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MERGER, PARTIAL COLLUSION AND RELOCATION

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Merger, partial collusion and relocation[*]

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Abstract

We set up a three-firm model of spatial competition to analyse how a merger affects the incentives for relocation, and conversely, how the possibility of relocation affects the profitability of the merger, particularly for the non-participating firm. The analysis is carried out for the assumptions of both mill pricing and price discrimination, and we also consider the case of partial collusion. For the case of mill pricing, a merger will generally induce the merger participants to relocate, but the direction of relocation is ambiguous, and dependent on the degree of convexity in the consumers' transportation cost function. We also identify a set of parameter values for which the free-rider effect of a merger vanishes, implying that the possibility of relocation could solve the 'merger paradox', even in the absence of price discrimination.

Keywords: Spatial competition; Merger; Relocation; Partial collusion.

JEL classification: L13, L41, R30

1   Introduction

In imperfectly competitive markets, an important part of the strategic interaction among firms occurs along a spatial dimension. More specifically, the profitability of a given firm is in many cases highly dependent

on the firm's *location*, relative to its competitors. Thus, to the extent that a firm is able to influence its own location, this is one of the most important decisions to be made by firms. If we interpret location in a *physical space*, this decision involves the location of production plants or outlets. This is an especially important consideration in industries in which physical transportation costs are high. In many industries, though, location in the *product space* plays an even more important role. In this case, the strategic decision involves which type of product the firm should produce.

The purpose of this paper is to analyse the strategic importance of spatial competition for firms' incentives to merge or collude. More specifically, we want to examine how a merger, or partial collusion along one or more dimensions, affects firms' incentives to *relocate* from an initial position. The possibility of relocation will, in turn, affect the incentives for merger or collusion.

In the literature on purely anti-competitive horizontal mergers, a merger is normally assumed strategically to affect only the firms' pricing or output decisions. The seminal contributions are Salant *et. al.* (1983) for the case of Cournot competition, and Deneckere and Davidson (1985) for the case of Bertrand competition. A striking feature of these models is the so-called 'merger paradox': a merger between two or more firms is always more beneficial for the firms not participating in the merger. We often observe in real-life, though, that a corporate merger is accompanied by some other structural changes, particularly in the spatial dimension. Typical examples are relocations of production facilities, or changes in the product range offered by the merging firms.[1] It is reasonable to believe that relocations of this kind would affect the profitability of a merger, also for non-participating firms in the industry.

We set up a model where firms can undertake a costly investment in order to relocate from an initial position. This assumption should fit a broad interpretation of location. If we interpret location in the product space, it is perhaps most natural to think of the relocation cost as investment in product R&D. With this interpretation, our paper is also related to Lin and Saggi (2002), who analyse firms' incentives to invest in product R&D as a way of increasing the degree of product differentiation in a symmetrically differentiated industry. By assuming a symmetric Chamberlinian demand system, product R&D has two different effects

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[1] In a related, but quite different, paper, Lommerud and Sørgard (1997) analyse the possibilities of introducing a new product, or withdrawing an existing brand, in a context of horizontal merger. In another study, Berry and Waldfogel (2001) analyse empirical evidence of the effect of mergers on variety and product repositioning in US local radio broadcasting markets.

in their model. In addition to the differentiation effect, product R&D by one firm also increases the demand for *all products* in the industry by an equally large amount, which is a somewhat extreme assumption. In the present paper we choose a model set-up which focuses exclusively on the differentiation effect.

With a few exceptions, the effect of mergers on relocation, and *vice versa*, has received relatively little attention in the literature. Rothschild (2000) and Rothschild et al. (2000) analyse the case where three firms are initially located on a Hotelling line and can relocate in the anticipation of a merger between two of the firms. A problem with this set-up is that the structure of the industry is *ex ante* asymmetric, so that the choice of merger candidates is somewhat arbitrary. Norman and Pepall (2000a, 2000b) solve this problem by assuming that all firms are initially located at the market centre, which is a Nash equilibrium in the no-merger game. The main result in these studies is that the 'merger paradox' could be solved by allowing for the possibility of relocation. However, this result is obtained under the assumption that firms are able to engage in price discrimination. Assuming Cournot competition, this means that the firms compete in a continuum of segmented markets.[2]

In the present paper, we consider a two-firm merger in a model where three firms are initially equidistantly located on a circle. The firms are price setters, and we consider both the cases of mill pricing and spatial price discrimination. Regarding the former case, the most related paper is probably Levy and Reitzes (1992) who show that a side-by-side merger is always profitable in a model of this kind. However, they do not consider the possibility of relocation, which is the main objective of our paper.

We find that a merger gives the merger participants incentives to relocate under the assumption of mill pricing, but not in the case of price discrimination. The direction of relocation in the former case is crucially dependent on the characteristics of consumers' transportation costs. Adopting a disutility function with both a linear and a quadratic component, we find that the merger participants will relocate towards the outsider if the weight attached to the linear part is sufficiently high. In this case, we also identify the existence of a set of parameter values for which a merger will be more profitable for an insider than for the non-participant. Thus, we show that the possibility of relocation could possibly solve the 'merger paradox' even in the absence of price discrimination. Regarding welfare considerations, we also show that re-

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[2]Similar assumptions are also used by Matsushima (2001) in a Salop model. Reitzes and Levy (1995) consider the case of Bertrand competition with price discrimination in a Salop model.

location could in some cases improve locational efficiency, thus reducing the negative impact of the merger.

We also extend the model to consider partial collusion in either location or price setting. In this case we find that partial collusion of either kind will always provide incentives for relocation, and the direction of relocation is not dependent on whether or not the firms are able to price discriminate. A non-trivial observation is further that for the case of price discrimination, full collusion (or merger) is always preferred to partial collusion in terms of locational efficiency.

The remainder of the paper is organised as follows: The basic ingredients of the model are introduced in Section 2, where the effect of a merger, and partial collusion, is analysed under the assumption of mill pricing. Section 3 then replicates the analysis for the case of price discrimination. Finally, some concluding remarks are offered in Section 4.

2   A model of spatial competition with mill pricing

Consider a population of consumers uniformly distributed, with a constant density of 1, on a circle with circumference l. Three firms are located on the circle, with the location of firm i given by x_i. Assuming unit demand, the utility of a consumer located at $z \in [0, l]$, and buying from firm i, is given by

$$U\left(z, v, x_i, p_i\right) = v - p_i - t\left(\psi_i\right), \tag{1}$$

where

$$\psi_i = \min\{|z - x_i|, l - |z - x_i|\}, \tag{2}$$

v is the reservation utility, assumed to be equal for all consumers, p_i is the price charged by firm i and $t\left(\cdot\right)$ is a transportation cost function. We also assume that v is sufficiently high for the market always to be covered, i.e. all consumers are active.

Regarding transportation costs, the standard approach is to assume these costs to be either linear or quadratic in distance. We will adopt a functional form that encompasses both the linear and the quadratic variant as special cases. The costs of travelling a distance Δ is given by[3]

$$t\left(\Delta\right) = a\Delta + b\Delta^2, \quad a, b \geq 0. \tag{3}$$

We introduce the notation $\widehat{z}_i$ for the location of the consumer who is indifferent between buying the good from the two neighbouring firms i

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[3]A similar cost function is used by Lambertini (2001).

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and $i + 1$.[4] The location of this consumer is implicitly given by

$$U\left(\widehat{z}_i, v, x_i, p_i\right) = U\left(\widehat{z}_i, v, x_{i+1}, p_{i+1}\right).$$

Given the locations of the indifferent consumers, the market share of firm i is given by

$$M_i = \widehat{z}_i - \widehat{z}_{i-1}. \tag{4}$$

We also assume that the firms can undertake an investment in order to change their location. We assume that relocation costs are convex in distance. The cost for firm i of relocating a distance d_i is given by kd_i^2, where k is a positive constant.

The marginal cost of production is assumed to be constant and equal for all firms and, without loss of generality, set equal to zero. Firm i's (pre-investment) profits are then given by

$$\pi_i = p_i M_i - kd_i^2. \tag{5}$$

The game is played in two stages:

Stage 1: The firms simultaneously choose the level of investment, d_i.

Stage 2: The firms simultaneously set prices, p_i.

2.1 Merger

As a benchmark for comparison, we will first consider the case in which all firms make independent decisions about prices and investments. In this case the model is completely symmetric. It is easily shown that each firm, operating independently, would prefer to be located as far away from its competitors as possible. Thus, given initial equidistant locations, the firms have no incentives to invest in relocation.

Solving for the Nash equilibrium, with $d_i = 0$, yields the following solution for prices and profits:

$$p_i = \frac{l\left(3a + bl\right)}{9}, \tag{6}$$

$$\pi_i = \frac{l^2\left(3a + bl\right)}{27}. \tag{7}$$

The main focus of the analysis in this subsection is to investigate how a merger may influence the incentives for relocation. Given equidistant initial locations, we can assume, without loss of generality, that the merger participants (firms 1 and 2) are located at 0 and $\frac{l}{3}$, with the outsider (firm 3) located at $\frac{2l}{3}$. Obviously, any relocation for the merging

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[4] Because of the geometry of the model any firm referred as $j \pm 3n$ is the same as firm j, for every $n \in N$.

firms must be symmetric across both plants (products), thus $d_1 = -d_2$.[5] We will focus on the relocational incentives of the plant/product located at 0. Let d denote the distance of relocation, measured in the clockwise direction. Hence, $d < 0$ implies that the merger participants relocate in the direction of the outside firm. Obviously, the outsider has no incentives to relocate.[6] Since the merger participants coordinate their price setting, the symmetric feature of the model enables us to solve for the equilibrium by identifying the location of one indifferent consumer only. Consider the consumer who is indifferent between buying from firm 1 and firm 3. Her location, $\widehat{z}_3$, is found by solving

$$p_1 + t\left(l + d - \widehat{z}_3\right) = p_3 + t\left(\widehat{z}_3 - \frac{2l}{3}\right).$$

Using (3), this yields

$$\widehat{z}_3 = \frac{1}{6}\left(5l + 3d\right) + \frac{3}{2}\left(\frac{p_1 - p_3}{3a + bl + 3bd}\right). \tag{8}$$

Due to symmetry and coordinated price setting, the consumer who is indifferent between buying from either of the merger participants is located at $\widehat{z}_1 = \frac{l}{6}$. Furthermore, symmetry also ensures that the market shares of the merged firm and the outsider, respectively, are

$$M_1 + M_2 = 2\left(l - \widehat{z}_3 + \frac{l}{6}\right) \tag{9}$$

and

$$M_3 = 2\left(\widehat{z}_3 - \frac{2l}{3}\right). \tag{10}$$

Equilibrium prices, as functions of the optimal degree of relocation, is found by inserting (8)-(10) into the profit functions, (5), and maximising with respect to prices. This yields

$$p_1 = p_2 = \frac{1}{27}\left(5l - 3d\right)\left(3a + bl + 3bd\right), \tag{11}$$

$$p_3 = \frac{1}{27}\left(4l + 3d\right)\left(3a + bl + 3bd\right), \tag{12}$$

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[5]This assumption of symmetry regarding the relocation distances is made to facilitate the analysis and it is not imposed as an exogenous condition. The symmetric outcome can be obtained by explicitly solving the game for d_i, $i = 1, 2, 3$.

[6]Again, besides being an argument derived from the symmetry of the model, this result can also be obtained as an equilibrium outcome of the relocation game.

with corresponding profits given by

$$\pi_1 = \pi_2 = \frac{1}{486}(5l - 3d)^2(3a + bl + 3bd) - kd^2, \qquad (13)$$

$$\pi_3 = \frac{1}{243}(4l + 3d)^2(3a + bl + 3bd). \qquad (14)$$

Let us first consider the effects of a merger between two firms, without relocation. With $d = 0$ the following result can be stated:

Proposition 1 *With three firms initially located equidistantly from each other, then*
(i) a merger between two firms is always jointly profitable,
(ii) profits are higher for the non-participating firm.

Proof. (i) Comparing (13) and (7) we find that

$$\pi_1(d = 0) - \pi_i = \frac{7}{486}l^2(3a + bl) > 0.$$

(ii) A comparison of (13) and (14) reveals that

$$\pi_1(d = 0) - \pi_3(d = 0) = -\frac{7}{486}l^2(3a + bl) < 0.$$

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This is a restatement of Levy and Reitzes (1992), and corresponds to the well known results in Deneckere and Davidson (1985). The gain from price setting coordination, resulting in higher prices, more than outweighs, in terms of profits, the loss of market shares for the merger participants. However, the outside firm enjoys both higher prices and a higher market share, implying that free-rider incentives are present: rather than participating in a merger, each firm would prefer the other firms to merge.

Let us now see how a merger between two firms affects the incentives to relocate. Since the merging firms would only spend resources to relocate their plants/products if it increases profits, relocation obviously increases the profitability of a merger. The question is, however, whether the merging firms would relocate away from, or in the direction of, the outside firm. The optimal distance of relocation is given by

$$d^* = \arg\max\{\pi_1 + \pi_2\}.$$

Using (13), we find the explicit value of the interior solution to be

$$d^* = \frac{18bl + 108k - 6a - 6\sqrt{4bl(a + bl + 27k) + (a - 18k)^2}}{18b}. \qquad (15)$$

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In order to secure an interior solution[7] we make the assumption that $k \geq \overline{k}$, i.e. that relocation is sufficiently costly.[8]

Proposition 2 *The merger participants will relocate towards (away from) the outside firm if $a > (<) \frac{1}{2}bl$.*

Proof. Follows immediately from (15). ∎

The first observation to be made is that d is generally non-zero: a merger between two firms creates incentives for relocation. Furthermore, the direction of relocation is generally ambiguous, and depends on the specifics of the transportation cost function. It is easy to verify, though, that $\frac{\partial d}{\partial a} < 0$, $\frac{\partial d}{\partial b} > 0$ and $\frac{\partial d}{\partial k} < 0$ if $a < \frac{1}{2}bl$.

The merged firm faces a trade-off in deciding on the direction of relocation: by moving away from the outside firm price competition is reduced, at the expense of a lower market share. Alternatively, the merged firm can gain a larger share of the market by relocating towards its competitor. The nature of this trade-off is determined by the characteristics of the transportation cost function. If there is a relatively high degree of convexity in transportation costs, the degree of price competition is highly dependent on the distance between the firms. The further apart the firms are located, the more costly it is to 'steal' market shares from the competitors, implying that the degree of competition is relatively lower. Consequently, relocating further away from their competitor is an effective way for the merger participants to reduce the degree of price competition.

On the other hand, if there is a relatively low degree of convexity in transportation costs, the degree of price competition is not sufficiently reduced to compensate for the reduction of market share by moving further away from the competing firm. In this case, the market share effect dominates the competition effect, and the merged firm can increase profits by moving closer to the outside firm, thereby controlling a larger share of the total market.

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[7]I.e., to prevent that the indifferent consumers are pushed outside the market segment between the two firms, and to avoid that firms overtake one another when they relocate.

[8]To ensure that

$$|d| < \max\left\{\frac{l}{6}, l - \widehat{z}_3\right\}$$

we have to make the assumption that

$$k > \overline{k} = \max\left\{-\frac{1}{2}a + \frac{1}{8}bl, \frac{3}{4}a - \frac{21}{40}bl\right\}.$$

2.1.1 A special case: linear transportation costs

The transport cost function specified in (3) encompasses the two most commonly used specifications in the literature on spatial competition: linear $(b = 0)$ and quadratic $(a = 0)$ transportation costs. In our model, an interesting result appears for the special case of linear transportation costs.[9] From Proposition 2 it follows that linear transportation costs implies relocation towards the outside firm. Comparing the cases with and without relocation, we find that relocation always leads to higher prices for the merged firm and lower prices for the outsider. From the viewpoints of the merging firms, the cost of charging higher prices is a loss of market share to the outsider. However, the merger participants can partly compensate for this effect by moving closer to the outside firm, which enables the colluding firms to charge even higher prices. The non-participant, on the other hand, now faces a higher degree of competition, and is forced to reduce its price in order to soften the loss in market share. Thus, the possibility of relocation for the merged firm implies a reduction of both price and market share for the outsider, and this could potentially cause the well-known free-rider effect to vanish.

Proposition 3 *When transportation costs are linear in distance, a merger participant earns higher profits than a non-participant if the cost of relocation is sufficiently small.*

Proof. Inserting $\lim_{b \to 0} d^*$ from (15) into (13)-(14), we find that

$$\pi_3 - \pi_1 < 0$$

if

$$\frac{(119 - 5\sqrt{385})a}{252} < k < \frac{(119 - 5\sqrt{385})a}{252}.$$

Imposing the restriction $k \geq \overline{k}$, we have that

$$\pi_3 - \pi_1 < 0$$

if

$$\frac{3}{4}a < k < \frac{(119 - 5\sqrt{385})a}{252} (\approx 0.86a).$$

∎

The Proposition identifies a (small) range of k for which each firm would like to participate in the merger, rather than waiting for the other firms to merge.[10]

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[9]The relevant equilibrium expressions for this case is easily found by inserting $\lim_{b \to 0} d^*$ into (8)-(14). Note also that linear transportation costs implies $\overline{k} = \frac{3}{4}a$.

[10]Although we have shown that the free rider effect might vanish when the transportation cost function is linear, this result is not restricted to this particular case.

2.2 Welfare

We apply the standard definition of social welfare, W, as the sum of consumers' and producers' surplus, which in our case reduces to:

$$W = vl - \sum_{i=1}^{3} \int_{\widehat{z}_{i-1}}^{\widehat{z}_i} t\left(\psi_i\right) dz - 2kd^2.$$

With the assumptions of unit demand and a non-binding reservation price for consumers, social welfare does not depend on prices directly, but is given by the sum of consumers' gross valuation, vl, net of total transportation and relocation costs. Thus, a welfare analysis in this kind of model is basically an analysis along one dimension only, namely *locational efficiency*.

Using the symmetry properties of the model, the expressions for social welfare in the merger and no-merger cases, respectively, are found to be

$$W_m = vl - \Gamma - 2kd^2, \tag{16}$$

where

$$\Gamma = \frac{1674ad^2 - 270bd^3 - 72adl + 486bd^2l + 87al^2 - 18bdl^2 + 11bl^3}{972},$$

and

$$W_{nm} = vl - \frac{9a + bl}{108}l^2. \tag{17}$$

Assume first that relocation is not possible. Comparing (16) and (17), we find that

$$W_m\mid_{d=0} - W_{nm} = -\frac{3a + bl}{486}l^2 < 0.$$

Thus, a merger is socially harmful even if it does not lead to any relocation. Post-merger there is a price difference between the merger participants and the non-participant which implies that a larger share of consumers is buying from the outside firm. This causes an increase in the total outlay on transportation costs.

A closer inspection of (16) and (17) also reveals that $W_m - W_{nm} < 0$ for the equilibrium value of d, implying that a merger is always socially harmful. However, once two firms have merged welfare is not maximised at $d = 0$. Thus, from society's point of view there are incentives for relocation, as long as this is in the right direction. The possibility of

For example, if we consider $b \neq 0$ and costless relocation ($k = 0$), a merging firm also obtains higher profit that the outsider if $0.64bl < a < 0.70bl$.

relocation means that the negative impact of a merger, in terms of social welfare, could be reduced if the merger participants relocates away from the outsider. The exact condition is given by the following Proposition.

Proposition 4 *Given that a merger takes place, relocation leads to a welfare improvement if $d^* \in \left(0, \overline{d}\right)$.*

Proof. From (16) we find that

$$W_m\left(d\right) - W_m\left(d = 0\right) = \frac{1}{54}d\left(4al + bl^2 - 93da + 15d^2b - 27dbl - 108dk\right).$$

It follows that

$$W_m\left(d\right) - W_m\left(d = 0\right) > 0 \text{ iff } 0 < d < \overline{d},$$

where

$$\overline{d} = \frac{108k + 27bl + 93a - \sqrt{\left(27bl + 108k + 93a\right)^2 - 60bl\left(4a + bl\right)}}{30b}.$$

∎

This result, which is not immediately obvious, can be explained as follows: consider the location of the consumer who is indifferent between buying from firm 1 and 3, given by $\widehat{z}_3$. For any set of prices, the optimal location of this indifferent consumer is mid-way between firms 1 and 3. With a merger, but without relocation, $\widehat{z}_3$ gets too close to firm 1, because of the merger-induced price increase. If firm 1 relocates (marginally) away from firm 3, then $\widehat{z}_3$ moves in the same direction, but by a smaller distance than firm 1. This implies that $\widehat{z}_3$ gets *relatively* closer to firm 3, and thus closer to the 'new' midpoint, which is a welfare improvement.

Combining Propositions 2 and 4, it is apparent that $a < \frac{1}{2}bl$ is a necessary condition for welfare improving relocations. It is difficult, though, to provide a further general characterisation of the condition given in Proposition 2, in terms of the parameters of the model. However, we can use the expression for $\overline{d}$ to analyse three special cases. If relocation is costless ($k = 0$), we find that $d^* \in \left(0, \overline{d}\right)$ if $a \in (0.44bl, 0.50bl)$. If transport costs are linear in distance ($b = 0$), relocation is always welfare detrimental as the condition $a < \frac{1}{2}bl$ cannot be satisfied. For quadratic transportation costs ($a = 0$), we know that the firms relocate in the 'right' direction. However, it turns out that the distance of relocation is always excessive, i.e. $d^* > \overline{d}$, and thus socially undesirable, for every value of b and k within the valid ranges.

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2.3 Partial collusion

So far we have assumed that the merger participants coordinate both the price setting and the relocation decisions. These are obvious assumptions if we regard the merged firm as a new fully integrated entity. However, the analysis of mergers when the different plants are maintained is similar to an analysis of collusion, as long as other effects, like e.g. cost synergies or defection, are not considered. Thus, the model presented in the previous section might also be interpreted as a cartel where the participants coordinate their decisions with respect to both strategic variables. Therefore, it is also interesting to ask the question of how the analysis would change if firms were able to coordinate decisions with respect to only one of the variables. There are several reasons why partial collusion might be relevant. For example, antitrust legislation may make price coordination infeasible, or at least difficult. It is reasonable to assume, though, that a coordination of relocation decisions is much less likely to be prohibited by antitrust authorities. Other examples where partial collusion might be relevant include franchises or regulation in which the franchiser, or the regulator, decides locations (prices) of the firms, but let these compete in prices (locations). As another example of partial collusion in prices, we can think of a situation in which the firms independently make relocation investments, anticipating that two of the firms might merge or collude in the future.[11]

2.3.1 Collusion in prices

To carry out this analysis we should firstly notice that we cannot *a priori* apply an argument of symmetry for the relocation distances of the colluding firms, since they must be treated as independent variables. Thus, let d_i denote the distance of relocation, measured in the clockwise direction, with respect to its original position for firm i. Consequently, the location of the indifferent consumers between firm i and firm $i+1$, $\widehat{z}_i$, is found by solving

$$p_i + t\left(\widehat{z}_i - \left(\frac{i-1}{3}\right)l + d_i\right) = p_{i+1} + t\left(\frac{i}{3}l + d_{i+1} - \widehat{z}_i\right),$$

while the profits are given by

$$\pi_i = p_i M_i - k d_i^2 = p_i(\widehat{z}_i - \widehat{z}_{i-1}) - k d_i^2. \tag{18}$$

At stage two of the game, firms 1 and 2 are assumed to coordinate their price setting. Profit maximisation leads to a system of equilibrium prices

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[11]In a somewhat different setting, the case of partial collusion in prices is also considered in Friedman and Thisse (1993), who analyse a location-then-price game when the firms anticipate collusion in prices.

$p_i\,(d_1, d_2, d_3)$, $i = 1, 2, 3$. By substituting $p_i\,(d_1, d_2, d_3)$ back into (18), we can express profits as functions of the relocation distances alone.

In the first stage of the game the colluding firms act independently, so that each firm maximises individual profits by choosing d_i. Using the fact that, by symmetry, $d_2 = -d_1$ and $d_3 = 0$, profit maximisation yields the following solution for d_1:

$$d_1 \equiv d_p = \frac{1944ak + 171abl + 648bkl + 87b^2l^2 - 9\sqrt{A}}{18b(9a + 5bl)}, \qquad (19)$$

where $A > 0$ is a function of the parameters of the model.[12]

Equilibrium prices and profits are found by substituting d_p for d in (11)-(14). It is straightforward to show that d_p is always non-negative,[13] which establishes the following Proposition:

Proposition 5 *Under partial collusion in prices, the colluding firms will relocate, if at all, away from the outside firm.*

The intuition is found by comparing with the case of full collusion, or merger. Consider the decision of firm 1 to possibly relocate as a response to price collusion with firm 2. When the firms do not coordinate their location decisions, there is an extra cost associated with moving away from this firm (i.e. moving towards firm 3). The gain in market share vis-à-vis firm 3 is accompanied by a loss of market share to firm 2. Consequently, the competition effect always dominates, and the firms engaged in price collusion will move closer together.

It is worth noting that the special case of linear transportation costs $(b = 0)$ implies no relocation. From (19) we find that

$$\lim_{b \to 0} d_p = 0.$$

The intuition is relatively straightforward. In this case price competition is not reduced by moving further away from firm 3, and there is no net gain of market share by moving in either direction.

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[12] $A = 46656a^2k^2 + 8208a^2bkl + 31104abk^2l + 121a^2b^2l^2 + 6912ab^2kl^2 + 5184b^2k^2l^2 + 154ab^3l^3 + 1392b^3kl^3 + 49b^4l^4$.

[13] It can be shown that

$$k > \frac{5a + bl}{16(3a + bl)}bl$$

must be satisfied to ensure an interior solution.

2.3.2 Collusion in locations

When the firms coordinate their location decisions but compete in prices the analysis is similar. The two main differences are that at the second stage firms maximise individual profits, whereas at the first stage the colluding firms maximise joint profits with respect to the relocation decisions. Following the same procedure as in the previous section and again applying arguments of symmetry, it is directly shown that prices are given by

$$p_1 = p_2 = \frac{(5l - 3d_l)(3a + 3bd_l + bl)(3a - 6bd_l + bl)}{9(15a - 12bd_l + 5bl)}, \qquad (20)$$

$$p_3 = \frac{(3a + 3bd_l + bl)(15al - 15bld_l + 5bl^2 + 18ad_l - 9bd_l^2)}{9(15a - 12bd_l + 5bl)}, \qquad (21)$$

with corresponding profits

$$\pi_1 = \pi_2 = \frac{(5l - 3d_l)^2(6a - 3bd_l + 2bl)(3a - 6bd_l + bl)(3a + 3bd_l + bl)}{54(15a - 12bd_l + 5bl)^2} - kd_l^2, \qquad (22)$$

$$\pi_3 = \frac{(3a + 3bd_l + bl)(15al - 15bld_l + 5bl^2 + 18ad_l - 9bd_l^2)^2}{27(15a - 12bd_l + 5bl)^2}, \qquad (23)$$

where d_l is the interior solution of the fifth-degree polynomial defined by $\partial(\pi_1 + \pi_2)/\partial d_l = 0$.

Unfortunately, and due to the fifth-degree nature of the problem, it is impossible to find an explicit expression for the interior solution. It can be shown, though, that $d_l < 0$ for every permissible value of the parameters. Again, the intuition is clearly tractable. If the firms do not coordinate their location and price decisions at all, we know that neither firm has any incentive to relocate, since the increased competition with the closer neighbouring firm more than offsets, in terms of profits, the decrease in competition with the other neighbour. However, if two of the firms are able to coordinate their location decisions, they can make sure, by both moving in the direction of the third firm, that the decrease in the degree of competition between them is sufficiently reduced to more than compensate for the increase in the degree of price competition with the third firm.[14] Moreover, as there is not any agreement between the colluding firms to increase their price, the outsider faces stronger competition and a lower market share, which eliminates any free-riding

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[14]The unique case which permits tractable analysis is the one with linear transportation costs ($b = 0$), in which $d_l = -\frac{5al}{3(25k-a)}$, where $k > \frac{9}{25}a$ ensures an interior solution. The quadratic case ($a = 0$) with no relocation costs ($k = 0$) implies $d_l = -0.027l$.

effect and even lower its profits compared with the situation with no collusion. The next proposition summarises these results:

Proposition 6 *With partial collusion in location, then*

(i) the colluding firms relocate towards the outsider and make higher profits than this firm,

(ii) the outsider makes less profits, compared with the case without collusion.

2.3.3 Welfare and profit comparisons

For the case of partial collusion in locations, it is easily shown that social welfare is maximised at $d = 0$. This is an obvious result. Since prices are set non-collusively, total transportation costs are always minimised with symmetric locations. Furthermore, it is also possible to show that partial collusion in locations is always preferred, from a welfare point of view, to full collusion (or merger). For the special cases of linear and quadratic transportation costs, it is also possible to show that partial collusion in *prices* is preferred to total collusion. Again, this is not too surprising.

Comparing welfare for the two different kinds of partial collusion, it can also be shown, for the case of linear transportation costs, that partial collusion in locations is socially preferred to partial collusion in prices if the cost of relocation is sufficiently large.[15]

The private incentives for the different kind of collusion do not necessarily correspond with the social incentives. For the colluding firms, full collusion is preferred to price collusion, which is preferred to collusion in location. For the outsider, full collusion and price collusion are both preferred to collusion in location. However, collusion in prices might be preferred to full collusion, at least for linear transportation costs.

3 Price discrimination

The model considered in the previous section has been set up in the classical Salop-Hotelling view of spatial product differentiation, where transportation costs are borne by consumers. In the taste-space interpretation the transportation cost is seen as a kind of *disutility* suffered by a consumer when she has to buy a product that differs from her most favoured one.

In this section we want to reproduce the previous analysis under the assumption that transportation costs are paid by the firms, and not the

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[15]For the case of $b = 0$ we find that social welfare is higher with partial collusion in locations, compared with partial collusion in prices, if and only if $k > \frac{1}{25}(19 + 6\sqrt{31})a$.

consumers. We maintain the standard assumption in this kind of models that firms are able to price discriminate among consumers, implying that they can charge different prices according to the cost of delivery. The physical-space interpretation of this model is straightforward. A taste-space interpretation is a bit more subtle, though. In this case one could think of the location of a firm as a particular product design or model to which the firm makes modifications according to customer preferences.[16] Thus, the cost of supplying the product to a particular consumer increases with the amount of changes required by the consumer.

We can apply the same model apparatus and notation as in the previous section, with the exception that transportation costs, $t\left(\cdot\right)$, are paid by the firms. Thus a consumer located at $z \in [0, l]$ and buying from firm i derives a utility given by

$$U = v - p_i,$$

where p_i represent the price charged by the firm at point z.[17] Firm i, located at x_i, pays per-unit transportation costs equal to $t\left(\psi_i\right)$ for deliveries to a consumer located at z, where ψ_i is given by (2). Since we have assumed zero production cost, $t\left(\psi_i\right)$ can be seen as the marginal cost of production for firm i at location z.

In order to analyse the price decisions at the second stage of the game, for any given location z we can make a ranking of the firms in terms of distance from z. Starting with the closest firm, we use the indices i, j and k. Thus, at location z there are three potential suppliers with three different marginal costs engaging in Bertrand competition. Consequently, firm i will be the only supplier of the product at point z and will charge a price equal to the second lowest marginal cost, i.e., $t\left(\psi_j\right)$. By this reasoning, firm i will capture all the market segments for which it is the closest firm, and will make profits given by

$$\pi_i = \int_{\Omega_{ij}} [t\left(\psi_j\right) - t\left(\psi_i\right)]dz + \int_{\Omega_{ik}} [t\left(\psi_k\right) - t\left(\psi_i\right)]dz, \qquad (24)$$

where Ω_{ij} (Ω_{ik}) represents the market segment for which firm i is the closest and firm j (k) is the second closest firm.

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[16] This also corresponds to the interpretation of Eaton and Schmitt (1994), where transportation costs are interpreted as the cost of producing variations on a basic product.

[17] As we will see later on, this price is in general a function of the consumer's location and the locations of the firms.

3.1 Merger

Once more, we want to use the no-merger equilibrium as a benchmark. In this case the model is completely symmetric, the firms have no incentives for relocation $(d_i = 0)$,[18] and total profits for firm i is given by

$$\pi_i = \frac{l^2(3a + bl)}{54}. \tag{25}$$

Let us now assume that firms 1 and 2 merge, or fully collude. Apart from jointly choosing locations in the first stage of the game, the two firms also agree not to invade each other markets in the price game, and face competition only from firm 3. We will refer to this kind of collusion as Market Sharing Agreement.[19] It is easy to see that it is in the best interest of the merger participants to divide the market according to an efficiency rule: firm 1 will only supply the market segment for which it has the lowest marginal delivery cost, and *vice versa*. By symmetry, and the fact that the merger participants coordinate their location decisions, we can *a priori* assume $d_1 = d$, $d_2 = -d$ and $d_3 = 0$. Using the fact that the market limits between any two firms are at the middle points, profits are given by

$$\pi_1 = \pi_2 = \frac{1}{108}(3d + l)(-27ad + 9bd^2 + 9al - 12bdl + 4bl^2) - kd^2, \tag{26}$$

$$\pi_3 = \frac{1}{54}(3d + l)^2(3a + 3bd + bl). \tag{27}$$

Maximising $(\pi_1 + \pi_2)$ with respect to d, we find that the optimal distance of relocation is given by $d^* = 0$. Thus,

$$\pi_1 = \pi_2 = \frac{(9a + 4bl)l^2}{108}, \tag{28}$$

$$\pi_3 = \frac{(3a + bl)l^2}{54}. \tag{29}$$

Proposition 7 *With three firms initially located equidistantly from each other, then*
    (i) a merger (full collusion) between two firms is always profitable,
    (ii) the non-participant's profits and the firms' locations are unaffected by the merger.

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[18]It is indeed straightforward to show that the symmetric outcome $x_i = \frac{(i-1)l}{3}$ is a Nash equilibrium in locations.

[19]A general treatment of this kind of collusion, albeit in a very different setting, is given by Belleflamme and Bloch (2001).

The only effect of the merger is that competition is reduced for the market segment between the merger participants. In this segment, the merged entity can set prices equal to the marginal delivery costs of the outside firm, and use the nearest located plant for deliveries. There is no scope for any strategic response from the outside firm. Furthermore, as the merging firms are not able to charge higher prices at any point in the market if they relocate,[20] total profits for the merger participants are maximised at locations where total transportation costs for the market segment controlled by the merged firms are minimised. Thus, there are no incentives to relocate away from the initial symmetric locations. This explains the results in Proposition 7, which also implies that there is no free-rider effect.[21]

3.2 Welfare

Using the previously established definition, social welfare in the model with discriminatory pricing is given by

$$W = vl - \sum_{i=1}^{3} \int_{\Omega_{ij}+\Omega_{ik}} t\left(\psi_i\right) dz - \sum_{i=1}^{3} kd_i^2. \tag{30}$$

Since, by symmetry, $d_3 = 0$ and $d_1 = -d_2 = d$ we can get an explicit expression for (30) as

$$W = vl - \frac{1}{108}(162ad^2 - 54bd^3 + 54bd^2l + 9al^2 + bl^3) - 2kd^2. \tag{31}$$

Since a merger does not affect locations, welfare is unaffected as well. It is also easily verified that (31) is maximised at $d = 0$, yielding

$$W = vl - \frac{9a + bl}{108}l^2, \tag{32}$$

which is identical to welfare in the model of mill pricing with no merger.

3.3 Partial collusion

3.3.1 Market Sharing Agreement

In this section we will assume that the colluding firms agree on an efficient sharing of the market segment that they are able to control, but make independent decisions about location. As the outcome of this analysis must be symmetric, and relocation investment of firm 3 is an

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[20]These prices are determined by the distance to firm 3, which remains constant.
[21]This is similar to the results in Reitzes and Levy (1995) for a merger between neighbouring firms.

independent variable, we can *a priori* make the assumption that $d_3 = 0$. However, d_1 and d_2 must be treated as independent variables. The profits of firm 1 are in this case given by

$$\pi_1 = \frac{1}{108}(3d_1 + l)(-9ad_1 + 18ad_2 + 9bd_2^2 + 9al + 12bd_2l + 4bl^2) - kd_1^2. \quad (33)$$

Maximising π_1 with respect to d_1, and using the fact that by symmetry $d_2 = -d_1$, we can easily solve for d_1 to obtain

$$d_1 \equiv d_m = \frac{2bl + 12k + 6a - \sqrt{6}\sqrt{6a^2 + 24ak + 24k^2 + 3abl + 8bkl}}{3b}, \quad (34)$$

where $k > \frac{3}{16}bl$ ensures an interior solution. It is easily verified that $d_m > 0$, implying that a market sharing agreement leads the colluding firms to relocate away from the outsider. Thus, the equivalent result for the model of mill pricing is replicated. These incentives arise because firm 1 can gain some market share from firm 2 by moving closer to this firm. This is accompanied by a corresponding loss of market share to firm 3. However, the market share gained from firm 2 is much more valuable since firm 1 does not face competition from firm 2, and is thus able to charge higher prices in this market segment. Consequently, firm 1 has an incentive to relocate towards firm 2, and *vice versa*. Both colluding firms would be better off, though, by forming an agreement to remain at the original locations.

Equilibrium profits for the colluding firms and the outsider are found by substituting d for d_m in (26)-(27). It is then easily verified that the market sharing agreement is profitable for the colluding firms. Moreover, since $\frac{\partial \pi_3}{\partial d} > 0$ and $d_m > 0$ this kind of collusion is also always profitable for the outsider. When the competing firms move further away, firm 3 is allowed to charge higher prices and serve a larger market segment.

The comparison between the outsider and the colluding firms is less direct, though, but we are clearly able to identify a possible free-rider effect for some combinations of parameter values. For the case of linear transportation costs $(b = 0)$ we have that $\pi_1 > \pi_3$ if $k > 0.89a$, whereas quadratic transportation costs $(a = 0)$ implies that $\pi_1 > \pi_3$ if $k > 0.64bl$. Thus, a free-rider effect is present for sufficiently low relocation costs, which is quite intuitive, given that competition is considerably reduced for the outside firm in this case.

3.3.2 Collusion in locations

Assume that firms do not reach any market sharing agreement, but coordinate their locational decisions. As d_1 and d_2 are not independent variables we can *a priori* assume that $d_1 = -d_2 = d$ and $d_3 = 0$. In this

case, profits for the colluding firms are given by

$$\pi_1 = \pi_2 = \frac{1}{108}(6d - l)(3d + l)(-6a + 3bd - 2bl) - kd^2, \qquad (35)$$

whereas profits for the outside firm is given by (27). Maximising $(\pi_1 + \pi_2)$ with respect to d, we find the optimal distance of relocation, d_l, to be given by

$$d_l = \frac{6bl + 24k + 24a - \sqrt{(24a + 24k + 6bl)^2 + 72b(2al + bl^2)}}{36b}. \qquad (36)$$

It is easily verified that $d_l < 0$, so once more, the equivalent result from the model of mill pricing is replicated. By moving towards the outsider, firm 1 gains some market share from this firm, without losing any customers to firm 2 since these firms coordinate locations. Furthermore, although the delivery cost to consumers between the merging firms increases they can also be charged higher prices since firm 2 also moves away from this market segment.

From (27) we know that $\frac{\partial \pi_3}{\partial d} > 0$. Thus, this kind of collusion always harms the outside firm, since it faces increased competition from both neighbours. Obviously, and by construction, partial collusion in location is always profitable for the colluding firms.

3.3.3 Welfare and profit comparisons

Using the measure of social welfare given by (30), it is possible to show that partial collusion in the price game (market sharing agreement) is socially more harmful than partial collusion in locations. More interesting, though, is a welfare comparison between full collusion and partial collusion.

Proposition 8 *When firms engage in price discrimination, full collusion (or a merger) between two firms is always preferred to partial collusion of either kind.*

The proof is straightforward. We know that social welfare is always maximised at symmetric locations, i.e. $d = 0$, which minimises total transportation costs. Since a merger, or full collusion, implies $d = 0$, whereas partial collusion yields $d \neq 0$, the result follows immediately.

Regarding the privates incentives, the colluding firms always prefer total collusion over any kind of partial collusion, and a market sharing agreement over collusion in location. The outsider, on the other hand, prefers a market sharing agreement over total collusion, which, in turn, is preferred to collusion in location.

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It may seem surprising that full collusion should be socially preferred to partial collusion. However, we have to be somewhat cautious with the interpretation when we perform a welfare analysis in this kind of models. With unit demand there is no efficiency loss associated with a price in excess of marginal costs. A price increase is just a one-to-one utility transfer from consumers to producers. Thus, we should perhaps be particularly careful about distributional issues when we consider welfare effects of collusion in this model.

One way to introduce a distributional dimension to the analysis is to assume the existence of a social planner that attaches weights α and $(1 - \alpha)$, respectively, to consumers' and producers' surplus. In the following, we will assume that $\alpha > \frac{1}{2}$, implying that the social planner puts a relatively stronger emphasis on consumers' surplus.

With the preferences of the social planner given by the parameter α, social welfare when two firms relocate a distance d and engage in a market sharing agreement is given by

$$W_m = \alpha \left[vl - \frac{1}{108} (18dl \, (2a + lb) + 54d^2 \, (a + bd + lb) + 11l^2 \, (3a + lb)) \right]$$

$$+ (1 - \alpha) \left[\frac{1}{54} (3d + 1)(-18ad + 18bd^2 + 12al - 6bdl + 5bl^2) - 2kd^2 \right].$$
$$(37)$$

On the other hand, if two firms relocate a distance d but do not engage in any market sharing agreement, social welfare is given by

$$W_{nm} = \alpha \left[vl - \frac{1}{108} (108bd^3 + 54bd^2l + 27al^2 + 7bl^3) \right]$$

$$+ (1 - \alpha) \left[\frac{1}{18} (3d + l)(-9ad + 9bd^26 + 3al - 3bdl + bl^2) - 2kd^2 \right]. \quad (38)$$

From the previous results in this section, we know that $W_m(d = 0)$ corresponds to merger, or full collusion, whereas $W_{nm}(d = 0)$ corresponds to no collusion.

Comparing (37) and (38) we can confirm that a merger is always harmful:

$$W_m(d = 0) - W_{nm}(d = 0) = \frac{1}{54}(1 - 2\alpha)(3a + 2bl) < 0.$$

From (37) it is easily verified that W_m is maximised at $d \neq 0$ for every $\alpha \neq \frac{1}{2}$. Letting d_w denote the optimal distance of relocation in the case of a market sharing agreement between two firms, it can be shown that $d_w < 0$ for $\alpha > \frac{1}{2}$. Thus, given that a market sharing agreement

has taken place, its negative effect on consumers can be reduced if the colluding firms relocate towards the outsider. However, from Section 3.3.1 we know that partial collusion in the price game implies $d > 0$. Thus, partial collusion, in the form of a market sharing agreement, is still more harmful than total collusion for every $\alpha > \frac{1}{2}$.

Regarding partial collusion in location we can easily show that W_{nm} is maximised at $d = 0$ for every value of α. Thus, partial collusion in location, which implies $d_l < 0$, is always socially harmful, irrespective of the social planner's preferences. However, partial collusion in location is preferred to full collusion if α is sufficiently large. For instance, with linear transportation costs ($b = 0$), a comparison of (37) and (38) shows that

$$W_{nm}(d = d_l) > W_m(d = 0)$$

if

$$\alpha > \frac{19a^2 + 36ak + 16k^2}{35a^2 + 68ak + 32k^2} > \frac{1}{2}.$$

4 Concluding remarks

The purpose of this paper has been to analyse how horizontal mergers might create incentives for relocation within a framework of spatial competition, and conversely, how the possibility of relocation might affect the profitability of non-participating firms, as well as locational efficiency (social welfare). In order to facilitate analytical tractability, we have used a rather simple set-up, where we consider a two-firm merger in an industry with three price-setting firms initially located in symmetric fashion on a circle. Given this specific industry structure, we have covered a variety of different assumptions about price setting and coordinating behaviour, including both the cases of *mill pricing* and *price discrimination*, as well as distinguishing between *merger* and *partial collusion* in either price setting or relocation decisions.

We have found that whether or not a merger creates incentives for relocation depends crucially on whether or not the firms engage in price discrimination. If firms are not able to price discriminate, a merger will generally lead to a relocation of the plants (products) of the merger participants, but the direction of relocation is ambiguous, and depends on the characteristics of the transportation cost (disutility) function. Regarding the effects of a merger on the profits of the non-participating firm, the possibility of relocation implies that the well-known free-rider effect could be either mitigated or reinforced, depending on the direction of relocation. If a merger leads to a relocation in the direction of the outside firm, we have shown the existence of a set of parameter values for which the free-rider effect vanishes. Thus, the possibility of

relocation could solve the 'merger paradox' even in the absence of price discrimination.

Except for the special case of linear transportation costs, partial collusion will always provide incentives for relocation, and the direction of relocation is not dependent of whether or not the firms are able to price discriminate. Perhaps the most interesting result in this dimension of the analysis is that total collusion (or merger) could be preferred to partial collusion, from a viewpoint of social welfare, if the firms engage in price discrimination. This result holds also for the case of a social planner who puts more weight on consumers' surplus than firm profits.

Due to the potential complexities involved in performing a joint analysis of the questions of merger and location choices in a spatial framework, we have been forced to consider a fairly particular set of assumptions. Important questions that are not touched on in our analysis include the possibility of entry to the industry. We have also made the analysis tractable by setting up a three-firm analysis, which implies that the non-participating firm has no incentives to relocate. Generally, though, we would expect the relocation incentives of non-participating firms also to be affected by a merger. Thus, the present analysis should perhaps be seen as a first stepping stone towards a more comprehensive understanding of the effects of merger and collusion in a spatial framework.

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