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Abstract

In many two-sided markets, platforms use intermediary agents to reach consumers at one side of the market. In addition to the usual externalities in two-sided markets, the use of agents creates an additional externality for the platforms. We study if and how competing platforms can internalize the externalities by imposing resale price maintenance (RPM) on the agents. By the appropriate use of RPM, the platforms can induce the fully integrated outcome. Using a specific example, we show that consumers' surplus is reduced when the equilibrium involves the use of minimum RPM, and consumers benefit when maximum RPM is used.

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1 Introduction

A two-sided market is characterized by a set of platforms and two distinct consumer groups that value each other's participation. Many industries— particularly within 'the new economy'–fit this description; video games, payment cards and online auctions are typical examples of two-sided markets, and companies such as MasterCard and eBay are well-known platforms. To maximize profits, a platform must consider the externalities between consumer groups as well as any horizontal externalities among platforms. An optimal pricing strategy will balance these concerns.

However, not all platforms can control the prices charged to both consumer groups. The present article studies two-sided markets with this property. Our key modeling feature is that platforms, while selling directly to one consumer group (the *direct* side), must use intermediary agents to reach the other group (the *retail* side). Many real-world two-sided markets have this structure. For example, a firm producing video games and consoles may contract directly with software developers, whereas it sells hardware and software through retail stores. In a similar vein, newspapers and commercial TV channels normally sell advertising slots directly, but may use distributors to reach readers and viewers. Because pricing is a complicated matter in any two-sided market, one could expect this feature to create additional problems. This conjecture turns out to be true. However, and most importantly, this article will also show that the presence of intermediary agents on one side of the market enables platforms to use sophisticated vertical contracts to internalize the multiple externalities. In particular, we will study how *resale price maintenance* (RPM) can be used for this purpose.[1]

As an illustration of our argument, consider the simplest case with a monopoly platform. The platform's best strategy is to charge a low price on the side that creates the largest total surplus and a high price on the other side. In the standard case without agents, the platform can adjust its margins optimally and maximize the industry profit. However, this is not possible if it needs an intermediary agent to reach one of the sides. The reason is that the platform's margin on the retail side now does two things: it determines the price charged by the agent, but it also affects the platform's pricing incentive on the direct side. Suppose for example that the platform wants to subsidize consumers on the agent's side. The agent should then be given a negative wholesale margin. However,

[1]There is some evidence of RPM practices in two-sided markets, as follows. Cover pricing on newspapers and magazines is widely used in Europe and the US. When launching Windows 95, Microsoft used a system of minimum advertised prices for its retailers. In a Japanese case from 2001 (*Sony Computer Entertainment Inc. v. JFTC*), Sony was faulted for imposing recommended retail prices for PlayStation software for wholesalers and retailers.

the negative margin will induce the platform to change the price on the direct side, which distorts the final outcome.[2] Note however that this problem can be resolved by restricting the agent's pricing flexibility. This is the role of RPM. By imposing an RPM clause on the agent, the platform can use the margin to correct its own pricing incentive on the direct side. This strategy allows it to again maximize the industry profit.

RPM is a much disputed practice in antitrust policy. In general, policymakers have been lenient toward maximum RPM, but frowned upon minimum and fixed RPM.[3] On the other hand, several authors have warned that two-sided markets require a different antitrust approach from ordinary markets (Posner, 2001; Evans, 2002; Wright, 2004). A second aim of this article is thus to investigate how the current antitrust policy in the EU and the US toward RPM holds up in a two-sided market.

The monopoly platform example above gives the basic intuition for why RPM can improve profits in a two-sided market. We build on this insight to analyze several important questions in a more general model. Our main aim is to study whether and how RPM can be implemented to maximize the industry profit when there are several competing platforms, which is less obvious than in the monopoly example above.[4] We analyze a noncooperative contracting game between two differentiated platforms that can use a set of intermediary agents when selling to one side of the market.

With competing platforms our main results are as follows. Without RPM there is no pair of bilateral, quantity-based contracts that enables the platforms to maximize the industry profit. This result changes when RPM is allowed. We show that with RPM, platforms can sustain the fully integrated prices on both sides of the market in a subgame-perfect equilibrium. The optimal nonlinear contracts and RPM clauses on the retail side are determined by the cross-group externalities and the level of platform substitutability. In general, platforms are tempted to compete too fiercely when making their sales to their direct customers.

When the cross-group externalities between the two sides of the market are both positive, this incentive is reduced by setting low margins for sales on the retail side. The

[2]A related issue arises if platforms deal sequentially with the two sides (Hagiu, 2006) or if one side lacks information about prices on the other side (Belleflamme and Peitz, 2014; Hagiu and Halaburda, 2014).

[3]A recent case—*Leegin Creative Leather Products, Inc. v. PSKS, Inc., S. Ct.*—has involved a softer treatment of minimum and fixed RPM in the US. These practices are still considered 'hard-core' infringements of EU law.

[4]In fact, previous work has found that competing platforms have little to gain by fixing the price on only one side of the market because the positive cross-group externalities will trigger fierce competition on one side if the price is high on the other side (Evans and Noel, 2005; Armstrong and Wright, 2007).

fiercer the platform competition, the lower the required margins. In turn, low margins on the retail side result in excessively low prices to the final customers on this side, which may be resolved with minimum RPM. If the platforms are poor substitutes for the direct customers, the competitive prices on this side will be too high. To correct for this, the margins on the retail side should be increased. However, this may cause the final price on this side of the market to be too high, and this incentive is curbed with maximum RPM. Hence, platforms will use minimum RPM if they are close substitutes and maximum RPM if they are substantially differentiated.

We also consider the case where the externality from the direct side to the retail side is negative. Now competition on the direct side needs to be curbed always. This involves setting a positive margin for the agents, and a need for maximum RPM to prevent final prices being too high on this side.

These results are derived with a very general demand specification. To understand how RPM may affect consumer welfare and to make a clear policy statement, we adopt a specific utility function and focus on the case with only positive cross-group externalities. Here, we find that when minimum RPM is used, its effect on the consumer surplus is negative, whereas when maximum RPM is used, the effect is positive. This suggests that the logic behind antitrust policy toward RPM in ordinary markets also applies in two-sided markets.

The present article extends the industrial organization literature on two-sided markets, in which Caillaud and Jullien (2003), Armstong (2006) and Rochet and Tirole (2006) are the seminal contributions. In particular, our article is related to an emerging strand of literature that studies how two-sided platforms can make use of vertical restraints. Research in this area, surveyed by Evans (2013), has shown for example that tying can soften platform competition (Amelio and Jullien, 2012) and that vertical integration and exclusivity contracts can facilitate platform entry (Lee, 2013). Our contribution is to consider RPM, which has not been done before.

However, a related issue is studied by Hagiu and Lee (2011). In a model of the videogame industry, they distinguish between two business strategies: under 'outright sale', a content provider lets his content distributor set the content price, whereas the provider retains price control under 'affiliation'. These regimes correspond to the cases with and without RPM in our paper.[5] Hagiu and Lee (2011) show that content providers will tend to contract with one distributor exclusively whenever they give up price control.

[5] Note that Hagiu and Lee (2011) label firms in a different way to us. In particular, our platforms correspond to their content providers, and our agents correspond to their content distributors.

A related result arises in our model because competing platforms are more likely to enter a common agency equilibrium if they can use RPM.

Outside the two-sided markets field, our article relates to the general literature on RPM. Recent work here has found that RPM can be used to sustain retail prices at the monopoly level in markets with multilateral externalities, for example when there is competition at both the upstream and downstream levels (Innes and Hamilton, 2009; Rey and Vergé, 2010). We prove a similar result for two-sided markets with externalities between consumer groups. However, unlike these articles, we find that maximum RPM is the most common restraint.

The rest of the article is structured as follows. Section 2 outlines the formal model. Section 3 considers the case with a monopoly platform. Section 4 contains our main analysis of the case with competing platforms and an extension with quantity competition. Section 5 conducts a welfare analysis with a linear demand system. Section 6 briefly discusses some of our assumptions. Section 7 then concludes. Most of the formal proofs are in the appendix.

2 The basic framework

We analyze a market with either a single monopolist platform, or two similar but differentiated platforms $i \in 1, 2$ that sell their products in a two-sided market. On one side of the market each platform sells directly to its customers. We will refer to this as 'the direct side', denoted by d. On the other side of the market, we will assume that the platform selects one among many homogenous and equally efficient agents, which will resell the product on the platforms' behalf to the final customer. We will refer to this as 'the retail side', denoted by r. Each platform incurs a constant (and symmetric) marginal cost, equal to c_d when selling to side d and c_r when selling to side r. Fixed costs are normalized to zero. The agents incur no costs except the prospective payments they make to the platform(s).

We will assume throughout that the final customers on each side pay linear prices (no fixed fees). We denote by p_s^i the price charged to customers on side $s \in d, r$ for platform $i \in 1, 2$, and we denote by $q_s^i = D_s^i(\mathbf{p}_s, \mathbf{q}_{-s})$ the resulting quantity demanded and consumed at side s for platform i, as a function of the price(s) charged on side s, $\mathbf{p}_s = (p_s^1, p_s^2)$, and the consumption or 'participation' on the opposite side $-s$, $\mathbf{q}_{-s} = \left(q_{-s}^1, q_{-s}^2\right)$.

For the case of two platforms our demand system comprises four products. For this system to be invertible and stable, it is required that the feedback loops between the two sides are convergent. It can be shown that this holds as long as the cross-group network externalities are not too strong.[6] We will assume that this is the case, and that our demand system therefore has a unique solution $\widetilde{\mathbf{q}}(\mathbf{p}) = (\widetilde{q}_s^i)$ in quantities demanded as a function of the prices set on each side. In the following, we will omit the tildes and denote these reduced-form quantities simply by $q_s^i(\mathbf{p}_s, \mathbf{p}_{-s})$ for $i \in 1,2$ and $s \in d,r$.

To ensure that the reduced-form demands are well behaved, we will impose some additional regularity conditions. First, we will assume that they are continuously differentiable in all prices and that the partial derivatives of q_s^i have the signs that we would expect. Specifically, we will assume that the goods consumed on side $s \in d,r$ are 'gross' substitutes as defined by Vives (1999, p. 145), i.e., that we have both $\partial q_s^i/\partial p_s^i < 0$ and $\partial q_s^i/\partial p_s^j > 0$ for $i \neq j \in 1,2$, and that direct effects dominate on each side separately but also overall, i.e., we have $\sum_k \partial q_s^k/\partial p_s^i < 0$ as well as $\sum_h \sum_k \partial q_h^k/\partial p_s^i < 0$, for $i \in 1,2$ and $s \in d,r$. Moreover, we will assume that if the indirect network effect from side $-s$ to side s is negative ($\partial D_s^i/\partial q_{-s}^i < 0$), then this implies $\partial q_s^i/\partial p_{-s}^i > 0$, $\partial q_s^i/\partial p_{-s}^j < 0$ and $\sum_k \partial q_s^i/\partial p_{-s}^k > 0$; and that if the indirect network effect from side $-s$ to side s is positive ($\partial D_s^i/\partial q_{-s}^i > 0$), then this implies $\partial q_s^i/\partial p_{-s}^i < 0$, $\partial q_s^i/\partial p_{-s}^j > 0$ and $\sum_k \partial q_s^i/\partial p_{-s}^k < 0$. Our assumptions are common in models of two-sided markets. It also turns out that, for the case of linear demand, our assumptions are not more restrictive than needed; for the linear demand system we use in Section 5, for example, it can be shown that all of our assumptions hold as long as the restriction $\partial q_s^i/\partial p_s^i < 0$ holds.

Throughout the analysis we will assume that buyers on side d always attach a positive value to participation on side r, i.e., that $\partial D_d^i/\partial q_r^i > 0$. On the other hand, we will allow buyers on side r to either value or dislike participation on side d, i.e., we can have either $\partial D_r^i/\partial q_d^i > 0$ or $\partial D_r^i/\partial q_d^i < 0$.[7]

2.1 Timing of the game

We will consider the following three-stage game: at stage 1 each platform $i \in 1,2$ (if allowed) decides on the level of a fixed, minimum or maximum resale price, v^i, which applies to any agent distributing i's product to side r. If RPM is not allowed, then the game simply starts at stage 2, which consists of two steps: at the first step, the agents engage in a bidding game to compete for the platforms' patronage. Each agent's bid

[6] See Filistrucchi and Klein (2013) for a formal analysis of this problem.

[7] These assumptions correspond to many real life markets. For example, advertisers on TV, in newspapers or in magazines always attach a positive value to consumption on the other side, whereas viewers or readers may either dislike or value advertisements.

consists of a (nonlinear) tariff $T^i(q_r^i)$, which is a function of the quantity of the product it distributes to side r.[8] At step two, each platform $i \in 1,2$ accepts at most one bid. Finally, at stage 3 active firms (i.e., platforms and agents) set prices, which are observed by all customers before demand is realized, and then payments are completed according to the terms of trade. All information is common knowledge.

Given our assumptions, we can write the profit of any agent a as $\pi^a = \sum_{k \in 1,2} \left\{ p_r^k q_r^k - T^k \right\}$ if it sells the goods of both platforms, and simply $\pi^a = p_r^i q_r^i - T^i$ if it sells the goods of platform i only. Similarly, platform i's profit can be written as $\pi^i = (p_d^i - c_d)\, q_d^i + T^i - c_r q_r^i$. The tariff T^i can take a wide variety of forms but is assumed to be differentiable almost everywhere.

We continue to solve the game in the usual way, looking for a subgame-perfect equilibrium $(\mathbf{v}^*, \mathbf{T}^*, \mathbf{p}^*)$. For the case with competing platforms, we will assume throughout that the firms can make their terms (v^i, T^i) at stages 1 and 2 conditional on whether the agent will serve one or two platforms at stage 3. This is a natural assumption, as the optimal contract terms generally will depend on whether the agent serves more than one platform. If the contract terms were not conditional on market configurations, then the platform and its agent may sometimes be stuck with an 'inefficient' contract, in which case they would like to renegotiate.

3 The case of a monopoly platform

We start by analyzing the situation with one monopoly platform. Because we are dealing with a single platform, we will simply drop the superscripts $i, j \in 1,2$ for the remainder of this section.

We first consider the fully integrated (industry-profit maximizing) outcome as a benchmark, i.e., the situation in which the platform is able to sell directly to *both* sides of the market simultaneously. Industry profits are $\Pi = \sum_{s \in d,r} (p_s - c_s)\, q_s$, and reach a maxi-

[8] One could assume that the tariffs depend on the prices and quantities on the opposite side of the market. However, prices and quantities are often more volatile on one side of the market compared with the other. For example, in a newspaper market the advertising price and/or quantity may change and fluctuate widely throughout the year, whereas the conditions on the reader side are more stable. This indicates that it may be very inefficient for the platform to fix prices or quantities on the direct side of the market, as compared with fixing prices on the retail side. It also indicates that it may be either prohibitively costly or very difficult for an agent to verify (for a court) the quantities and prices set by the platform on the opposite side of the market. Hence, we simply assume that the tariffs cannot depend on these variables. Moreover, our results demonstrate that if it is feasible for the firms to fix the prices on the retail side of the market, then it may not be beneficial for them to have the tariffs depend on the prices and quantities set on the direct side of the market as well.

mum at some price vector that we will denote by $\mathbf{p}^M = \left(p_d^M, p_r^M\right)$. We let $\Pi^M = \Pi\left(\mathbf{p}^M\right)$ denote industry profits when prices are set equal to $\mathbf{p}^M$.

The fully integrated firm's first-order conditions, evaluated at $\mathbf{p}^M$, are then given by

$$q_d^M + \sum_{s \in d,r} \left(p_s^M - c_s\right) \left.\frac{\partial q_s}{\partial p_d}\right|_{\mathbf{p}^M} = 0, \tag{1}$$

for the price on side d, p_d, and analogously,

$$q_r^M + \sum_{s \in d,r} \left(p_s^M - c_s\right) \left.\frac{\partial q_s}{\partial p_r}\right|_{\mathbf{p}^M} = 0, \tag{2}$$

for the price on side r, p_r. Here, q_d^M and q_r^M represent the participation on each side when prices are equal to $\mathbf{p^M}$. We assume that the monopoly markup on side d is nonnegative, $p_d^M - c_d \geq 0$.[9] On the other hand, we will assume that the monopoly markup on the retail side can take any value, positive or negative, $p_r^M - c_r \lesseqgtr 0$.[10]

Now suppose that the platform must use an agent in order to sell to side r. First we consider the case when RPM is not allowed. At stage 2 the agents compete by offering tariffs to the monopolist platform. Suppose the platform accepts the bid T. Given the contract T, the platform sets its price to side d while the agent sets the price to side r. We let (p_r^*, p_d^*) be the equilibrium prices at stage 3 as functions of the accepted contract terms. Given that the agents are homogenous, we know that T has to maximize the platform's profit subject to the winning agent's zero-profit condition.

$$\begin{aligned} \max_T \Big\{ (p_d^* - c_d)\, q_d\left(p_d^*, p_r^*\right) - c_r q_r\left(p_r^*, p_d^*\right) + T\left(q_r\right) \Big\} \\ \text{such that} \;\; p_r^* q_r\left(p_r^*, p_d^*\right) - T\left(q_r\right) = 0 \end{aligned} \tag{3}$$

We can then state the following lemma.

Lemma 1. *If (T^*, p_r^*, p_d^*) forms a subgame-perfect equilibrium, then the tariff T^* is continuous and differentiable at the quantity q_r^* induced by (p_r^*, p_d^*).*

Proof. See Appendix A.

[9] This assumption is made here for convenience. The case $p_d^M - c_d < 0$ is covered in Appendix B.

[10] Platforms sometimes incur a loss on one side of the market while making a profit on the other side. It is well known that Sony, for example, often makes a loss when selling videogame consoles to their final customers, while they make up for it by charging their videogame developers. In the same way, many newspapers make a loss when distributing their papers to readers (e.g., we see many free newspapers) while they make up for it by charging their advertisers.

In addition to simplifying the rest of the analysis, Lemma 1 also provides valuable insights into which contract arrangements *do not* occur in any equilibrium. Lemma 1 says that, in equilibrium, a slight increase or decrease in the quantity q_r sold to side r cannot induce a discontinuous change in the payment from the agent to the platform. The accepted tariff may have discontinuities, but the point of discontinuity (the threshold value) of, say, q_r cannot be equal to the equilibrium quantity $q_r^* = q_r(\mathbf{p}^*)$, because then $\mathbf{p}^*$ would not be immune to profitable deviations. The intuition for this is straightforward: in a two-sided market, the quantity sold to side r is a function of the quantity sold to the buyers on the direct side of the market. Hence, if for example T^* were to 'jump up' at the equilibrium quantity q_r^*, then the platform could induce a discrete increase in its profit by slightly adjusting its price or the quantity sold to side d (either up or down depending on the sign of the cross-group effect from side d to side r), so as to cause a slight increase in the quantity sold to side r. Obviously, the payment T^* cannot 'jump down', otherwise it would be profitable for the agent to increase the quantity sold by charging a slightly lower price to side r.[11] Note that this result gives us a reason to focus on price restraints, as we do here, and not, for example, quantity restraints or sales-forcing contracts.

Given that the contract $T(q_r)$ is accepted at stage 2, the firms proceed to set their prices at stage 3. We may note that the platform and the agent will not able to achieve the fully integrated outcome, even if they are both monopolists. To see this, notice that Lemma 1 together with our assumptions on demand imply that the agent's and the platform's profit functions are differentiable at the equilibrium point. The agent's first-order condition for profit maximization is therefore

$$(p_r - T')\frac{\partial q_r}{\partial p_r} + q_r = 0, \tag{4}$$

while the platform's first-order condition is

$$(p_d - c_d)\frac{\partial q_d}{\partial p_d} + (T' - c_r)\frac{\partial q_r}{\partial p_d} + q_d = 0. \tag{5}$$

In order for $\left(p_d^M, p_r^M\right)$ to form an equilibrium, (2) and (4) have to be aligned when evaluated at the optimal prices. By using the implicit function theorem[12], we obtain the

[11] Note that the intuition for this result resembles the intuition for Lemma 1 in O'Brien and Shaffer (1992).

[12] We can use the implicit function theorem because our demand system is invertible under the assumption that cross-group externalities are not too strong.

condition

$$T' - c_r < -\left(p_d^M - c_d\right)\frac{\partial D_d}{\partial q_r} < 0, \tag{6}$$

which says that the platform's markup on the retail side should be negative in order for the agent to fully take into account the positive feedback on the platform's sales on the direct side. On the other hand, it is easy to see that to induce the optimal price to side d, the platform needs to earn the full monopoly rent on the last unit sold to its agent. That is, we require that $T' - c_r = p_r^M - c_r$ when $q_r = q_r^M$. These two conditions generally do not hold simultaneously. We can therefore state the following result.

Proposition 1. *Without RPM, a nonlinear tariff is insufficient to induce the fully integrated outcome* Π^M*.*

An important insight from the literature on vertical restraints is that a successive monopoly, and also common agency settings with competing suppliers, can achieve the first-best level of profit by using simple nonlinear contracts, e.g., a two-part tariff with a marginal wholesale price equal to the manufacturer's marginal cost. Such a sellout contract will avoid double marginalization and allow the agent to maximize industry profits. The monopoly profit can then be shared or collected through a positive fixed fee. Proposition 1 shows that in a two-sided market this does not work. The reason is that the marginal wholesale price needs to be set high enough for the platform to fully take into account the indirect network effects when setting the price on side d—and a high marginal wholesale price will cause the agent to set the price on side r too high. The second-best contract (without RPM) therefore involves setting the marginal wholesale price below the monopoly price to side r, yet above the level that would secure the monopoly price to side r. All else being equal, this will cause too few sales to the retail side of the market, and either too many (in the case of a negative indirect network effect $\partial D_r/\partial q_d < 0$) or too few (in the case of a positive indirect network effect $\partial D_r/\partial q_d > 0$) sales to the direct side of the market. The platform is therefore left unable to extract its full monopoly profit. This suggests that there might be some scope for improving profits by letting the platform impose an RPM clause at stage 1 of the game, which leads us to our first main result.

Proposition 2. *If RPM is allowed, then a subgame-perfect equilibrium exists in which the platform imposes a fixed or maximum resale price* $v = p_r^M$ *at stage 1 and collects the fully integrated profit* Π^M *at stage 3.*

Note that the platform always imposes (a fixed or) *maximum* price at stage 1, never a minimum price. A maximum resale price imposed at stage 1 solves the problem fully for the platform by committing its agent not to set the price too high on the retail side. At stage 2 this will cause the agents to bid up the marginal wholesale price for the last unit sold on the retail side, T', which again will cause the platform to set the correct price p_d^M on the direct side of the market.

4 The case of competing platforms

Our results above show that there is a rationale for the use of RPM in two-sided markets and that maintained prices may be good for the platform's customers. When imposing a maximum resale price on its agent, the platform is allowed to internalize all the indirect network externalities, which may improve welfare.[13] One should perhaps expect this intuition to carry over to the case of competing platforms. Yet this is not necessarily the case, as we now demonstrate.

We start again with the fully integrated (industry-profit maximizing) outcome as a benchmark, i.e., the situation with a single firm that is fully integrated both horizontally and vertically. The overall profit is now given by $\Pi = \sum_{s\in d,r} \sum_{k\in 1,2} \left(p_s^k - c_s\right) q_s^k$, and we will again assume that it reaches its maximum for some unique price vector, which we will denote by $\mathbf{p}^M = \left(p_d^M, p_d^M, p_r^M, p_r^M\right)$. In the same way as before, we will denote by $\Pi^M = \Pi\left(\mathbf{p}^M\right)$ the industry profits when the prices are set equal to $\mathbf{p}^M$.

The fully integrated firm's first-order conditions, evaluated at the optimal prices $\mathbf{p}^M$, are now equal to

$$\rho_r^M := q_r^M + \sum_{s\in d,r} \sum_{k\in 1,2} \left(p_s^M - c_s\right) \left.\frac{\partial q_s^k}{\partial p_r^i}\right|_{\mathbf{p}^M} = 0, \tag{7}$$

for the price on side r, and

$$\rho_d^M := q_d^M + \sum_{s\in d,r} \sum_{k\in 1,2} \left(p_s^M - c_s\right) \left.\frac{\partial q_s^k}{\partial p_d^i}\right|_{\mathbf{p}^M} = 0, \tag{8}$$

for the price on side d, for $i \in 1,2$. We will continue to assume that the fully integrated monopoly markups are nonnegative on side d, $p_d^M - c_d \geq 0$, and that they can take any value on side r, $p_r^M - c_r \lesseqgtr 0$.[14]

Suppose next that the platforms must use an agent(s) in order to sell their goods to

[13] This is also largely confirmed by the linear demand example we use in Section 5, where we find that the use of maintained prices (weakly) increases the overall surplus for the buyers when platforms do not compete.

[14] The case $p_d^M - c_d < 0$ is again covered in Appendix B.

side r. We solve the game first for the case without RPM. We take advantage of the fact that Lemma 1 easily extends to the case of competing platforms.[15]

Lemma 2. *If* $(\mathbf{T}^{*}, \mathbf{p}^{*})$ *forms a subgame-perfect equilibrium in the game with competing platforms, then for each platform* $i \in 1,2$ *the accepted tariff* T^{i*} *is continuous and differentiable at the quantity* q_r^{i*} *induced by* $\mathbf{p}^{*}$.

Proof. See Appendix A.

Lemma 2 allows us to restrict the analysis to two-part tariffs, without loss of generality. Hence, in the following we will assume that the firms use two-part tariffs of the form $T^i(q_r^i) = w^i q_r^i + F^i$, for $i \in 1,2$, where w^i is the marginal wholesale price and F^i is a fixed fee.

In the game with competing platforms, the outcome at stage 3 depends on whether the platforms select a common agent or different exclusive agents at stage 2 of the game. Suppose first that the platforms select a common agent. We may then notice that in any candidate equilibrium with a common agent, the accepted bids $\mathbf{T}$ will have to maximize the sum of the platforms' profits, subject to the agent's zero-profit condition.

$$\max_{\mathbf{w},\mathbf{F}} \sum_{i\in 1,2} \left\{ \left(p_d^{i*} - c_d\right) q_d^i\left(\mathbf{p}_d^*, \mathbf{p}_r^*\right) + \left(w^i - c_r\right) q_r^i\left(\mathbf{p}_r^*, \mathbf{p}_d^*\right) + F^i \right\} \tag{9}$$

$$\text{such that} \sum_{i\in 1,2} \left\{ \left(p_r^{i*} - w_i\right) q_r^i\left(\mathbf{p}_r^*, \mathbf{p}_d^*\right) - F^i \right\} = 0$$

If this were not the case, then, all else being equal, any rival agent could obtain positive profits by offering and having both platforms accept slightly more attractive contract terms. However, just like in the case with a monopoly platform, a pair of nonlinear tariffs is generally not sufficient to induce the overall first-best outcome for the platforms and the agent. To see this, note that the agent's first-order condition with respect to retail prices is

$$\rho_r^i := q_r^i + \sum_{k\in 1,2} \left(p_r^k - w^k\right) \frac{\partial q_r^k}{\partial p_r^i} = 0, \tag{10}$$

for $i \in 1,2$, while for platform $i \in 1,2$ the first-order condition is

$$\rho_d^i := q_d^i + \left(p_d^i - c_d\right) \frac{\partial q_d^i}{\partial p_d^i} + \left(w^i - c_r\right) \frac{\partial q_r^i}{\partial p_d^i} = 0. \tag{11}$$

[15] This holds for any subgame, whether the platforms use a common agent or exclusive agents.

To induce the fully integrated prices on side r, we know that (7) and (10) have to be aligned. Evaluating the two expressions at the prices $\mathbf{p}^M$, and using the implicit function theorem, we obtain the condition

$$w^i - c_r = -\left(p_d^M - c_d\right) \sum_{k \in 1,2} \left. \frac{\partial D_d^k}{\partial q_r^i} \right|_{\mathbf{p}^M} < 0. \tag{12}$$

On the other hand, to induce the fully integrated prices on side d, (8) and (11) have to be aligned. When evaluating the two expressions at the optimal prices $\mathbf{p}^M$, we obtain the condition

$$w^i - c_r = \left(p_r^M - c_r\right) + \left. \frac{\sum_{s \in d,r} \left(p_s^M - c_s\right) \frac{\partial q_s^j}{\partial p_d^i}}{\frac{\partial q_r^i}{\partial p_d^i}} \right|_{\mathbf{p}^M} \lesseqgtr 0. \tag{13}$$

(12) and (13) are generally not the same, and we can therefore state the next result.[16]

Proposition 3. *In a common agency situation without RPM, a pair of nonlinear tariffs is generally not sufficient to induce the fully integrated outcome* Π^M. *As a consequence, a common agency equilibrium may not exist.*

Proof. See Appendix A.

Proposition 3 is an extension of Proposition 1 to the case of competing platforms. To induce the agent to take into account the positive indirect network effects exerted on the direct side of the market, the platforms should sell their goods at a wholesale price below cost according to (12). On the other hand, the wholesale margin should also take into account the platform's incentives when selling to the direct side of the market, which (because of platform competition) now includes the incentive to set low prices in order to steal customers from the rival. The latter implies that the appropriate wholesale margin (13) could be either positive or negative, depending on the indirect network externalities and the degree of competition between the platforms. Obviously, the wholesale margin cannot (on a general basis) achieve both goals simultaneously, and the platforms are therefore unable to extract monopoly rents.

Note finally that because the platforms are unable to achieve the fully integrated profit when using a common agent, this opens the possibility that the platforms will opt to use

[16] Note that another version of this result appears in an earlier version of Kind et al. (2015). They use a linear demand model to analyze a TV industry with viewers that dislike advertising.

exclusive agents instead. The intuition for this is as follows: in a one-sided market, e.g., if the platforms were to sell to side r only, we know that the platforms would always i) select a common agent and ii) supply this agent at cost ($w_i^i = c_r$), hence eliminating any upstream margins and maintaining retail prices at the monopoly level, as for example in Bernheim and Whinston (1985). The platforms could then collect the monopoly profits through fixed fees. In a two-sided market, however, supplying the distributor at cost is not necessarily optimal, because, in the presence of indirect network externalities, the wholesale margin $w^i - c_r$ also affects the platforms' optimal quantities and prices on the direct side of the market. Hence, without RPM, when choosing the wholesale margin, there is a trade-off between the concerns about the prices on the two sides of the market. This trade-off can sometimes lead to excessively high prices on the retail side, and to such an extent that the total industry profit would be greater with exclusive distribution—which, all else being equal, would involve lower prices on the retail side.

Proposition 3 suggests again that there is scope for improving profits by letting the platforms impose RPM clauses, which leads us to our main result.

Proposition 4. *If RPM is allowed, then a subgame-perfect equilibrium exists in which the platforms i) impose a resale price equal to p_r^M at stage 1, ii) adopt a common agent at stage 2 and iii) collect half the fully integrated profit Π^M at stage 3.*

- *If the indirect network effect on side r is negative ($\partial D_r^i / \partial q_d^i < 0$), then the appropriate RPM clause is always a (fixed or) maximum price.*
- *If the indirect network effect on side r is positive ($\partial D_r^i / \partial q_d^i > 0$), then the appropriate RPM clause is a (fixed or) minimum price iff the diversion ratio between the platforms on side d $\gamma_{dd}^{ij} := -\dfrac{\partial q_d^j}{\partial p_d^i} / \dfrac{\partial q_d^i}{\partial p_d^i}$ is sufficiently large, and a (fixed or) maximum price otherwise.*

Proof. See Appendix A.

Proposition 4 says that, even when platforms compete, an equilibrium always exists in which the platforms impose RPM clauses at the first stage of the game and extract the fully integrated profit at the final stage. Moreover, depending on the signs of the indirect network externalities and the degree of substitution between the platforms, the appropriate RPM clause may now turn out to be a minimum price. Recall from Proposition 2 that if the platform is a monopolist the appropriate RPM clause is always a (fixed or) maximum price.

In equilibrium the marginal wholesale price w^i is set according to (13). For the case $\partial D_r^i/\partial q_d^i > 0$, if we substitute (13) into $\rho_r^i - \rho_r^M$ and evaluate the resulting expression at the fully integrated prices, we find that $\rho_r^i - \rho_r^M < 0$ (in which case the appropriate RPM clause is a *minimum* price) as long as

$$\gamma_{dd}^{ij} > \frac{\gamma_{rd}^{ii} + \gamma_{rd}^{ij}}{1 - \gamma_{rr}^{ij}} \gamma_{dr}^{ii} - \frac{p_r^M - c_r}{p_d^M - c_d} \left(\gamma_{dr}^{ii} + \gamma_{dr}^{ij} \right). \tag{14}$$

In (14), $\gamma_{ss}^{ij} = \partial q_s^j/\partial p_s^i / \left(-\partial q_s^i/\partial p_s^i\right)$ is the (same-side) diversion ratio between the platforms on side $s \in d, r$; $\gamma_{dr}^{ii} = \partial q_r^i/\partial p_d^i / \left(-\partial q_d^i/\partial p_d^i\right)$ and $\gamma_{rd}^{ii} = \partial q_d^i/\partial p_r^i / \left(-\partial q_r^i/\partial p_r^i\right)$ are the cross-side diversion ratios from side d to r and side r to d, respectively, for platform $i \in 1, 2$—i.e., the fraction of the quantity lost by platform i on one side that is captured by platform i on the opposite side; and $\gamma_{dr}^{ij} = \partial q_r^j/\partial p_d^i / \left(-\partial q_d^i/\partial p_d^i\right)$ and $\gamma_{rd}^{ij} = \partial q_d^j/\partial p_r^i / \left(-\partial q_r^i/\partial p_r^i\right)$ are the cross-side diversion ratios between platforms, from side d to r and side r to d, respectively—i.e., the fraction of the quantity lost by platform i on one side that is captured by platform j on the opposite side.

Condition (14) says that, for the case of positive indirect network effects, the resale price has to be imposed as a (fixed or) *minimum* price whenever the diversion ratio between the platforms is sufficiently large on side d.[17] The intuition for this is straightforward. Suppose the platforms' wholesale markups are positive, i.e., $w^i - c_r > 0$ for $i \in 1, 2$. Given that the indirect network externality on the retail side of the market is positive, for each new customer captured on side d the platform gains $\partial D_r^i/\partial q_d^i$ new customers on side r, and each of them will net the platform $w^i - c_r$. This creates an additional incentive for the platform to set a low price on side d, and this incentive grows stronger the larger the indirect network externality $\partial D_r^i/\partial q_d^i$ becomes and the stronger the diversion between the two platforms becomes. To counter this incentive, the marginal wholesale price w^i should therefore be reduced. However, for a sufficiently low marginal wholesale price w^i, the agent would like to set the retail price p_r^i below p_r^M. The platform can prevent this by imposing a minimum price at stage 1 of the game.

[17] Note that the diversion ratio in a two-sided market is a more complex variable than the diversion ratio in a one-sided market. The reason is that the diversion ratio in a two-sided market not only reflects the degree of substitution between the products on one side of the market; it also reflects feedback effects from demand shifting to the other side of the market.

4.1 The case of quantity competition

An assumption sometimes imposed in the two-sided literature, either explicitly or implicitly, is that the platforms choose quantities or participation rates on one side of the market and prices on the other. This assumption guarantees the uniqueness of equilibria in the customer market, and hence the platform can achieve any desired allocation of goods to its customers on both sides of the market. See, e.g., Anderson and Coate (2005), Weyl (2010) and White and Weyl (2012).[18]

To see how the assumption of quantity competition affects our results, imagine that each platform $i \in 1,2$ chooses a quantity q_d^i on the direct side of the market, selling it at the market clearing price $p_d^i = P_d^i(\mathbf{q}_d, \mathbf{q}_r)$, while the agent still charges prices, facing demand functions $q_r^i = D_r^i(\mathbf{p}_r, \mathbf{q}_d)$ for $i \in 1,2$ on the retail side of the market. We will assume that $\partial P_d^i/\partial q_d^i < 0$, $\partial P_d^i/\partial q_d^j < 0$, $\partial P_d^i/\partial q_r^i > 0$ and $\partial P_d^i/\partial q_r^j = 0$.[19] The overall industry profit is now $\Pi = \sum_{k\in 1,2}\left(P_d^k(\mathbf{q}) - c_d\right)q_d^k + \sum_{k\in 1,2}\left(p_r^k - c_r\right)D_r^k$. We let $\left(\mathbf{p}_r^M, \mathbf{q}_d^M\right)$ denote the profit-maximizing prices and quantities for the fully integrated firm. The first-order conditions are

$$\rho_r^M := \sum_{k\in 1,2}\frac{\partial P_d^k}{\partial q_r^k}\frac{\partial D_r^k}{\partial p_r^i}q_d^k + \sum_{k\in 1,2}\left(p_r^k - c_r\right)\frac{\partial D_r^k}{\partial p_r^i} + D_r^i = 0, \tag{15}$$

with respect to the price p_r^i, and

$$\begin{aligned}\rho_d^M := P_d^i - c_d + \sum_{k\in 1,2}\left(\frac{\partial P_d^k}{\partial q_d^i} + \frac{\partial P_d^k}{\partial q_r^k}\frac{\partial D_r^k}{\partial q_d^i}\right)q_d^k & \\ + \sum_{k\in 1,2}\left(p_r^k - c_r\right)\frac{\partial D_r^k}{\partial q_d^i} &= 0,\end{aligned} \tag{16}$$

with respect to the quantity q_d^i, for $i \in 1,2$.

Consider a candidate equilibrium in which the platforms use a common agent, and let $(\mathbf{p}_r^*, \mathbf{q}_d^*)$ be the firms' equilibrium prices and quantities at stage 3. We then know that in any candidate equilibrium with a common agent, the accepted bids $\mathbf{T}$ will again have to

[18] Alternatively, one has to rely on the invertibility of the full demand system (comprising four products in our setting) and the contraction mapping approach used by Affeldt et al. (2013) and Filistrucchi and Klein (2013).

[19] In a newspaper advertising market, for example, $\partial P_d^i/\partial q_r^j = 0$ amounts to the assumption that the marginal profit of advertising in newspaper i is independent of the number of readers of newspaper 2, for given advertising volumes $\left(q_d^1, q_d^2\right)$ and for a given number of readers q_r^i of newspaper i, which seems reasonable.

maximize the sum of the platforms' profits, subject to the agent's zero-profit condition.

$$\max_{\mathbf{w},\mathbf{F}} \sum_{i\in 1,2} \left\{ \left(P_d^i\left(\mathbf{q}_d^*, \mathbf{q}_r^*\right) - c_d\right) q_d^{i*} + \left(w^i - c_r\right) D_r^i\left(\mathbf{p}_r^*, \mathbf{q}_d^*\right) + F^i \right\} \tag{17}$$
$$\text{such that} \sum_{i\in 1,2} \left\{ \left(p_r^{i*} - w^i\right) D_r^i\left(\mathbf{p}_r^*, \mathbf{q}_d^*\right) - F^i \right\} = 0$$

If not, there would be scope for earning positive profits for at least one of the agents. Assuming that the tariffs $\mathbf{T}$ are both continuous and differentiable at the equilibrium point[20], the agent's first-order conditions at stage 3 of the game are

$$\rho_r^i := \sum_{k\in 1,2} \left(p_r^k - w^k\right) \frac{\partial D_r^k}{\partial p_r^i} + D_r^i = 0, \tag{18}$$

for $i \in 1, 2$, while the first-order condition for platform i is

$$\rho_d^i := P_d^i - c_d + \left(\frac{\partial P_d^i}{\partial q_d^i} + \frac{\partial P_d^i}{\partial q_r^i}\frac{\partial D_r^i}{\partial q_d^i}\right) q_d^i + \left(w^i - c_r\right) \frac{\partial D_r^i}{\partial q_d^i} = 0, \tag{19}$$

for $i \in 1, 2$. We may then note that for $p_r^{i*} = p_r^M$ to hold, the condition $\rho_r^i = \rho_r^M$ has to hold when evaluated at the quantities and prices of the fully integrated firm. By symmetry, we can rewrite this condition as[21]

$$w^i - c_r = -q_d^M \left.\frac{\partial P_d^i}{\partial q_r^i}\right|_{\left(\mathbf{p}_r^M, \mathbf{q}_d^M\right)} < 0. \tag{20}$$

Again we may note that as long as $\partial P_d^i / \partial q_r^i > 0$, the marginal wholesale price should be set below the platform's marginal cost in order to induce the agent not to set the price too high.

On the other hand, we may note that for $q_r^{i*} = q_r^M$ to hold, the condition $\rho_d^i = \rho_d^M$ has to hold when evaluated at the quantities and prices of the fully integrated firm. We can

[20]The proof that the tariffs are in fact differentiable at the equilibrium point is analogous to the proof of Lemma 2 above. It is omitted here for the sake of brevity.

[21]Note that (20) is equivalent to condition (12), which we obtained for the case of price competition.

write this condition as[22]

$$w^i - c_r = \left(p_r^M - c_r\right) + \left.\frac{\left(\dfrac{\partial P_d^j}{\partial q_d^i} + \dfrac{\partial P_d^j}{\partial q_r^j}\dfrac{\partial D_r^j}{\partial q_d^i}\right) q_d^M + \left(p_r^M - c_r\right)\dfrac{\partial D_r^j}{\partial q_d^i}}{\dfrac{\partial D_r^i}{\partial q_d^i}}\right|_{\left(\mathbf{p}_r^M, \mathbf{q}_d^M\right)} \lesseqgtr 0. \quad (21)$$

We can see that the conditions (20) and (21) are generally not the same, and we may therefore conclude that simple nonlinear tariffs are not sufficient to sustain the fully integrated prices and quantities. We may then state the following result.

Proposition 5. *(Quantity competition) A pair of imposed resale prices equal to p_r^M, combined with a pair of marginal wholesale prices set according to condition (21), fully restores the integrated outcome Π^M at the final stage of the game.*

- *If the indirect network effect on side r is negative ($\partial D_r^i / \partial q_d^i < 0$), then the appropriate RPM clause is always a (fixed or) maximum price.*
- *If the indirect network effect on side r is positive ($\partial D_r^i / \partial q_d^i > 0$), then the appropriate RPM clause is a (fixed or) minimum price iff the degree of substitution between the platforms on side d, $\varphi_d := \dfrac{\partial P_d^i}{\partial q_d^j} / \dfrac{\partial P_d^i}{\partial q_d^i}$, is sufficiently large, and a (fixed or) maximum price otherwise.*

Proof. Given that the indirect network externality is positive, $\partial D_r^i / \partial q_d^i > 0$, that the marginal wholesale price is set according to (21) and that the platforms' quantities on side d are equal to q_d^M, the condition that the agent would like to set the price $p_r^i < p_r^M$ is equivalent to the condition that (21) < (20). With some rewriting we can express this condition as follows:

$$\varphi_d > \left.\frac{\left(\left(p_r^M - c_r\right) + q_d^M \dfrac{\partial P_d^i}{\partial q_r^i}\right) \displaystyle\sum_{k \in 1,2} \dfrac{\partial D_r^k}{\partial q_d^i}}{-\dfrac{\partial P_d^i}{\partial q_d^i} q_d^M}\right|_{\left(\mathbf{p}_r^M, \mathbf{q}_d^M\right)}, \quad (22)$$

[22]Note that condition (21) is analogous to condition (13), which we obtained for the case of price competition.

where $\varphi_d := \left. \frac{\partial P_d^i}{\partial q_d^j} / \frac{\partial P_d^i}{\partial q_d^i} \right|_{(\mathbf{p}_r^M, \mathbf{q}_d^M)}$ represents the degree of substitutability between the two platforms' services on side d when the prices and quantities are equal to $(\mathbf{p}_r^M, \mathbf{q}_d^M)$. The condition says that the appropriate price restraint is a *minimum* price as long as the degree of substitution between the platforms is sufficiently high on side d.

If the indirect network externality is negative, $\partial D_r^i / \partial q_d^i < 0$, condition (22) instead becomes the condition that the agent would like to set the price above the integrated price, $p_r^i > p_r^M$. Using symmetry, we may note that the fully integrated firm's first-order condition with respect to p_r^i, when evaluated at the optimum, can be expressed as

$$(p_r^M - c_r) + q_d^M \frac{\partial P_d^i}{\partial q_r^i} = \frac{D_r^i}{-\sum_{k \in 1,2} \frac{\partial D_r^k}{\partial p_r^i}} > 0. \tag{23}$$

According to this, the right-hand side of condition (22) is always negative whenever $\sum_{k \in 1,2} \partial D_r^k / \partial q_d^i < 0$. Hence, the appropriate price restraint is then *always* a maximum price. **Q.E.D.**

To show that the outcome described in Proposition 5 also is the outcome of a subgame-perfect equilibrium of the full game when RPM is allowed, the proof follows the same structure and is analogous to the proof of Proposition 4 above. It is omitted here for the sake of brevity.

We may note, similar to the case of price competition, that a positive network externality from side d to side r, $\partial D_r^i / \partial q_d^i > 0$, means that the appropriate RPM clause is sometimes a fixed or *minimum* price. The intuition for this result is the same as the intuition under price competition: with a positive indirect network externality, for each additional unit sold on side d the platform sells $\partial D_r^i / \partial q_d^i > 0$ additional units on side r. If the platform has a positive markup $w^i - c_r > 0$ on each unit sold to side r, then this may create an incentive to set the participation rate q_d^i on side d of the market too high, and this incentive grows stronger the larger the degree of substitution between the platforms becomes. This incentive can be dampened by reducing the upstream margin $w^i - c_r$, and by imposing a minimum resale price to prevent the agent from then setting the final price p_r^i too low.

5 A linear demand example

To get a sense of the potential implications of allowing RPM for the platforms' customers, we will consider an example with a single representative customer on each side of the market $s \in d, r$, each maximizing a surplus function equal to

$$V_s = \sum_{i \in 1,2} q_s^i - \frac{1}{2(1+\varphi_s)} \left(\sum_{i \in 1,2} \left(q_s^i\right)^2 + 2\varphi_s q_s^1 q_s^2 - 2n_s \sum_{i \in 1,2} q_{-s}^i q_s^i \right) - \sum_{i \in 1,2} p_s^i q_s^i. \tag{24}$$

From V_s we obtain the inverse demand functions

$$p_s^i = P_s^i\left(\mathbf{q}_s, \mathbf{q}_{-s}\right) = 1 - \frac{1}{1+\varphi_s}\left(q_s^i + \varphi_s q_s^j - n_s q_{-s}^i\right), \tag{25}$$

for $i \in 1,2$ and $s \in d, r$. In (24) and (25), $n_s \lesseqgtr 0$ is a measure of the indirect network effect and $\varphi_s \in (0,1)$ measures the degree of substitution between the platforms on side s.

When inverting the system (25) for side s, we obtain the following direct demand function as a function of prices on side s and quantities on side $-s$:

$$D_s^i\left(\mathbf{p}_s, \mathbf{q}_{-s}\right) = 1 + n_s \frac{q_{-s}^i - \varphi_s q_{-s}^j}{\left(1-\varphi_s\right)\left(1+\varphi_s\right)} - \frac{p_s^i - \varphi p_s^j}{1-\varphi_s}, \tag{26}$$

for $i \in 1,2$ and $s \in d, r$. To make the analysis analytically tractable, we have to impose some additional symmetry conditions. We therefore consider the case with positive and symmetric network externalities $n_d = n_r = n > 0$, and we will assume that the platforms are equally differentiated on both sides of the market, $\varphi_d = \varphi_r = \varphi$. To ensure a unique and economically meaningful solution for the reduced form quantities, our demand parameters must then satisfy $n + \varphi < 1$.

We focus on common agency equilibria in this example. This restriction does not affect our results in any significant way, but makes it easier to derive analytically tractable expressions for the whole parameter space. Equilibria with exclusive agents (see Proposition 3) could be ruled out formally by introducing small economies of scale on the retail side.[23] We also assume that the platforms' marginal production costs are the same on each side of the market, $c_d = c_r = c$.

For the case without RPM, we know that a unique Nash equilibrium $w^i = w^j = w_C^*$

[23] Allowing for equilibra with exclusive agents would only increase the welfare gains from allowing platforms to impose maximum prices, for the cases when the degree of substitution between platforms is very weak but positive.

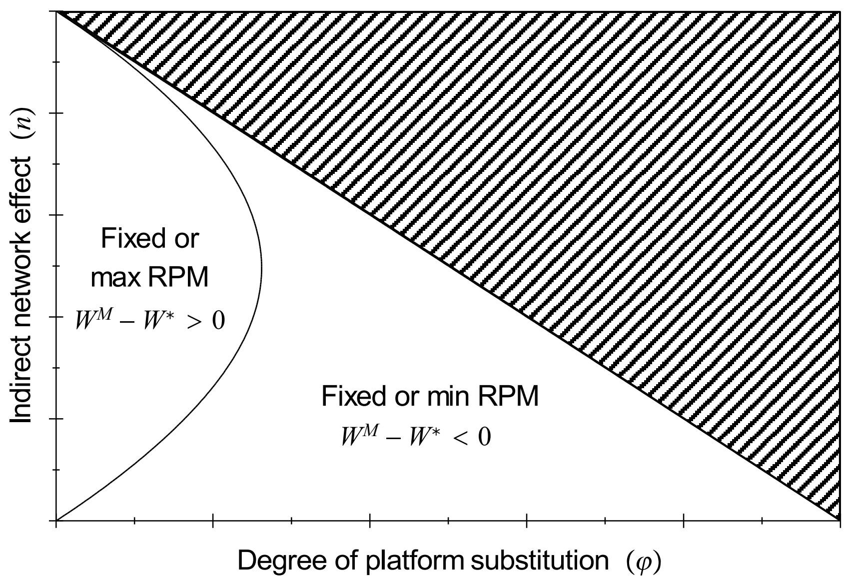

Figure 1: RPM regimes and their welfare effects when platforms compete by setting prices.

exists at stage 2 of the game, which is the solution to the maximization problems (9) and (17) under price and quantity competition, respectively. We define $W^M := \sum_{s \in d,r} V_s^M + \Pi^M$ and $W^* := \sum_{s \in d,r} V_s^* + \Pi^*$ as the overall welfare, with and without RPM, respectively. In Figures 1 and 2 we have plotted the loci for which maximum or minimum RPM is appropriate, and for which $W^M - W^* > 0$ and $W^M - W^* < 0$, for price and quantity competition between the platforms, respectively.

Notice that when the appropriate RPM clause is a minimum price, then the effect of RPM on overall welfare is negative, whereas when the appropriate RPM clause is a maximum price, the effect is positive. Banning fixed and minimum RPM clauses would therefore be beneficial in our example, whereas banning maximum prices would be detrimental.

Note that this is just an example, but the main intuition should apply more generally: when platform competition is strong, all else being equal, the platforms tend to set prices on the direct side that are too low compared with the prices a monopolist would set. When the wholesale prices are reduced, competition between the platforms is softened (given that the indirect network effects are positive) and the platforms will respond by increasing their prices. A reduction in the wholesale prices, however, will cause the agent to reduce his retail prices as well. This prevents the firms from fully eliminating competition, and as a result the equilibrium prices will end up below the monopoly level on both sides of the market. By imposing a minimum resale price, the platforms can prevent the agent

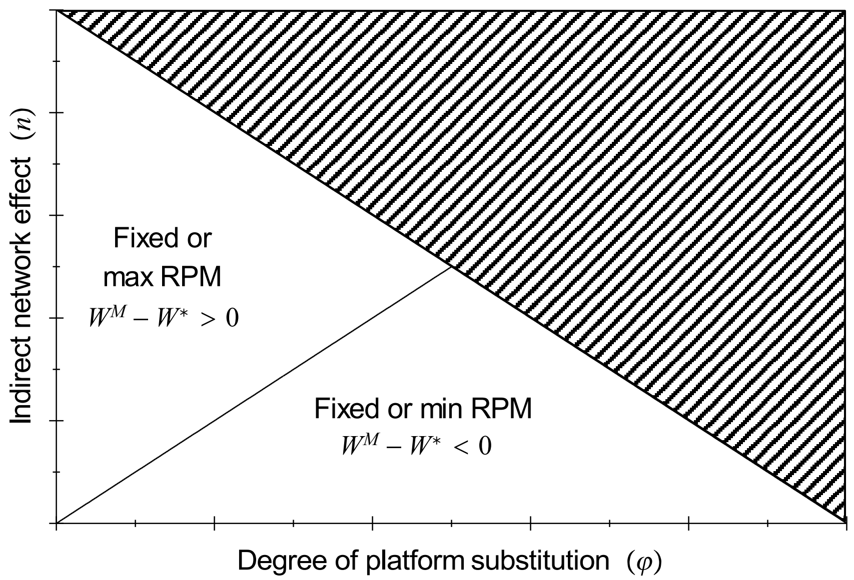

Figure 2: RPM regimes and their welfare effects when platforms compete by setting quantities.

from reducing his retail prices, and hence the prices can be increased to the monopoly level on both sides of the market simultaneously.

On the other hand, when platform competition is weak, the platforms tend to set prices on the direct side that are too high compared with what a monopolist would do. When the wholesale prices are increased, competition between the platforms is increased (given that the indirect network effects are positive) and the platforms therefore respond by reducing their prices. An increase in the wholesale prices, however, will cause the agent to increase his retail prices as well. This prevents the platforms and the agent from fully internalizing the positive network effects, and as a result the equilibrium prices will end up above the monopoly level on both sides of the market. By imposing a maximum resale price, the platforms can prevent the agent from increasing his retail prices, and hence the prices can be reduced on both sides of the market simultaneously.

Finally, we may note that the range for which minimum prices should be used is smaller when the platforms set quantities, compared with the range when they compete in prices. This is natural, given that competition is tougher when firms set prices compared with when they set quantities, all else being equal. The prices on both sides of the market are therefore generally lower in the Nash equilibrium without RPM when the platforms compete in prices, and a minimum price is therefore appropriate for a wider range of parameter values.

6 Discussion

Our analysis demonstrates how two rival two-sided platforms operating with a common agent may adopt RPM to achieve the fully collusive outcome. We believe that the main mechanism through which the platforms are able to achieve this in our model provides valuable insights for antitrust authorities and regulators, even though some of our assumptions may not fit particularly well in some markets. Below we briefly discuss the robustness of our results related to three specific assumptions.

First, real life retail markets often comprise imperfectly substitutable agents or outlets, and not isolated retail monopolies such as those in our model. Another layer of complexity would arise if we were to introduce multiple imperfectly substitutable retail locations in our model, as is done, for example, in Rey and Vergé (2010). As in Rey and Vergé (2010), however, we believe that not much would change in our results if we were to simply include a second retail location with its own set of agents. The fact that the second location is imperfectly substitutable for the first matters very little: the agents' bids would still have to satisfy their zero-profit condition; it would still be in the platforms' best interest to coordinate and use a common agent at each location (either because there are economies of scale or because the platforms can use RPM); and RPM would make it possible for the platforms to secure the first-best collusive outcome for exactly the same reasons as in our original model. However, because retail locations are competing, equilibrium prices will tend to be lower, which may imply that minimum resale prices will be appropriate over a wider range of parameter values.

Second, another contentious assumption is the premise that the agents are perfectly substitutable and therefore earn zero profit in equilibrium. The platforms in our model are therefore able to appropriate all of the profits created in the equilibrium. Yet, we will argue that our results would not change drastically if there were, for example, a single agent with monopoly power downstream: an equilibrium would still exist in which the platforms imposed RPM clauses at the first stage of the game, and the fully integrated prices and profits were maintained at the final stage. The only difference from our original framework would be that the platforms would have very little bargaining power vis-à-vis the agent, and would therefore be paid according to the value of their outside options—which in this case would be equal to zero because of the agent's monopoly power (assuming the platforms cannot bypass the agent). Hence, the platforms may not have particularly strong incentives to coordinate on the collusive equilibrium in this case.

Finally, a third possibility is, of course, a combination of the market conditions listed above: the downstream market may comprise imperfectly competing bottlenecks, as in the one-sided market studied in Rey and Vergé (2010, pp. 945–951). It is beyond the scope of this paper to describe what would happen in such a market. However, based on the results in Rey and Vergé (2010), in such a market an equilibrium may not exist in which all agents and platforms are active at the same time.

7 Concluding remarks

The existing literature on two-sided markets holds that rival two-sided platforms have little to gain by coordinating their prices at a high level on one side of the market only, as this will only induce them to compete more fiercely when selling to the other side. In this paper we have argued that this reasoning does not hold when platforms sell to one of the sides through an agent. More specifically, we have shown that two rival platforms can induce prices at the fully integrated level on both sides of the market by using resale price maintenance to fix their agent's prices on the retail side of the market. Moreover, we have shown that the appropriate RPM clause, i.e., whether it should be a minimum or maximum price, will depend on i) the sign of the indirect network effects on each side of the market, and ii) the degree of substitution between the two platforms.

Our paper adds to both the literature on two-sided markets and the literature on RPM. Regarding the literature on two-sided markets, our paper is the first to specifically study the effects of contractually determined RPM. Compared with a more conventional two-sided structure, where platforms sell directly to both sides, the presence of the intermediary agents in our framework opens the door for vertical restraints as efficient instruments. Regarding the RPM literature, we confirm that the efficiency, and possible necessity, of RPM as an instrument for internalizing multiple externalities and restoring monopoly prices might also carry over to two-sided markets.

Finally, we also investigate how the equilibrium use of RPM in our setting affects consumer surplus. Using a specific utility function and with positive cross-group effects, we find that when minimum RPM is used, its effect on consumer surplus is negative, whereas when maximum RPM is used, the effect is positive. This suggests that the logic behind antitrust policy toward RPM in ordinary markets applies also in two-sided markets.

Appendix A

Proof of Lemma 1 (Monopoly). The proof is structurally identical to the proof of Lemma 2 (see below) and is proved by deleting all superscrips i in the proof of Lemma 2. **Q.E.D**.

Proof of Lemma 2 (Duopoly). The proof is comprised of the following three steps:

Step 1. We show that T^{i*} is continuous at the quantities induced by $\mathbf{p}^*$. To see this, assume that T^{i*} is not continuous at the quantities induced by $\mathbf{p}^*$. Then, a marginal deviation, either positive or negative, from $q_r^i(\mathbf{p}^*) = q_r^*$ would cause a discrete change in T^i. If such a deviation causes T^i to 'jump up', then, since $\partial q_r^i/\partial p_d^i \neq 0$, platform i could adjust p_d^i slightly to change q_r^i, causing a discrete increase in his profits through a larger payment from the distributor. Since the jump up can be caused by both a positive and a negative marginal deviation from q_r^i, the appropriate adjustment of p_d^i depends on how q_r^i varies with p_d^i. For instance, to get $q_r^i > q_r^*$, platform i should reduce p_d^i slightly when q_r^i falls in p_d^i and increase p_d^i slightly when q_r^i rises in p_d^i. The opposite adjustments are needed to get $q_r^i < q_r^*$. If a marginal deviation causes T^i to 'jump down', platform j and the distributor could change T^j, i.e. q_r^j, slightly, resulting in a discrete increase in their bilateral profits. Thus, in both cases of discontinuity, at least one player has a profitable deviation. Hence, T^{i*} must be continuous at the quantities induced by $\mathbf{p}^*$.

By step 1., T^{i*} has both a right-hand (+) and a left-hand (−) partial derivative wrt. q_r^*, $\left(\frac{dT^{i*}}{dq_r^i}\right)_+$ and $\left(\frac{dT^{i*}}{dq_r^i}\right)_-$ respectively. For T^{i*} to be differentiable in equilibrium, we require that $\left(\frac{dT^{i*}}{dq_r^i}\right)_+ = \left(\frac{dT^{i*}}{dq_r^i}\right)_-$. We show this in two steps, as follows:

Step 2. We show that $\left(\frac{dT^{i*}}{dq_r^i}\right)_+ \geq \left(\frac{dT^{i*}}{dq_r^i}\right)_-$. For the agent, profit maximization requires that

$$\left(\frac{\partial \pi^a}{\partial p_r^i}\right)_- = \frac{\partial q_r^i}{\partial p_r^i}p_r^i + q_r^i - \frac{\partial q_r^i}{\partial p_r^i}\left(\frac{dT^{i*}}{dq_r^i}\right)_+ \geq 0 \tag{A1}$$

which we can rewrite as

$$\left(\frac{dT^{i*}}{dq_r^i}\right)_+ \geq \left(\frac{\partial q_r^i}{\partial p_r^i}p_r^i + q_r^i\right)\left(\frac{\partial q_r^i}{\partial p_r^i}\right)^{-1} \tag{A2}$$

and

$$\left(\frac{\partial \pi^a}{\partial p_r^i}\right)_+ = \frac{\partial q_r^i}{\partial p_r^i}p_r^i + q_r^i - \frac{\partial q_r^i}{\partial p_r^i}\left(\frac{dT^{i*}}{dq_r^i}\right)_- \leq 0 \tag{A3}$$

which we can rewrite as

$$\left(\frac{dT^{i*}}{dq_r^i}\right)_- \leq \left(\frac{\partial q_r^i}{\partial p_r^i}p_r^i + q_r^i\right)\left(\frac{\partial q_r^i}{\partial p_r^i}\right)^{-1} \tag{A4}$$

for $i \in \{1,2\}$. The left hand-side derivative of π^a includes the right-hand side derivative of T^{i*} because q_r^i is decreasing in p_r^i. $\left(\frac{dT^{i*}}{dq_r^i}\right)_+ \geq \left(\frac{dT^{i*}}{dq_r^i}\right)_-$ then follows directly from the rearranged inequalities.

Step 3. We show that $\left(\frac{dT^{i*}}{dq_r^i}\right)_+ \leq \left(\frac{dT^{i*}}{dq_r^i}\right)_-$. For the platform(s), two cases for the cross-group externality from side d to side r must be considered.

(i) : With a positive cross-group externality ($\partial q_r^i/\partial p_d^i < 0$), profit maximization requires that

$$\left(\frac{\partial \pi^i}{\partial p_d^i}\right)_- = \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right) + q_d^i + \left(\left(\frac{dT^{i*}}{dq_r^i}\right)_+ - c_r\right)\frac{\partial q_r^i}{\partial p_d^i} \geq 0 \tag{A5}$$

which we can rewrite as

$$\left(\frac{dT^{i*}}{dq_r^i}\right)_+ \leq \left(c_r\frac{\partial q_r^i}{\partial p_d^i} - q_d^i - \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right)\right)\left(\frac{\partial q_r^i}{\partial p_d^i}\right)^{-1}, \tag{A6}$$

and

$$\left(\frac{\partial \pi^i}{\partial p_d^i}\right)_+ = \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right) + q_d^i + \left(\left(\frac{dT^{i*}}{dq_r^i}\right)_- - c_r\right)\frac{\partial q_r^i}{\partial p_d^i} \leq 0 \tag{A7}$$

which we can rewrite as

$$\left(\frac{dT^{i*}}{dq_r^i}\right)_- \geq \left(c_r\frac{\partial q_r^i}{\partial p_d^i} - q_d^i - \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right)\right)\left(\frac{\partial q_r^i}{\partial p_d^i}\right)^{-1}. \tag{A8}$$

for $i \in \{1,2\}$. Hence, $\left(\frac{dT^{i*}}{dq_r^i}\right)_+ \leq \left(\frac{dT^{i*}}{dq_r^i}\right)_-$ by the same logic as in step 2.

(ii) : With a negative cross-group externality ($\partial q_r^i/\partial p_d^i > 0$), profit maximization requires that

$$\left(\frac{\partial \pi^i}{\partial p_d^i}\right)_- = \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right) + q_d^i + \left(\left(\frac{dT^{i*}}{dq_r^i}\right)_- - c_r\right)\frac{\partial q_r^i}{\partial p_d^i} \geq 0 \tag{A9}$$

which we can rewrite as

$$\left(\frac{dT^{i*}}{dq_r^i}\right)_- \geq \left(c_r\frac{\partial q_r^i}{\partial p_d^i} - q_d^i - \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right)\right)\left(\frac{\partial q_r^i}{\partial p_d^i}\right)^{-1}, \tag{A10}$$

and

$$\left(\frac{\partial \pi^i}{\partial p_d^i}\right)_+ = \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right) + q_d^i + \left(\left(\frac{dT^{i*}}{dq_r^i}\right)_+ - c_r\right)\frac{\partial q_r^i}{\partial p_d^i} \leq 0 \tag{A11}$$

which we can rewrite as

$$\left(\frac{dT^{i*}}{dq_r^i}\right)_+ \leq \left(c_r\frac{\partial q_r^i}{\partial p_d^i} - q_d^i - \frac{\partial q_d^i}{\partial p_d^i}\left(p_d^i - c_d\right)\right)\left(\frac{\partial q_r^i}{\partial p_d^i}\right)^{-1} \tag{A12}$$

for $i \in \{1,2\}$. Note that left hand-side derivative of π^i now includes the left-hand side derivative of T^{i*} because q_r^i is increasing in p_d^i. Again it follows that $\left(\frac{dT^{i*}}{dq_r^i}\right)_+ \leq \left(\frac{dT^{i*}}{dq_r^i}\right)_-$.

Together step 2 and step 3 therefore implies that $\left(\frac{dT^{i*}}{dq_r^i}\right)_+ = \left(\frac{dT^{i*}}{dq_r^i}\right)_-$, and thus T^{i*} is differentiable at the quantity q_r^{i*} induced by $\mathbf{p}^*$. **Q.E.D.**

Proof of Proposition 3.

Because the overall profit without RPM in a common agency situation is smaller than the fully integrated monopoly profit, there is the possibility that a subgame perfect equilibrium with common agency does not exist. Define Π_C^* as the overall equilibrium industry profit in the subgame with common agency and Π_N^* as the overall equilibrium profit in the subgame with exclusive agents. Without RPM we then have both $\Pi_C^* < \Pi^M$ and $\Pi_N^* < \Pi^M$. As a consequence we cannot rule out that $\Pi_C^* < \Pi_N^* < \Pi^M$; this is possible to show in an example with linear demand. In the latter case, a common agency equilibrium does not exist. The reason is that the maximum profit that the (least profitable) platform can make in the common agency situation, is $\Pi_C^*/2$. If $\Pi_C^* < \Pi_N^*$, then any agent whose offer was not accepted at stage 2 could have offered to the (least profitable) platform a contract that had secured the platform a profit of at least $\Pi_C^*/2$ if it accepted, and the agent could have kept the residual, $\left(\Pi_N^* - \Pi_C^*\right)/2 > 0$. **Q.E.D.**

Proof of Proposition 4. Without loss of generality, assume that the RPM clauses are fixed prices. In the following we will denote with subscript C the contract terms designed for common agency and with subscript N the contract terms designed for 'exclusive agency'.

First define $p_d^{1*} = p_d^1(\mathbf{v}_C, \mathbf{w}_C)$ and $p_d^{2*} = p_d^2(\mathbf{v}_C, \mathbf{w}_C)$ as the Nash equilibrium prices at the final stage of the game in the common agency situation, as functions of the resale prices $\mathbf{v}_C = (v_C^1, v_C^2)$ imposed at stage 1 and the unit wholesale prices $\mathbf{w}_C = (w_C^1, w_C^2)$ accepted at stage 2. Moreover, define (w_C^{1*}, w_C^{2*}) as the wholesale prices that solve

$$\Pi_C\left(v_C^1, v_C^2\right) = \max_{w_C^1, w_C^2} \sum_{i\in\{1,2\}} \left\{ \left(p_d^{i*} - c_d\right) q_d^i\left(\mathbf{p}_d^*, \mathbf{v}_C\right) + \left(v_C^i - c_r\right) q_r^i\left(\mathbf{v}_C, \mathbf{p}_d^*\right) \right\} \tag{A13}$$

Note that if $(v_C^1, v_C^2) = \left(p_r^M, p_r^M\right)$, then, according to (13), we have that

$$w_C^{i*} = p_r^M + \left.\frac{\sum_{s\in\{d,r\}} \left(p_s^M - c_s\right) \dfrac{\partial q_s^j}{\partial p_d^i}}{\dfrac{\partial q_r^i}{\partial p_d^i}}\right|_{\mathbf{p}^M} \tag{A14}$$

for $i \in \{1,2\}$, which, given that they were implemented, would induce the monopoly outcome $\Pi_C^* = \Pi^M$.[24]

Next, define $p_d^{1**} = p_d^1(\mathbf{v}_N, \mathbf{w}_N)$ and $p_d^{2**} = p_d^2(\mathbf{v}_N, \mathbf{w}_N)$ as the Nash equilibrium prices at the final stage of the game in a situation with exclusive agents, as functions of the resale prices $\mathbf{v_N} = (v_N^1, v_N^2)$ imposed at stage 1 and the unit wholesale prices $\mathbf{w}_N = (w_N^1, w_N^2)$ accepted at stage 2. Moreover, define (w_N^{1*}, w_N^{2*}) as the wholesale prices that simultaneously solve

$$\pi_N^i\left(v_N^i, v_N^j\right) = \max_{w^{iN}} \left\{ \left(p_d^{i**} - c_d\right) q_d^i\left(\mathbf{p}_d^{**}, \mathbf{v}_N\right) + \left(v_N^i - c_r\right) q_r^i\left(\mathbf{v}_N, \mathbf{p}_d^{**}\right) \right\} \tag{A15}$$

for $i \in \{1,2\}$. At stage 2, it is then an equilibrium that each agent bids (w_C^{i*}, F_C^{i*}) and (w_N^{i*}, F_N^{i*}), where

$$F_C^{i*} = \frac{\Pi_C\left(v_C^1, v_C^2\right)}{2} - \left[\left(p_d^{i*} - c_d\right) q_d^i\left(\mathbf{p}_d^*, \mathbf{v}_C\right) + \left(w_C^{i*} - c_r\right) q_r^i\left(\mathbf{v}_C, \mathbf{p}_d^*\right)\right], \tag{A16}$$

[24] Note that, depending the per-unit costs, the degree of substitution between the platforms and the size and signs of the indirect network externalities, the per unit price in (A14) could, in theory, become either positive or negative. In the latter case, we will assume that the platforms can require the agent to sell all that it purchases or, alternatively, that it can require the return of unsold units at the same per unit price w_i^{C*}.

and

$$F_N^{i*} = \pi_N^i\left(v_N^i, v_N^j\right) - \left[\left(p_d^{i**} - c_d\right) q_d^i\left(\mathbf{p}_d^{**}, \mathbf{v}_N\right) + \left(w_N^{i*} - c_r\right) q_r^i\left(\mathbf{v}_N, \mathbf{p}_d^{**}\right)\right], \tag{A17}$$

for $i \in \{1,2\}$, and where the platforms i) accept the offer of the same agent as long as $\Pi_C\left(v_C^1, v_C^2\right) \geq \sum_i \pi_N^i\left(v_N^i, v_N^j\right)$, and ii) accept the offers of different agents otherwise.

In order to break this stage 2 candidate equilibrium, an agent whose offer was not accepted would have to offer either $F_N^i > F_N^{i*}$, to attract one of the platforms, or $F_C^i > F_C^{i*}$ for $i \in \{1,2\}$ in order to attract both platforms simultaneously. We may note that neither is possible, as these offers would violate the agent's break even constraints. As all agents break even in the candidate equilibrium, an agent cannot increase his profit by offering $F_C^i < F_C^{i*}$ and/or $F_N^i < F_N^{i*}$ (and hence losing the contest). Hence, (w_C^{i*}, F_C^{i*}) and (w_N^{i*}, F_N^{i*}) form an equilibrium strategy at stage 2 for all agents. In the proposed equilibrium, each platform $i \in \{1,2\}$ therefore collects the profit $\Pi_C\left(v_C^1, v_C^2\right)/2$ as long as $\Pi_C\left(v_C^1, v_C^2\right) \geq \sum_i \pi_N^i\left(v_N^i, v_N^j\right)$, and $\pi_N^i\left(v_N^i, v_N^j\right)$ otherwise.

Next, define $v_N^i = p_r^E$ as the resale price that simultaneously maximizes $\pi_N^i\left(v_N^i, v_N^j\right)$ with respect to v_N^i for each platform $i \in \{1,2\}$ (we assume this includes the option not to fix the resale price), and define $\pi_N^1 = \pi_N^2 = \pi_N^*$ as the profit of each platform in the subgame with exclusive agents if the maintained prices are set according to $v_N^1 = v_N^2 = p_r^E$.[25]

Given the equilibrium at stage 2 described above, consider the platform's choice at stage 1. Suppose platform j's choice is for $\left(v_C^j, v_N^j\right) = \left(p_r^M, p_r^E\right)$. Note then that as long as the continuation equilibrium involves common agency, platform i's problem $\Pi_C\left(v_C^i, p_r^M\right)/2$ is maximized for $v_C^i = p_r^M$ as well. The only reason to pick $v_C^i \neq p_r^M$ would therefore be to induce exclusive agency at stage 2. The latter cannot be profitable, however, as π_N^* is the maximum profit that the platform may earn in the subgame with exclusive agents, and because we have that $\Pi^M > 2\pi_N^*$ per definition. Hence, $\left(v_C^i, v_N^i\right) = \left(p_r^M, p_r^E\right)$ is in this case an optimal response to $\left(v_C^j, v_N^j\right) = \left(p_r^M, p_r^E\right)$, and therefore forms an equilibrium choice for each platform at stage 1.

Next, note that, when the wholesale prices are set according to (A14), at the fully integrated prices $\mathbf{p}^M$ the condition that the agent would wish to reduce the price p_r^i is that (13) < (12). We have two cases to consider, depending on whether the indirect network effect $\partial D_r^i / \partial q_d^i$ is negative or positive.

[25] Note that this subgame is similar in structure to the set-up in Bonanno and Vickers (1988), with the difference that the two manufacturers (platforms) in our model may use RPM in addition to two-part tariffs, and that they sell to two sides of the market (and through a distributor to only one side).

Case 1. Suppose the indirect network effect is negative ($\partial D_r^i / \partial q_d^i < 0$). The condition that (13) > (12) and therefore that the appropriate resale price v_C^i is a maximum price, is

$$-\frac{\partial q_r^i}{\partial p_d^i} \sum_{k \in \{1,2\}} \frac{\partial q_r^k}{\partial p_r^i} \left(p_r^M - c_r\right) - \sum_{k \in \{1,2\}} \frac{\partial q_r^k}{\partial p_r^i} \sum_{s \in \{d,r\}} \left(p_s^M - c_s\right) \frac{\partial q_s^j}{\partial p_d^i} - \left(p_d^M - c_d\right) \frac{\partial q_r^i}{\partial p_d^i} \sum_{k \in \{1,2\}} \frac{\partial q_d^k}{\partial p_r^i} > 0 \tag{A18}$$

From the fully integrated monopolist's first-order condition with respect to p_r^i, we have that

$$p_r^M - c_r = \frac{q_r^M + \left(p_d^M - c_d\right) \sum_{k \in \{1,2\}} \frac{\partial q_d^k}{\partial p_r^i}}{-\sum_{k \in \{1,2\}} \frac{\partial q_r^k}{\partial p_r^i}} \tag{A19}$$

which is negative iff $q_r^M < -\left(p_d^M - c_d\right) \sum_{k \in \{1,2\}} \frac{\partial q_d^k}{\partial p_r^i}$. Substituting (A19) into (A18) and rearranging, we obtain the condition

$$q_r^M \sum_{k \in \{1,2\}} \frac{\partial q_r^k}{\partial p_d^i} + \left(p_d^M - c_d\right) \frac{\partial q_r^j}{\partial p_d^i} \sum_{k \in \{1,2\}} \frac{\partial q_d^k}{\partial p_r^i} - \left(p_d^M - c_d\right) \frac{\partial q_d^j}{\partial p_d^i} \sum_{k \in \{1,2\}} \frac{\partial q_r^k}{\partial p_r^i} > 0 \tag{A20}$$

Given that $\sum_{k \in \{1,2\}} \frac{\partial q_r^k}{\partial p_d^i} > 0$ and $\frac{\partial q_r^j}{\partial p_d^i} \leq 0$ (a negative indirect network effect), the left-hand side of (A20) is positive. Hence, the condition always holds and the appropriate resale price v_C^i is a maximum price.

Case 2. Suppose the indirect network effect is positive ($\partial D_r^i / \partial q_d^i > 0$). The condition that (13) < (12) and therefore that the appropriate resale price v_C^i is a *minimum* price, is the condition (A18). Adding $\left(p_r^M - c_r\right) \sum_{k \in \{1,2\}} \frac{\partial q_r^k}{\partial p_r^i} \sum_{k \in \{1,2\}} \frac{\partial q_r^k}{\partial p_d^i}$ to both sides of the inequality sign and dividing through by $p_d^M - c_d$, we obtain

$$-\frac{\partial q_d^j}{\partial p_d^i} \sum_{k \in \{1,2\}} \frac{\partial q_r^k}{\partial p_r^i} - \frac{\partial q_r^i}{\partial p_d^i} \sum_{k \in \{1,2\}} \frac{\partial q_d^k}{\partial p_r^i} > \frac{p_r^M - c_r}{p_d^M - c_d} \sum_{k \in \{1,2\}} \frac{\partial q_r^k}{\partial p_r^i} \sum_{k \in \{1,2\}} \frac{\partial q_r^k}{\partial p_d^i} \tag{A21}$$

Dividing through by $\frac{\partial q_d^i}{\partial p_d^i} \sum_{k\in\{1,2\}} \frac{\partial q_r^k}{\partial p_r^i}$, setting $\gamma_{ss}^{ij} = \partial q_s^j / \partial p_s^i / \left(-\partial q_s^i / \partial p_s^i\right)$, $\gamma_{dr}^{ii} = \partial q_r^i / \partial p_d^i / \left(-\partial q_d^i / \partial p_d^i\right)$ and $\gamma_{dr}^{ij} = \partial q_r^j / \partial p_d^i / \left(-\partial q_d^i / \partial p_d^i\right)$, and and rewriting, we get

$$\gamma_{dd}^{ij} > -\frac{\sum_{k\in\{1,2\}} \frac{\partial q_d^k}{\partial p_r^i}}{\sum_{k\in\{1,2\}} \frac{\partial q_r^k}{\partial p_r^i}} \gamma_{dr}^{ii} - \frac{p_r^M - c_r}{p_d^M - c_d} \left(\gamma_{dr}^{ii} + \gamma_{dr}^{ij}\right) \tag{A22}$$

Finally, may note that $-\sum_{k\in\{1,2\}} \frac{\partial q_d^k}{\partial p_r^i} / \sum_{k\in\{1,2\}} \frac{\partial q_r^k}{\partial p_r^i} = \frac{\gamma_{rd}^{ii} + \gamma_{rd}^{ij}}{1 - \gamma_{rr}^{ij}}$, when dividing by $\left(-\frac{\partial q_r^i}{\partial p_r^i}\right)^{-1}$ above and below the line, with $\gamma_{rd}^{ii} = \partial q_d^i / \partial p_r^i / \left(-\partial q_r^i / \partial p_r^i\right)$ and $\gamma_{rd}^{ij} = \partial q_d^j / \partial p_r^i / \left(-\partial q_r^i / \partial p_r^i\right)$. Hence, when the indirect network effects are positive, we arrive at the following condition that the appropriate resale price v_C^i is a minimum price,

$$\gamma_{dd}^{ij} > \frac{\gamma_{rd}^{ii} + \gamma_{rd}^{ij}}{1 - \gamma_{rr}^{ij}} \gamma_{dr}^{ii} - \frac{p_r^M - c_r}{p_d^M - c_d} \left(\gamma_{dr}^{ii} + \gamma_{dr}^{ij}\right), \tag{A23}$$

which says that the diversion ratio between the platforms on the direct side must be sufficiently high. **Q.E.D.**

Appendix B

In the following we will briefely consider the case where $p_r^M - c_r > 0$ and $p_d^M - c_d < 0$. Note that this only has implications for the consideration of whether the appropriate resale price is a maximum or minimum price. There are two cases to consider, depending on whether $\partial D_r^i / \partial q_d^i < 0$ or $\partial D_r^i / \partial q_d^i > 0$. We now show that the first case is already covered by Case 1 in the proof of Proposition 4 in Appendix A.

Case 1. Suppose the indirect network effect is negative ($\partial D_r^i / \partial q_d^i < 0$). We may note that the fully integrated monopolist's first-order condition with respect to p_d^i is

$$p_d^M - c_d = \frac{q_d^M + \sum_{k\in\{1,2\}} \frac{\partial q_r^k}{\partial p_d^i} \left(p_r^M - c_r\right)}{-\sum_{k\in\{1,2\}} \frac{\partial q_d^k}{\partial p_d^i}} \tag{B1}$$

The right-hand side of (B1) is positive as long as $p_r^M - c_r > 0$, $q_d^M > 0$ and $\sum_{k\in\{1,2\}} \frac{\partial q_d^k}{\partial p_d^i} < 0$. Hence, $p_d^M - c_d > 0$ always, and this case is therefore covered by Case 1 in the proof of Proposition 4 in Appendix A. In practice we therefore only have one case to consider, which we do next.

Case 2. Suppose the indirect network effect is positive ($\partial D_r^i/\partial q_d^i > 0$). Same as before, we find that the condition that (13) < (12) and therefore that the appropriate resale price v_C^i is a *minimum* price, is the condition (A18). Rewriting the condition, keeping in mind that $p_d^M - c_d < 0$, we find that the inequality (A23) is reversed. Hence, the condition that the appropriate resale price is a minimum price now becomes

$$\gamma_{dd}^{ij} < \frac{\gamma_{rd}^{ii} + \gamma_{rd}^{ij}}{1 - \gamma_{rr}^{ij}} \gamma_{dr}^{ii} - \frac{p_r^M - c_r}{p_d^M - c_d} \left(\gamma_{dr}^{ii} + \gamma_{dr}^{ij}\right) \tag{B2}$$

This condition is a bit harder to interpret than (A23). Yet, we can still show that, for a minimum price to be appropriate, one needs some degree of substitution between the platforms. To see this, consider the case with no substitution on either side of the market, $\gamma_{ss}^{ij} = \gamma_{rd}^{ij} = \gamma_{dr}^{ij} = 0$. The condition (B2) is reduced to

$$0 < \gamma_{rd}^{ii}\gamma_{dr}^{ii} - \frac{p_r^M - c_r}{p_d^M - c_d}\gamma_{dr}^{ii} \tag{B3}$$

Using the first-order condition with respect to p_d^i for the fully integrated monopolist, we can rewrite condition (B3) as

$$0 < \gamma_{rd}^{ii}\gamma_{dr}^{ii} - \frac{p_r^M - c_r}{\left((p_r^M - c_r) + \frac{q_d^M}{\frac{\partial q_r^i}{\partial p_d^i}}\right)} \tag{B4}$$

which never holds given that $p_r^M - c_r > 0$, $q_d^M > 0$, $\partial q_r^i/\partial p_d^i < 0$ and $\gamma_{rd}^{ii}\gamma_{dr}^{ii} < 1$. Hence, for a minimum price to be appropriate, we need some substitution between the platforms. However, different from the case $p_d^M - c_d > 0$, condition (B2) tells us that we now also need that the diversion ratio on the direct side, γ_{dd}^{ij}, is not too high compared to the diversion ratio on the retail side, γ_{rr}^{ij}.

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