expected profits. This reflects the previously discussed effect of outsourcing reducing the incentives of the lowest-cost firm to win the contest.

For the three-firm case, we can also use our parametric example to compare total contest effort in the different regimes that we have considered. For $n = 3$, the equilibrium levels of total effort in the benchmark (BM), outsourcing with price competition (PC), Nash bargaining (NB) and cooperative bargaining (CB), respectively, are given by

$$S_3^{BM} = \frac{2\alpha^2 v}{1+\alpha+\alpha^2}, \quad S_3^{PC} = \frac{2\alpha v}{3}$$
$$S_3^{NB} = \frac{v}{3}\left(\alpha\left(1+\alpha\right)+\left(1-\alpha\right)\left(2+\alpha\right)\beta\right), \quad S_3^{CB} = \frac{v}{9}\left(3+\alpha\left(2+\alpha\right)\right).$$

Straightforward comparisons show that $S_3^{CB} > S_3^{PC} > S_3^{BM}$. Total effort in the Nash bargaining case relies heavily on the bargaining parameter β. Assuming equal bargaining strength between the winner and the subcontractor, $\beta = 1/2$, the ranking of equilibrium contest effort, for all $\alpha \in (0,1)$, is given by

$$S_3^{CB} > S_3^{NB} > S_3^{PC} > S_3^{BM}.$$

5 Contest design and welfare

Should a contest designer allow ex post outsourcing of production? This depends, naturally, on the objective of the contest designer. A reasonable and widely used assumption in the contest design literature is the maximization of total contest effort. The effect of outsourcing on total effort has been analyzed in great detail in previous sections. The discussion so far suggests that, by allowing for outsourcing, aggregate contest effort increases if *i)* there are more than only a few potential contestants and *ii)* the high cost firms have a fair degree of ex post bargaining strength relative to the most efficient firm. With only two contestants, however, the possibility of outsourcing *reduces* total effort if the low-cost firm has sufficient bargaining power. There may be more instruments available to the contest administrator, though. In the first part of this Section we extend the analysis to consider also the case where a contest designer can collect entry fees from the contestants, along the lines of Fullerton and McAfee (1999).

However, increased contest effort may not always be desirable from a viewpoint of social welfare. Bribery is an obvious example, but socially excessive effort is also a possibility in, say, a research contest. Long patent protection or licensing periods (i.e., high prizes) may induce inefficiently high levels of effort, and in any case, higher effort also means lower aggregate profits. In the extreme case, were all effort is considered to be socially wasteful, the relevant welfare measure is aggregate profits. Outsourcing improves ex post allocative efficiency, but it may also induce more wasteful effort in the contest. In the latter part of this Section we highlight the trade-off between improved allocative efficiency and potential excessive effort, induced by ex post outsourcing.

5.1 Entry fees

The analysis in this subsection is closely related to Fullerton and McAfee (1999). They analyze a situation where the contest designer is able to collect entry fees, aiming to minimize the costs of inducing a given level of effort. Assuming a uniform entry fee, E, if the contest designer wants n firms to participate, he must choose the entry fee such that firm n makes non-negative expected profits while firm $(n+1)$ makes negative expected profits. Of course, there is no reason to let the n'th firm have any surplus, so the expected profits of this firm equals the optimal entry fee, i.e., $\pi_n = E$. Fullerton and McAfee (1999) show that, for a large class of contests, the optimal number of contestants which should be chosen in order to induce a given total effort at lowest possible procurement costs, $\Omega := V - nE$, is two.[26] Here, we extend their analysis by asking whether ex post outsourcing should be allowed, restricting our attention to the two-firm case.

Benchmark: No outsourcing

In the non-outsourcing benchmark case with two firms, the prize V needed to induce aggregate contest effort S is given by

$$S = \frac{(V - c_1)(V - c_2)}{2V - c_1 - c_2}. \tag{34}$$

[26] The intuition goes as follows. A lower number of contestants increases the spending (prize) needed to induce a given level of effort, but this is outweighed by the possibility to set higher entry fees due to higher expected profits of the remaining contestants. Similar results are found by Taylor (1995) and Che and Gale (2003).

Solving for V yields

$$V = S\left[1 + \frac{c_2 + c_1}{2S} + \sqrt{1 + \left(\frac{c_2 - c_1}{2S}\right)^2}\right]. \tag{35}$$

The optimal entry fee is given by $E = \pi_2$, yielding

$$E = \frac{(V - S - c_2)^2}{V - c_2}, \tag{36}$$

where V is given by (35). Total procurement costs for inducing effort S in the benchmark case are then given by

$$\Omega^{BM} = c_2 + S\left[1 + \frac{\left(\sqrt{1+\lambda^2} - \lambda\right)(1 + 2\lambda) - 1}{1 - \lambda + \sqrt{1+\lambda^2}}\right], \tag{37}$$

where

$$\lambda := \frac{c_2 - c_1}{2S} > 0.$$

Outsourcing

Now consider the possibility of allowing ex post outsourcing. With two contestants, asymmetric Nash bargaining for the price of the subcontract encompasses all feasible possibilities, yielding outsourcing prices in the interval $[c_1, c_2]$, depending on the bargaining parameter β. From (22), total contest effort is given by

$$S = \frac{1}{2}\left[V - c_1 - (1 - \beta)(c_2 - c_1)\right]. \tag{38}$$

Thus, inducing a level of effort S necessitates a contest prize

$$V = 2S + (1 - \beta) c_2 + \beta c_1, \tag{39}$$

and the optimal entry fee is found to be

$$E = \frac{S}{2}. \tag{40}$$

Perhaps surprisingly, we see that inducing a given level of effort always yields the same profits for firm 2, regardless of the ex post outsourcing arrangements. In other words, regardless of the division of ex post surplus, if a contest administrator wants to induce effort S, the highest entry fee she can take is $S/2$ to ensure that two firms will participate in the contest.

Total procurement costs with ex post outsourcing are then given by

$$\Omega^{NB} = S + \beta c_1 + (1 - \beta) c_2. \tag{41}$$

The effect of outsourcing on total procurement costs are given by a comparison of (37) and (41). We see that $\Omega^{NB} < \Omega^{BM}$ for all $\beta \in (0, 1)$ if the terms in square brackets in (37) exceed 1. It is easily verified that this is always the case.[27] Thus, ex post outsourcing should always be allowed in the two-firm case when the contest administrator can collect entry fees.

The intuition is the following. Consider first the case where β is relatively high. Then outsourcing raises total contest effort, due to increased effort incentives for the high-cost firm, implying that a given level of effort can be induced by offering a lower prize. Furthermore, a larger share of aggregate profits can be captured by the entry fee. In the limit case of $\beta \to 1$, both firms have the same expected profits, all of which are captured by the optimal entry fee. In sum, total procurement costs decline.

On the other hand, if β is relatively low, we know that outsourcing always reduces total effort, due to the reduced incentives of the low-cost firm to exert effort in the contest. This must be compensated for by a higher contest prize, which – all else equal – increases total procurement costs. However, the reduction of firm 1's contest effort raises the expected profits of firm 2 in equilibrium, which implies that a higher entry fee can be collected while still inducing firm 2 to participate in the contest. This more than offsets the prize increase, implying that total procurement costs decline also in this case.

We summarize our results as follows.

[27] We need to have $\frac{\left(\sqrt{1+\lambda^2}-\lambda\right)(1+2\lambda)-1}{1-\lambda+\sqrt{1+\lambda^2}} > 0$. It can be verified that this expression has a single positively valued maximum for $\lambda = \frac{3}{4}$ and approaches zero in the limits ($\lambda \to 0$ and $\lambda \to \infty$).

Proposition 5 *With two contestants, the possibility of ex post outsourcing reduces total procurement costs when entry fees can be collected.*

5.2 Socially wasteful effort

Finally, to deal with the question of socially wasteful effort, we now make the extreme assumption that all contest effort is considered socially wasteful or unwanted. We can think of such effort as lobbying or bribery undertaken by firms in order to increase the probability of being awarded, e.g., a government contract.[28] Is the possibility of ex post outsourcing likely to be socially beneficial in this case?

When effort is socially wasteful, the reasonable measure of social welfare is expected aggregate profits. The possibility of outsourcing improves ex post allocative efficiency, which is unambiguously positive from a welfare perspective. However, it may also increase incentives for socially wasteful effort. We can capture this trade-off by maintaining our two-firm example with asymmetric Nash bargaining for the subcontract.

Without outsourcing, individual and total contest efforts, respectively, are given by (4) and (5), yielding expected aggregate profits

$$\sum_{i=1}^{2} \pi_i^{BM} = \frac{V(V - c_1 - c_2) + c_1^2 + c_2^2 - c_1 c_2}{2V - c_1 - c_2}. \tag{42}$$

On the other hand, with ex post outsourcing, we saw in Section 4 that the contest becomes perfectly symmetric with two firms. Total effort is given by (38), while expected aggregate profits are

$$\sum_{i=1}^{2} \pi_i^{NB} = \frac{1}{2}\left[V - c_1 + (1-\beta)(c_2 - c_1)\right]. \tag{43}$$

[28] Clark and Riis (2000) study allocational efficiency in bribery contests for governmental contracts. Ex post outsourcing of the contract is not an issue, though.

Comparing the two cases, outsourcing is socially beneficial if

$$\sum_{i=1}^{2}\left(\pi_i^{NB}-\pi_i^{BM}\right)=\frac{(c_2-c_1)}{2}\left[\frac{3\,(V-c_2)}{2V-c_1-c_2}-\beta\right]>0. \tag{44}$$

For the special case of $\beta=0$, where the most efficient firm dictates the terms of the subcontract, the possibility of ex post outsourcing unambiguously improves social welfare. Wasteful effort is reduced, due to the reduced effort incentives of the most efficient firm, and ex post allocative efficiency is improved.

However, outsourcing is less likely to be socially beneficial the higher is β. The reason is that a higher level of β increases incentives for socially wasteful effort when ex post outsourcing is a possibility. More bargaining strength to the least efficient firm increases this firm's net valuation of winning the contest, with a corresponding stronger incentive to exert effort. At the same time, the low-cost firm also gets a stronger incentive to win the contest, since a higher β implies that the subcontract becomes less profitable for this firm if it does not win the contest.

From (44) we see that outsourcing will in fact reduce expected aggregate profits for sufficiently high levels of β if

$$\frac{3\,(V-c_2)}{2V-c_1-c_2}<1, \tag{45}$$

which can be expressed as

$$V-c_2<c_2-c_1. \tag{46}$$

In words, this condition states that the net valuation of the high-cost firm in the absence of outsourcing must be lower than the cost difference between the firms. If this is the case, the high-cost firm has low incentives to exert effort in the contest, absent outsourcing, relative to the low-cost firm. Thus, the probability that the most efficient firm will win the contest anyway is relatively high. This implies, in turn, that the improved allocative efficiency due to outsourcing is relatively moderate, and outweighed by the effect of increased total effort for sufficiently high level of β. In other words, if $V - c_2 < c_2 - c_1$, there exists a critical value $\beta^* < 1$ such that outsourcing is socially detrimental if $\beta \in (\beta^*, 1)$.

6 Concluding remarks

We have analyzed the strategic effects and implications of ex post outsourcing in situations where competition between firms take on the characteristics of an imperfectly discriminating contest. While horizontal outsourcing is often thought to facilitate collusion, we have shown that such arrangements might instead increase competition between firms in a majority of cases. With respect to contest design, whether or not such competition is desirable depends both on the interpretation of the model and the objective of the contest organizer. In a procurement contest, allowing outsourcing might increase the quality of the procured good, for example through higher R&D investments by the contestants, but it might also increase incentives for lobbying and bribery.

In order to improve the tractability of our analysis, we have made some simplifying assumptions. Among these is the assumption that, in case of ex post outsourcing, *all* production is outsourced to a lower-cost firm. In reality, though, we usually observe that only part of the initial contract is outsourced. However, the effect of such partial outsourcing can quite easily be interpreted within our modelling framework. In general, the lower share of production that is subject to ex post outsourcing, the higher are the effective ex post production costs for all but the most efficient firm. In the Nash bargaining version of our model, this is equivalent to a higher relative bargaining strength for the subcontractor, which – all else equal – reduce total contest effort.

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