WORKING PAPERS IN ECONOMICS

No. 10/08

EIRIK N. CHRISTENSEN

LEVELLING THE FIELD THROUGH SCORING AUCTIONS

Department of Economics
UNIVERSITY OF BERGEN

Levelling the Field through Scoring Auctions*

Eirik N. Christensen†

Department of Economics, University of Bergen

August 4, 2008

Abstract

This paper considers how price auctions compare with two-dimensional bidding on price and quality, when bidders have comparative advantages. Two-dimensional bids are evaluated by a scoring rule decided by the auctioneer and three auction types are evaluated: a) a scoring auction reflecting the auctioneer's true preferences; b) a scoring auction with "optimal" distortion of quality in the scoring rule; and c) a price-only auction with optimal quality threshold. The main findings are: 1) while the auctioneer always prefers the scoring auction, bidders may favour the price auction to the scoring auction and vice versa, depending on underlying conditions of the type space and cost parameters; and 2) the auctioneer can exploit firms' comparative advantages to level the field. An optimal scoring auction can, in some circumstances, extract all rent from bidders, leaving the auctioneer with all the efficiency gain from the bidding process. There even exists a knife-edge situation where the auctioneer can extract all rent when using his true preferences as the scoring rule.

JEL classification numbers: D44 (auctions), H57 (procurement)

Keywords: scoring auctions, private values, comparative advantages.

---

*Comments by Steinar Vagstad, Tommy Staahl Gabrielsen, and seminar participants at the University of Bergen, EEA 2007 in Budapest, and EAIRE 2007 in Valencia are gratefully acknowledged.

†Department of Economics, University of Bergen, Fosswinckels Gate 14, 5007 Bergen, Norway (eirik.christensen@econ.uib.no)

1 Introduction

In recent years, both practitioners and theorists have given more focus to scoring auctions, i.e., auctions where contestants are bidding on price and quality, and these multidimensional bids are ranked by a scoring function that generates a single dimensional value: "the score".[1] In the academic literature, large strides have been taken, particularly by Che (1993), Branco (1997), and Asker and Cantillon (2008). Nevertheless, there is still a lot to be done before we fully understand the properties of this auction format. Milgrom (2004) points out:

> "The idea that scoring can increase bidders' profits without reducing the auctioneer's value has been one of the main appeals of multidimensional bidding in procurement. Bidders (sellers) dislike bidding in price-only auctions in which their special advantages and characteristics receive no weight. By encouraging a more complete comparison of the attributes of suppliers and products, scoring may increase bidders' expected profits and encourage participation by more bidders, serving the interests of all parties. The theory does not give unqualified support to this intuitive argument. The conditions under which scoring benefits bidders and auctioneers alike remain an open question".

This paper builds upon the above-mentioned papers and extends their work to study how scoring auctions work when firms have comparative advantages, meaning that they have different costs associated with a given improvement of quality. Three auctions are compared: a) a scoring auction reflecting the auctioneer's true preferences for price versus quality; b) a scoring auction where the auctioneer is able to distort the scoring rule away from his true preferences towards price and quality;

[1]Public procurement accounted for nearly 16% of GDP in Europe in 2002. In the directive of public procurement in the EU, Article 53 states that contracting authorities can award public contracts either by "the tender most economically advantageous" or "the lowest price only". The contracting authorities shall specify in the contract documents the relative weighting that it gives to each of the criteria chosen to determine the most economically advantageous tender. In case it is not possible to weight these criteria (for demonstrable reasons), the authorities shall, as a minimum, rank the criteria in descending order of importance.

and c) a price auction with minimum quality threshold. I dub the first format the "naïve" scoring auction and the second the "optimal" scoring auction.[2]

When firms only differ in marginal costs of quality, i.e., there are no fixed cost differences, both bidders and the auctioneer prefer the naïve scoring auction to the price auction, but the auctioneer can improve his outcome by committing to an optimal scoring auction, not preferred by the bidders. This result is shown by Che (1993) and is replicated towards the end of section 3 of this paper.[3] Che (1993) considers a model with independent types (costs) and three variations of scoring auctions: the first-score (the winning firm delivers its bid combination); the second-score (the winning firm delivers any combination yielding the same score as the runner up); and the second-preferred-offer (the winning firm must deliver the runner up's exact combination). Che shows that with a scoring rule reflecting the buyer's true preferences, these three auction types give the same expected utility for the buyer, and, hence, it is a two-dimensional version of the revenue equivalence theorem (RET) (Vickrey (1961), Myerson (1981), and Riley and Samuelson (1981)). However, a scoring rule reflecting the true preferences of the buyer entails excessive quality under the first two formats, since they do not account for the informational costs associated with higher quality. This corresponds well to the mechanism-design literature (Laffont and Tirole (1987), (1993)). Che therefore considers other scoring rules and finds that the first- and second-score auctions can implement the optimal outcome (the second-preferred-offer is unable to do so), and this optimal scoring rule systematically discriminates against quality.[4]

The papers that are closest to the present one are Asker and Cantillon (2005) and (2008). In both papers, they study scoring auctions in which price enters linearly into the scoring rule and suppliers' private information about costs are multidimen-

[2] Note the quotation mark, as I only consider one way of distorting preferences, and there may exist more or less intricate ways to do the same.

[3] A path that is not followed in this paper is the effect of correlated costs. Branco (1997) studies this, and contrary to what Che (1993) finds, one-stage multidimensional mechanisms will not implement the first-best outcome. One will need a two-stage mechanism, where the first stage evaluates bids according to a scoring function, and a second round where the first-round winner bargains with the buyer.

[4] See Klemperer (2004) for a brief discussion of the similarities between Laffont and Tirole (1987) and Che (1993) and the connection to the Linkage Principle (Milgrom and Weber (1982))

sional, so they are able to consider situations where firms differ in their fixed and variable (marginal) costs. In this paper, suppliers' private information about costs are one-dimensional since I wish to highlight the effect of comparative advantages. Asker and Cantillon (2008) derive two sets of results. First, they describe equilibrium behaviour, and show the correspondence between scoring auctions and single dimension auctions for independent private values. Second, they compare the scoring auctions with other commonly used procedures to buy differentiated goods. They show that from the buyer's viewpoint, scoring auctions always strictly dominate price-only auctions with minimum quality standards, and dominate menu auctions and beauty contests depending on the auction format.[5] However, Asker and Cantillon (2008) do not discuss how the auctioneer can use comparative advantages to level the field through scoring auctions. In Asker and Cantillon (2005), comparative advantages is studied, but restricted to discrete distributions, while this paper looks at a continuous cost function. However, they do find, as this paper does, that scoring auctions do well compared with other commonly used procedures.

This paper shows that, for a certain set of preferences and cost functions, even the naïve scoring auction can level the field in such a way that all bidder profit is competed away. That means that it is a first-best solution, and the scoring auction reflecting the auctioneer's true preferences is indeed the optimal one. Moving away from this knife-edge situation, the scoring auctions can benefit both bidders and the auctioneer under certain conditions. The reason being that the scoring auction allows firms to exploit their comparative advantages in the bidding process. This results in an efficiency gain compared with the price auction, and this can be shared between the auctioneer and the bidders. However, in an optimal scoring auction, the auctioneer can distort his preferences in such a way that he is able to capture all the gains, and bidders will then prefer the price auction. This is most likely to happen when the type space is large and there exist comparative advantages.

The paper is organized as follows: Section 2 outlines the model where two firms compete. In Section 3, I show the outcome in the three auction formats, when firms

[5]For a discussion of price auction with quality thresholds, see Cripps and Ireland (1994) and Cabizza and De Fraja (1998), who also discuss quality considerations in auctions.

have comparative advantages or only differ in marginal costs. Section 4 concludes.

2 Model

Two risk-neutral firms compete to win the right to deliver a specific project. Firms' costs are given by:

$$c_i = c(\theta_i, s) = \frac{A}{\theta_i} + \theta_i s, \tag{1}$$

where $A \geq 0$ is a common cost parameter, s is quality, and θ_i is the firm's type, $i = 1, 2$. Let $\theta_i \sim U[\theta_L, \theta_H]$ and types are stochastically independently drawn. Firms have private knowledge about their own cost parameters. The first term in the cost function is a type-specific fixed cost, while the other term is variable costs. Costs are then not necessarily monotonically increasing in type, but can be decreasing for a sufficiently large A, reflecting comparative advantages. Some firms might be good at delivering high quality to a low variable cost (low θ_i), while others can deliver low fixed cost, associated with the project (high θ_i).

A risk-neutral auctioneer seeks to maximize his utility, given as the consumer surplus, V. The quantity delivered in the project is normalized to 1, so V is decided by the relationship between price, p, and quality, s. Because the quality parameter, being multidimensional by nature, can be computed as a one-dimensional number in monetary terms, price and quality can be evaluated along the same dimension. The consumer surplus is described as:

$$V(p, s) = v_0 + \sqrt{s} - p, \tag{2}$$

where $v_0 > 0$ is a common parameter for the consumers' surplus function. Because quality is concave in the consumer surplus function and linear in the cost function, I use first-order conditions to search for optimal quality. Based on his preferences, the auctioneer creates a scoring auction to maximize consumer surplus. Let the scoring function be given by:

$$\Omega(p, s, \alpha) = \alpha\sqrt{s} - p. \tag{3}$$

When $\alpha = 1$, the scoring function reflects the auctioneer's true preferences, while $\alpha \neq 1$ distorts from his true preferences. Price and quality can then be mapped into a one-dimensional score. Because the auctioneer and the bidders are risk neutral and the bidders' types are distributed according to an atomless continuous distribution, it follows:

Remark 1 *For any given scoring rule, first-score and second-score sealed bid auctions, English score auctions, and Dutch score auctions will all yield the same expected revenue, hence, the RET is satisfied.*

Satisfying the RET, English auctions are used in the setup for computational convenience. The timing of the model is as follows: first, the auctioneer decides which mechanism to use and publicly announces his decision; and second, the bidders post their bids, and the mechanism selects a winner. The competition is run as a scoring auction (bidding on price and quality) or as a price auction with minimum quality requirements (a quality level above the minimum requirement receives no weight in the auction).[6]

In the next section, I analyse the three auction types given the setup described above.

3 Auction Outcomes

In this section, I first look at the special case where $A = \frac{1}{4}$, and it turns out that the naïve scoring auction and the optimal scoring auction coincide. Next, I show that when $0 < A < \frac{1}{4}$, this is no longer true, and that the optimal scoring auction yields a higher expected consumer surplus than the naïve scoring auction and the optimal price auction. Lastly, I briefly show the case where firms only differ in marginal costs of quality, $A = 0$

[6] Technically, the price auction is a special case of the scoring auctions, where:

$$\Omega(p, s) = \begin{cases} \sqrt{s_0} - p & s \geq s_0 \\ 0 & \text{otherwise,} \end{cases}$$

and s_0 is the minimum quality threshold.

1) $A = \frac{1}{4}$

a) and b): The naïve (and optimal scoring) auction First, consider the English scoring auction where the auctioneer states his true preferences as the scoring function ($\alpha = 1$) and let t^W denote the winning score. To maximize consumers' surplus, the auctioneer needs bidders to set $p_i = c_i$, and maximize quality s_i. Each bidder maximizes:

$$\max_{(p,s)} \{p - c(s, \theta_i)\} \;\; s.t. \; \sqrt{s} - p = t^W, \tag{4}$$

and by substituting for p in the objective function this simplifies to a maximization problem where bidders choose s condition on being the winning bidder:

$$\max_{s} \left\{\sqrt{s} - c(s, \theta_i) - t^W\right\}. \tag{5}$$

I define:

$$k(\theta_i) = \max_{s} \left\{\sqrt{s} - c(s, \theta_i)\right\}. \tag{6}$$

Asker and Cantillon (2008) dub $k(\theta_i)$ the bidders' pseudotypes, which show the maximum score that bidder i can generate, and will be well-defined once the scoring rule is given. Inserting for the cost function in the maximization problem, and solving with respect to quality, s, yields the expression for the pseudotypes:

$$k(\theta_i^*) = \frac{1 - 4A}{4\theta_i}. \tag{7}$$

Consumers' surplus is increasing in θ_i if $A > \frac{1}{4}$ and decreasing if $A < \frac{1}{4}$. If $A = \frac{1}{4}$, the consumer surplus is independent of θ_i and the naïve scoring auction yields a first-best outcome. Hence, the naïve scoring auction is the optimal one.

c) The optimal price auction Now, assume that $A = \frac{1}{4}$ and turn to the price auction. Assume that the auctioneer has announced a quality threshold, s_0, that all bids must meet to compete, and that the winner is determined solely by the lowest price. When bidding, firms optimize with respect to costs given s_0 and θ_i:

$$\frac{\partial c_i}{\partial \theta_i} = -\frac{1}{4\theta_i^2} + s_0 = 0 \Leftrightarrow \theta_i = \frac{1}{2\sqrt{s_0}}. \tag{8}$$

So, unlike the scoring auction, there is only one type that can maximize consumer surplus given the chosen quality threshold, s_0. I can then state the following:

Proposition 1 *If $A = \frac{1}{4}$, all firms are able to deliver the same maximal consumer surplus in a naïve scoring auction. This is independent of their type and the distribution of types. In a price auction, only the type $\theta_i = \frac{1}{2\sqrt{s_0}}$ is able to deliver this surplus.*

Proof. See Appendix B. ■

Regardless of the type distribution, there exists a knife-edge situation where the auctioneer, using his true preferences as a scoring function, is able to level the field resulting in a first-best outcome. The scoring auction allows firms to exploit their comparative advantages when posting bids, with the consequence that there is no expected profit for the winning firm. However, in a price auction, the auctioneer needs to design the auction to fit the most efficient firm to reach first-best, and to ensure competition adjusting quality is needed; thus, the auctioneer is forced to leave some rent to the most efficient firm.

The result for the scoring auction is quite intriguing since it makes apparently different firms compete the profit away. This is, however, dependent of the shape of the auctioneer's preferences and the firms' cost functions. Nevertheless, it indicates that the scoring auction has properties that make, at least, the auctioneer better off. The next sections discuss how this result carries over when $0 < A < \frac{1}{4}$ and $A = 0$.[7]

2) $0 < A < \frac{1}{4}$

For notational ease, denote the expected score from a naïve scoring auction $E(\Omega(p,s)) = \Omega^s$, and let Ω^* denote the expected score in an optimal scoring auction $E(\Omega(p,s,\alpha))$. Expected consumer surplus $E(V(p,s,\alpha))$ is written as V^s, V^*,

[7] I do not discuss the situation where $A > \frac{1}{4}$ as this mirrors the results for $0 < A < \frac{1}{4}$.

and V^p for the three auctions. Let the expected profit for the bidders be given by, $\pi^s, \pi^*, and \pi^p$ for the naïve score, optimal score, and optimal price auction, respectively.

a) The naïve scoring auction First, consider the naïve scoring auction reflecting true preferences, $\alpha = 1$ in (3). Generally, the expected score from the auction is given by:

$$\Omega^s = \int_{\theta_L}^{\theta_H} \int_{\theta_L}^{\theta_H} \min \left(k\left(\theta_i^*\right)\right) dF\left(\theta_1\right) dF\left(\theta_2\right) \;\; i = 1, 2, \tag{9}$$

where the double integral is the expected maximum score that the pseudotype ranked as second best can deliver. Types are independently uniformly distributed over the square $\left[\theta_L, \theta_H\right] \times \left[\theta_L, \theta_H\right]$. The type space is illustrated in Figure 1, and I use the symmetry to focus on the area above the diagonal, i.e., where $\theta_1 < \theta_2$, and write (9) as:

$$\Omega^s = \frac{2}{\Delta^2} \int_{\theta_L}^{\theta_H} \int_{\theta_L}^{\theta_H} \min k\left(\theta_i^*\right) d\theta_1 d\theta_2 \; i = 1, 2, \tag{10}$$

where $\Delta = \theta_H - \theta_L$. Because $A < \frac{1}{4}$, $k\left(\theta_1^*\right) > k\left(\theta_2^*\right)$, and the winning score, t^W, is then the maximum score firm 2 can deliver, his pseudotype is:

$$t^W = k\left(\theta_2^*\right) = \frac{1-4A}{4\theta_2}. \tag{11}$$

Equation (10) then simplifies to:

$$\Omega^s = \frac{(1-4A)}{2\Delta^2}\left(\Delta - \theta_L \Theta\right),$$

where $\Theta = \ln \frac{\theta_H}{\theta_L}$. The corresponding expected consumers' surplus is thus given as:

$$V^s = v_0 + \frac{(1-4A)}{2\Delta^2}\left(\Delta - \theta_L \Theta\right). \tag{12}$$

Turning to the bidders, the expected profit is generally given by:

$$\pi^s = \int_{\theta_L}^{\theta_H} \int_{\theta_L}^{\theta_H} \left(\max k\left(\theta_i^*\right) - \min k\left(\theta_i^*\right)\right) dF\left(\theta_1\right) dF\left(\theta_2\right), \; i = 1, 2, \tag{13}$$

which, with the uniformly distributed cost parameters, yields:

$$\pi^s = \frac{1-4A}{2\Delta^2}\left(\left(\theta_H+\theta_L\right)\Theta-2\Delta\right). \tag{14}$$

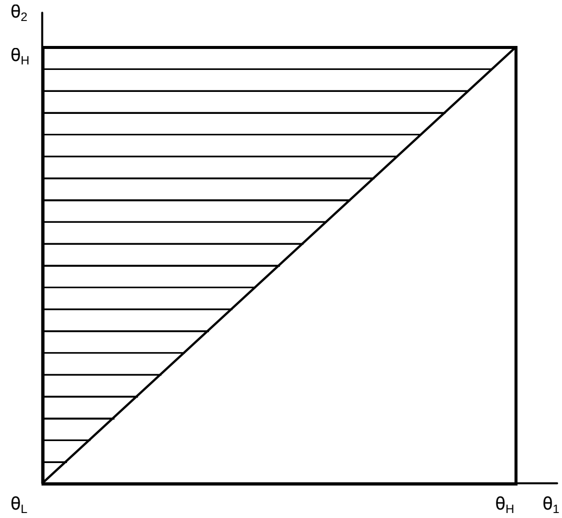


Figure 1: Illustration of the type space $[\theta_L, \theta_H] \times [\theta_L, \theta_H]$ .The shaded area represent the interval where $\theta_1 < \theta_2$.

b) The optimal scoring auction As pointed out by Che (1993), the auctioneer can, in certain environments, improve upon this outcome by distorting from his true preferences for quality. Specifically, the auctioneer distorts his preferences for quality downwards to limit the information rent associated with higher quality (Laffont and Tirole (1993)). In this case, let α in (3) take any value, and let α^* be the optimal α. Pseudotypes are given by:

$$\begin{aligned} k\left(\theta_i\right) &= \max_s \left\{\alpha\sqrt{s} - \frac{A}{\theta_i} - \theta_i s\right\} \\ k\left(\theta_i^*\right) &= \frac{\alpha^2-4A}{4\theta_i}, \end{aligned} \tag{15}$$

and since $0 < A < \frac{1}{4}$, firm 1 will have the lowest cost, and the winning score will be decided by firm 2's maximal score. The expected score in an optimal scoring auction is then given by:

$$\Omega^* = \frac{(\alpha^2 - 4A)}{2\Delta^2}(\Delta - \theta_L \Theta). \tag{16}$$

Firm 1's profit maximization problem is:

$$\begin{aligned} \pi_1^* &= \max_{p,s}\left\{p - \frac{A}{\theta_1} - \theta_1 s\right\} \text{ s.t. } \alpha\sqrt{s} - p = t^W = \frac{\alpha^2 - 4A}{4\theta_2} \\ &= \max_{s}\left\{\alpha\sqrt{s} - \frac{\alpha^2 - 4A}{4\theta_2} - \frac{A}{\theta_1} - \theta_1 s\right\}. \end{aligned} \tag{17}$$

The maximization of this yields the offered quality:

$$s_1 = \frac{\alpha^2}{4\theta_1^2}, \tag{18}$$

and, using the restriction in (17), yields the price:

$$p_1 = \alpha\sqrt{s_1} - t^W = \frac{\alpha^2}{2\theta_1} - \frac{\alpha^2 - 4A}{4\theta_2}, \tag{19}$$

and the expected profit is:

$$\pi_1^* = \frac{(\alpha^2 - 4A)}{4}\left(\frac{1}{\theta_1} - \frac{1}{\theta_2}\right). \tag{20}$$

The corresponding expected consumer surplus is:

$$V^* = v_0 + \frac{2}{\Delta^2}\int_{\theta_L}^{\theta_H}\int_{\theta_L}^{\theta_2}\left(\sqrt{s} - p\right)d\theta_1 d\theta_2,$$

and rewriting this yields:

$$V^* = v_0 + \frac{2}{\Delta^2}\int_{\theta_L}^{\theta_H}\int_{\theta_L}^{\theta_2}\left(\frac{\alpha(2-\alpha) - 4A}{4\theta_1} - \frac{(\alpha^2 - 4A)}{4}\left(\frac{1}{\theta_1} - \frac{1}{\theta_2}\right)\right)d\theta_1 d\theta_2. \tag{21}$$

The first term of the integrand in (21) relates to preferences towards quality, and the term is maximized when $\alpha = 1$. The last term reflects rent extraction. The auctioneer must balance his wish for optimal quality, on the one hand, against limiting the profit for the winning firm on the other. Also, from the last term in the integrand and (20), it becomes evident that the auctioneer can extract all rent from the bidders if he sets $\alpha = 2\sqrt{A}$. To see this, let $\tilde{\alpha}$ be the distortion that completely levels the field between pseudotypes:

$$\begin{aligned} k(\theta_1^*) &= k(\theta_2^*) \\ \frac{\tilde{\alpha}^2 - 4A}{4\theta_1} &= \frac{\tilde{\alpha}^2 - 4A}{4\theta_2}. \end{aligned} \tag{22}$$

For this to be the case, $\tilde{\alpha} = 2\sqrt{A}$, and the playing field is completely levelled. Bidders then compete all profit away, similar to the knife-edge result from proposition 1, where firms use their comparative advantages to win the price at the cost of not obtaining any expected profit. If the auctioneer sets $\alpha^* = 2\sqrt{A}$, the expected score is equal to zero, and the consumer surplus is:

$$\begin{aligned} V^*\left(\alpha^* = 2\sqrt{A}\right) &= v_0 + \frac{2\left(\sqrt{A} - 2A\right)}{\Delta^2} \int_{\theta_L}^{\theta_H} \int_{\theta_L}^{\theta_2} \frac{1}{\theta_1} d\theta_1 d\theta_2 \\ &= v_0 + \frac{2\left(\sqrt{A} - 2A\right)}{\Delta^2} \left(\theta_H \Theta - \Delta\right). \end{aligned} \tag{23}$$

If he sets $\alpha^* > 2\sqrt{A}$, he will maximize (21) with respect to α. Let $\hat{\alpha}$ denote this value. Integrating (21) yields the expected consumer surplus:

$$V^* = v_0 + \frac{1}{2\Delta^2}\left(2\alpha\left(\theta_H\Theta - \Delta\right) - \alpha^2\left(\left(2\theta_H + \theta_L\right)\Theta - 3\Delta\right) - 4A\left(\Delta - \theta_L\Theta\right)\right). \tag{24}$$

Maximizing (24) with respect to α yields:

$$\begin{aligned} \frac{dV^*}{d\alpha} &= 0 \\ &\Updownarrow \\ \alpha &= \hat{\alpha} \equiv \frac{\Theta\theta_H - \Delta}{\left(2\theta_H + \theta_L\right)\Theta - 3\Delta}, \end{aligned} \tag{25}$$

and, hence, the expected consumer surplus can be written as:

$$V^*(\alpha^* = \hat{\alpha}) = v_0 + \frac{1}{2\Delta^2}\left(\hat{\alpha}\left(\theta_H\Theta - \Delta\right) - 4A\left(\Delta - \theta_L\Theta\right)\right), \tag{26}$$

and the expected score will be:

$$\Omega^* = \frac{\left(\hat{\alpha}^2 - 4A\right)\left(\Delta - \theta_L\Theta\right)}{2\Delta^2}, \tag{27}$$

with bidders having an expected profit of:

$$\begin{aligned} \pi^* &= \frac{2\left(\hat{\alpha}^2 - 4A\right)}{4\Delta^2}\int_{\theta_L}^{\theta_H}\int_{\theta_L}^{\theta_2}\left(\frac{1}{\theta_1} - \frac{1}{\theta_2}\right)d\theta_1 d\theta_2 \\ &= \frac{\left(\hat{\alpha}^2 - 4A\right)}{2\Delta^2}\left(\left(\theta_H + \theta_L\right)\Theta - 2\Delta\right). \end{aligned} \tag{28}$$

Note that the auctioneer will never set $\alpha^* < 2\sqrt{A}$, as this would reverse the ranking of the firms, and make the firm with the highest cost the highest pseudotype. So $\alpha^* \geq 2\sqrt{A}$. However, since $\hat{\alpha}$ is independent of A, $\hat{\alpha}$ may be lower than $2\sqrt{A}$, and, if that is the case, the auctioneer will prefer to set $\alpha^* = \tilde{\alpha} = 2\sqrt{A}$. If $\hat{\alpha} > 2\sqrt{A}$, the auctioneer will prefer to set $\alpha^* = \hat{\alpha}$ since this yields a higher expected consumer surplus. The reason for $\hat{\alpha}$ being independent of A is that it relates to marginal cost, while A relates to fixed costs. The conditions for which the auctioneer will choose to level the field completely rather than let firm 1 receive some rent is dependent on:

$$2\sqrt{A} > \frac{\Theta\theta_H - \Delta}{\left(2\theta_H + \theta_L\right)\Theta - 3\Delta} \equiv \hat{\alpha}. \tag{29}$$

Holding θ_L in (29) fixed and taking limits indicate that when $\theta_H \to \theta_L$, $\hat{\alpha} \to 1$, and when $\theta_H \to \infty$, $\hat{\alpha} \to \frac{1}{2}$. When $A \to 0$, $2\sqrt{A} \longrightarrow 0$, and when $A \to \frac{1}{4}$, $2\sqrt{A} \longrightarrow 1$. So when the type space is small, there are smaller comparative advantages and the auctioneer will set $\alpha^* = \hat{\alpha}$, and in the limit his true preferences will be used in the optimal scoring auction. When the type space increases and there are comparative advantages (A increases), the auctioneer will be able to level the field and extract all rent. For this to be the case, $A > \frac{1}{16}$. So for low values of A, the auctioneer will always use $\hat{\alpha}$. Based on this, the auctioneer will set:[8]

$$\alpha^* = \max\left(2\sqrt{A}, \hat{\alpha}\right) = \max\left(2\sqrt{A}, \frac{\Theta\theta_H - \Delta}{(2\theta_H + \theta_L)\Theta - 3\Delta}\right). \tag{30}$$

The expected consumer surplus in the optimal scoring auction is:

$$V^* = \begin{cases} v_0 + \frac{1}{2\Delta^2}\left(\frac{(\Theta\theta_H - \Delta)^2}{(2\theta_H+\theta_L)\Theta - 3\Delta} - 4A(\Delta - \theta_L\Theta)\right) & \text{if} \quad \alpha^* = \frac{\Theta\theta_H - \Delta}{(2\theta_H+\theta_L)\Theta-3\Delta} \\ v_0 + \frac{2\left(\sqrt{A}-2A\right)}{\Delta^2}(\theta_H\Theta - \Delta) & \text{if} \quad \alpha^* = 2\sqrt{A}, \end{cases} \tag{31}$$

and the expected profit for the winner is:

$$\pi^* = \begin{cases} \frac{\left(\left(\frac{\Theta\theta_H - \Delta}{(2\theta_H+\theta_L)\Theta-3\Delta}\right)^2 - 4A\right)}{2\Delta^2}\left((\theta_H + \theta_L)\Theta - 2\Delta\right) & \text{if} \quad \alpha^* = \frac{\Theta\theta_H - \Delta}{(2\theta_H+\theta_L)\Theta-3\Delta} \\ 0 & \text{if} \quad \alpha^* = 2\sqrt{A}. \end{cases} \tag{32}$$

Proposition 2 *The optimal scoring auction generates a higher expected consumer surplus than the naïve scoring auction, and a lower expected profit for the bidders. The optimal scoring auction extracts all profit from bidders if* $2\sqrt{A} \geq \hat{\alpha}$.

Proof. See Appendix B. ■

The auctioneer is thus able to raise the expected consumer surplus by using an optimal scoring auction instead of a scoring auction reflecting his true preferences. However, this limits the information rent of the winning firm, which would prefer the naïve scoring auction. Because of the rent extraction, there will be an efficiency loss using the optimal auction, so the auctioneer needs strong commitment to be able to use the optimal form, for example, through legislation as within the EU directive for public procurement.

c) The optimal price auction The next question is then how the scoring auctions compare with the optimal price auction. In the optimal price auction, the auctioneer optimizes the quality threshold, s_0, before conducting the auction, i.e.,

[8]Note that for $0 < A \leq \frac{1}{4}$, the auctioneer will set $\alpha \leq 1$ with strict equality only when $A = \frac{1}{4}$, as the optimal scoring auction and the naive scoring auction then coincide.

he is maximizing the expected consumer surplus with respect to s_0. This auction is rather hard to solve analytically since what is the most cost efficient firm depends on the interval $[\theta_L, \theta_H] \times [\theta_L, \theta_H]$, the quality threshold, s_0, and the common fixed-cost parameter, A. To see this, assume that firm 1 has lower cost than firm 2:

$$\frac{A}{\theta_1} + \theta_1 s_0 < \frac{A}{\theta_2} + \theta_2 s_0.$$

For this to be the case, the following must hold:

$$\theta_1 > \frac{A}{s_0 \theta_2}.$$

It is clear that this will not always hold, and there might be intervals where firm 2 has the lowest cost. The optimal price auction then needs to be solved for four different intervals in the $[\theta_L, \theta_H] \times [\theta_L, \theta_H]$ space; for $s_0 > \frac{A}{\theta_L^2}$, $\frac{A}{\theta_L^2} > s_0 > \frac{A}{\theta_L \theta_H}$, $\frac{A}{\theta_L \theta_H} > s_0 > \frac{A}{\theta_H^2}$, and $\frac{A}{\theta_H^2} > s_0$. For the intervals $s_0 > \frac{A}{\theta_L^2}$ and $\frac{A}{\theta_H^2} > s_0$, I can find the expected consumer surplus and expected profit explicitly, while for the intervals $\frac{A}{\theta_L^2} > s_0 > \frac{A}{\theta_L \theta_H}$ and $\frac{A}{\theta_L \theta_H} > s_0 > \frac{A}{\theta_H^2}$, I need to use numerical simulations to solve the problem. The maximization problems are as follows:

$$V^P = \begin{cases} v_0 + \sqrt{s_0} - \frac{2}{\Delta^2} v_1 & \text{if} \quad \frac{A}{\theta_L^2} < s_0 \\ v_0 + \sqrt{s_0} - \frac{2}{\Delta^2} v_2 & \text{if} \quad \frac{A}{\theta_L \theta_H} < s_0 < \frac{A}{\theta_L^2} \\ v_0 + \sqrt{s_0} - \frac{2}{\Delta^2} v_3 & \text{if} \quad \frac{A}{\theta_H^2} < s_0 < \frac{A}{\theta_L \theta_H} \\ v_0 + \sqrt{s_0} - \frac{2}{\Delta^2} v_4 & \text{if} \quad s_0 < \frac{A}{\theta_H^2} \end{cases}, \tag{33}$$

and for expected profit:

$$\pi^p = \begin{cases} \frac{2}{\Delta^2} \beta_1 & \text{if} \quad \frac{A}{\theta_L^2} < s_0 \\ \frac{2}{\Delta^2} \beta_2 & \text{if} \quad \frac{A}{\theta_L \theta_H} < s_0 < \frac{A}{\theta_L^2} \\ \frac{2}{\Delta^2} \beta_3 & \text{if} \quad \frac{A}{\theta_H^2} < s_0 < \frac{A}{\theta_L \theta_H} \\ \frac{2}{\Delta^2} \beta_4 & \text{if} \quad s_0 < \frac{A}{\theta_H^2} \end{cases}, \tag{34}$$

the terms $v_1 - v_4$ and $\beta_1 - \beta_4$ are defined in Appendix A, where a full description of the solution and numerical simulation is given. The solution is also shown graphically in

Appendix A (Figures 2 - 5) Based on the numerical simulations of the price auction, I state the following:

Conjecture 1 *In an auction where bidders have comparative advantages, the optimal scoring rule always generates higher surplus than the naïve scoring rule and the optimal price auction for a given interval* $[\theta_L, \theta_H]$ *and* $A < \frac{1}{4}$*. The bidders can, however, realize a higher expected profit in the optimal price auction when* $A \to \frac{1}{4}$ *and the type interval grows larger.*

Summing up, I have found that there exist situations where both the auctioneer and bidders prefer the naïve scoring auction to the optimal price auction with quality threshold. However, the auctioneer can improve on the outcome by changing the weight attached to quality in the scoring auction. Then he can extract some, if not all, the rent from the bidders. This is done by allowing the firms to compete harder along their comparative advantages; thus, he is levelling the field and toughens competition.

3) $A = 0$.

To highlight some of the features of the scoring auction and show the relationship to Che (1993), I now turn to the case where firms only differ with respect to marginal costs, so $A = 0$. In this situation, the firms' cost function is given by:

$$c_i = \theta_i s, \; i = 1, 2. \tag{35}$$

a) The naïve scoring auction Assuming as above that $\theta_1 < \theta_2$, we focus on the shaded area above the 45° line in Figure 1. In the scoring auctions, the winning score will then be decided by firm 2's pseudotype. By setting $A = 0$ in equations (12) and (14), I find the expected consumer surplus and bidders' expected profit in the naïve scoring auction to be:

$$V^s = v_0 + \frac{1}{2\Delta^2} (\Delta - \theta_L \Theta), \tag{36}$$

$$\pi^s = \frac{1}{2\Delta^2} ((\theta_H + \theta_L) \Theta - 2\Delta). \tag{37}$$

b) The optimal scoring auction By the same reasoning, I find the expected consumer surplus and expected profit in the optimal scoring auction by setting $A = 0$ in equations (31) and (32):[9]

$$V^* = v_0 + \frac{(\Theta\theta_H - \Delta)^2}{2\Delta^2 ((2\theta_H + \theta_L)\Theta - 3\Delta)}, \quad (38)$$

$$\pi^* = \frac{(\Delta - \theta_H\Theta)^2 ((\theta_H + \theta_L)\Theta - 2\Delta)}{2\Delta^2 (3\Delta - (2\theta_H + \theta_L)\Theta)^2}. \quad (39)$$

c) The optimal price auction In this case, I can also solve the optimal price auction directly. The auctioneer maximizes the expected consumer surplus to find the quality threshold, s_0, before conducting the auction. Given the uniform distribution and firm 1 being the most cost effective one, the auctioneer must maximize the following expression:

$$\begin{aligned} V^p &= v_0 + \sqrt{s_0} - \frac{2}{\Delta^2}\int_{\theta_L}^{\theta_H}\int_{\theta_L}^{\theta_2} (s_0\theta_2)\, d\theta_1 d\theta_2 \quad (40) \\ &= v_0 + \sqrt{s_0} - \frac{(2\theta_H + \theta_L)\, s_0}{3}. \end{aligned}$$

Optimizing with respect to s_0 yields a quality threshold of:

$$s_0 = \frac{9}{4(2\theta_H + \theta_L)^2}, \quad (41)$$

which, in turn, gives us an expected consumer surplus of:

$$V^p = v_0 + \frac{3}{4(2\theta_H + \theta_L)}. \quad (42)$$

[9] Note that the auctioneer, in this case, always will set $\alpha^* = \widehat{\alpha}$, (see equation (25)) since this is always higher than $2\sqrt{A} = 0$.

Because higher θ_i indicates higher costs, the auctioneer should lower the quality threshold if θ_L or θ_H increases. Expected profit for the bidders will be given by:

$$\begin{aligned} \pi^p &= \frac{2s_0}{\Delta^2}\int_{\theta_L}^{\theta_H}\int_{\theta_L}^{\theta_2}(\theta_2-\theta_1)\,d\theta_1 d\theta_2 \\ &= \frac{3\Delta}{4\left(2\theta_H+\theta_L\right)^2}. \end{aligned} \tag{43}$$

I can then compare the three auction formats when $A = 0$, and state the following:

Proposition 3 *When there are only marginal cost differences, the expected consumer surplus is higher in an optimal scoring auction than in a naïve scoring auction, which, in turn, dominates the optimal price auction with quality threshold. The winning bidder's expected profit is higher in a naïve scoring than in the optimal scoring and price auction.*

Proof. See Appendix B ■

This means that both auctioneer and bidders will prefer the scoring auction to a price-only auction with quality threshold when there are only marginal cost differences. The reason is that there is an efficiency gain to be split between the auctioneer and the winning bidder by allowing quality to vary freely. So both the auctioneer and bidders can benefit from using a scoring auction instead of a price auction. For the auctioneer, this will always be the case even if he is not able to commit to an optimal scoring auction. The bidders will always prefer the naïve scoring auction to the optimal price auction. However, if the auctioneer is able to commit to an optimal scoring auction, there exist situations where the bidders would prefer an optimal price auction. It should come as no surprise that the naïve scoring auction results in higher total welfare than the two other mechanisms as these optimal mechanisms involve rent extraction from the bidders at the expense of an efficiency loss.

4 Conclusion

This paper shows that there exist situations where scoring auctions can benefit both the auctioneer and the bidders. When firms have comparative advantages, the scoring mechanism allows the bidders to exploit these advantages when forming their bids and toughens competition. If the auctioneer uses his true preferences as the scoring function, then both bidders and the auctioneer prefer the scoring auction to the optimal price auction when fixed costs are not too high and the type interval is small. There even exists a situation where the mechanism results in a first-best outcome and leaves the bidders with no expected profit. If the auctioneer can commit to an optimal scoring auction, then zero expected profit for the winners can be sustained for a broader interval, exploiting firms' comparative advantages. This will typically be the case when the common fixed-cost parameter is high (close to $\frac{1}{4}$) and the type space is large. For an auctioneer who maximizes consumer surplus, the optimal scoring auction dominates the naïve scoring auction that, in turn, dominates the optimal price auction. Bidders' profit will always be higher in a naïve scoring auction compared with an optimal scoring auction, and, up to a certain level of fixed cost, compared with the optimal price auction. Total welfare will always be higher in the naïve scoring auction since the two optimal mechanisms involve quality distortion and, thus, an efficiency loss.

References

Asker, J., and E. Cantillon (2005): "Optimal Procurement when Both Price and Quality Matter," *CEPR Discussion Paper*, No. 5276.

——— (2008): "Properties of Scoring Auctions," *RAND Journal of Economics*, 39, 69–85.

Branco, F. (1997): "The Design of Multidimensional Auctions," *RAND Journal of Economics*, 28, 63–81.

Cabizza, M. M., and G. De Fraja (1998): "Quality Considerations in Auctions for Television Franchises," *Information Economics and Policy*, 10, 9–22.

Che, Y. (1993): "Design Competition Through Multidimesional Auctions," *RAND Journal of Economics*, 24, 668–680.

Cripps, M., and N. Ireland (1994): "The Design of Auctions and Tenders with Quality Thresholds: The Symmetric Case," *The Economic Journal*, 104, 316–326.

Klemperer, P. (2004): *Auctions: Theory and Practice*. Princeton University Press.

Laffont, J.-J., and J. Tirole (1987): "Auctioning Incentives Contracts," *Journal of Political Economy*, 95, 921–937.

——— (1993): *A Theory of Incentives in Procurement and Regulation*. MIT Press.

Milgrom, P. (2004): *Putting Aution Theory to Work*. Cambridge University Press.

Milgrom, P., and R. J. Weber (1982): "A Theory of Auctions and Competitive Bidding," *Econometrica*, 50, 1089–1122.

Myerson, R. (1981): "Optimal Auction Design," *Mathematics of Operations Research*, 6, 58–73.

Riley, J., and W. Samuelson (1981): "Optimal Auctions," *American Economic Review*, 77, 375–387.

Vickrey, W. (1961): "Counterspeculation, Auctions and Competitive Sealed Tenders," *Journal of Finance*, 16, 8–37.

Appendices

A Optimal price auction

The full description of the maximization problems in (33) and (34) is for the consumer surplus:

$$V^P = \begin{cases} v_0 + \sqrt{s_0} - \frac{2}{\Delta^2} \underbrace{\int_{\theta_L}^{\theta_H} \int_{\theta_L}^{\theta_2} c_2 d\theta_1 d\theta_2}_{v_1} & \text{if} \quad \frac{A}{\theta_L^2} < s_0 \\ v_0 + \sqrt{s_0} - \frac{2}{\Delta^2} \underbrace{\begin{pmatrix} \int_{\theta_L}^{\sqrt{\frac{A}{s_0}}} \int_{\theta_L}^{\theta_2} c_1 d\theta_1 d\theta_2 \\ + \int_{\sqrt{\frac{A}{s_0}}}^{\frac{A}{\theta_L s_0}} \begin{pmatrix} \int_{\theta_L}^{\frac{A}{\theta_2 s_0}} c_1 d\theta_1 \\ + \int_{\frac{A}{\theta_2 s_0}}^{\theta_2} c_2 d\theta_1 \end{pmatrix} d\theta_2 \\ + \int_{\frac{A}{\theta_L s_0}}^{\theta_H} \int_{\theta_L}^{\theta_2} c_2 d\theta_1 d\theta_2 \end{pmatrix}}_{v_2} & \text{if} \quad \frac{A}{\theta_L \theta_H} < s_0 < \frac{A}{\theta_L^2} \\ v_0 + \sqrt{s_0} - \frac{2}{\Delta^2} \underbrace{\begin{pmatrix} \int_{\theta_L}^{\frac{\sqrt{A}}{\sqrt{s_0}}} \int_{\theta_L}^{\theta_2} c_1 d\theta_1 d\theta_2 \\ + \int_{\frac{\sqrt{A}}{\sqrt{s_0}}}^{\theta_H} \begin{pmatrix} \int_{\theta_L}^{\frac{A}{\theta_2 s_0}} c_1 d\theta_1 \\ + \int_{\frac{A}{\theta_2 s_0}}^{\theta_2} c_2 d\theta_1 \end{pmatrix} d\theta_2 \end{pmatrix}}_{v_3} & \text{if} \quad \frac{A}{\theta_H^2} < s_0 < \frac{A}{\theta_L \theta_H} \\ v_0 + \sqrt{s_0} - \frac{2}{\Delta^2} \underbrace{\int_{\theta_L}^{\theta_H} \int_{\theta_L}^{\theta_2} c_1 d\theta_1 d\theta_2}_{v_4} & \text{if} \quad s_0 < \frac{A}{\theta_H^2} \end{cases},$$

and for expected profit:

$$\pi^p = \begin{cases} \underbrace{\frac{2}{\Delta^2}\int_{\theta_L}^{\theta_H}\int_{\theta_L}^{\theta_2}(c_2-c_1)\,d\theta_1 d\theta_2}_{\beta_1} & \text{if} \quad \frac{A}{\theta_L^2} < s_0 \\ \underbrace{\frac{2}{\Delta^2}\left(\begin{array}{c} \int_{\theta_L}^{\sqrt{\frac{A}{s_0}}}\int_{\theta_L}^{\theta_2}(c_2-c_1)\,d\theta_1 d\theta_2 \\ +\int_{\sqrt{\frac{A}{s_0}}}^{\frac{A}{\theta_L s_0}}\left(\begin{array}{c}\int_{\theta_L}^{\frac{A}{\theta_2 s_0}}(c_2-c_1)\,d\theta_1 \\ +\int_{\frac{A}{\theta_2 s_0}}^{\theta_2}(c_1-c_2)\,d\theta_1\end{array}\right)d\theta_2 \\ +\int_{\frac{A}{\theta_L s_0}}^{\theta_H}\int_{\theta_L}^{\theta_2}(c_1-c_2)\,d\theta_1 d\theta_2 \end{array}\right)}_{\beta_2} & \text{if} \quad \frac{A}{\theta_L\theta_H} < s_0 < \frac{A}{\theta_L^2} \\ \underbrace{\frac{2}{\Delta^2}\left(\begin{array}{c} \int_{\theta_L}^{\frac{\sqrt{A}}{\sqrt{s_0}}}\int_{\theta_L}^{\theta_2}(c_2-c_1)\,d\theta_1 d\theta_2 \\ +\int_{\frac{\sqrt{A}}{\sqrt{s_0}}}^{\theta_H}\left(\begin{array}{c}\int_{\theta_L}^{\frac{A}{\theta_2 s_0}}(c_2-c_1)\,d\theta_1 \\ +\int_{\frac{A}{\theta_2 s_0}}^{\theta_2}(c_1-c_2)\,d\theta_1\end{array}\right)d\theta_2 \end{array}\right)}_{\beta_3} & \text{if} \quad \frac{A}{\theta_H^2} < s_0 < \frac{A}{\theta_L\theta_H} \\ \underbrace{\frac{2}{\Delta^2}\int_{\theta_L}^{\theta_H}\int_{\theta_L}^{\theta_2}(c_1-c_2)\,d\theta_1 d\theta_2}_{\beta_4} & \text{if} \quad s_0 < \frac{A}{\theta_H^2} \end{cases}.$$

In case $A > 0$, the optimal price auction is difficult to solve analytically since the most cost-efficient firm will change in the interval $[\theta_L, \theta_H] \times [\theta_L, \theta_H]$ depending on the relationship between the quality threshold s_0 and the common fixed-cost parameter, A. Therefore, I have solved the problem by numerical simulations. Performing the integrations given in equations (33) and (34), the expected consumer surplus and expected profit are given by:

$$V^p = \begin{cases} v_0 + \frac{3}{4(2\theta_H+\theta_L)} - \frac{2A(\Delta-\theta_L\Theta)}{\Delta^2} & \text{if} \quad \frac{A}{\theta_L^2} < s_0 \\ v_0 + \sqrt{s_0} - \frac{2}{\Delta^2}(a+b+c+d) & \text{if} \quad \frac{A}{\theta_L\theta_H} < s_0 < \frac{A}{\theta_L^2} \\ v_0 + \sqrt{s_0} - \frac{2}{\Delta^2}(e+b+f+d) & \text{if} \quad \frac{A}{\theta_H^2} < s_0 < \frac{A}{\theta_L\theta_H} \\ v_0 + \frac{3}{4(\theta_H+2\theta_L)} - \frac{2(A(\theta_H\Theta-\Delta))}{\Delta^2} & \text{if} \quad s_0 < \frac{A}{\theta_H^2} \end{cases}, \qquad (44)$$

$$\pi^p = \begin{cases} \frac{3\Delta}{4(2\theta_H+\theta_L)^2} - \frac{2A((\theta_H+\theta_L)\Theta-2\Delta)}{\Delta^2} & \text{if} \quad \frac{A}{\theta_L^2} < s_0 \\ \frac{2}{\Delta^2}(g+h+i+j) & \text{if} \quad \frac{A}{\theta_L\theta_H} < s_0 < \frac{A}{\theta_L^2} \\ \frac{2}{\Delta^2}(k+l+m+j) & \text{if} \quad \frac{A}{\theta_H^2} < s_0 < \frac{A}{\theta_L\theta_H} \\ \frac{2A((\theta_H+\theta_L)\Theta-2\Delta)}{\Delta^2} - \frac{3\Delta}{4(\theta_H+2\theta_L)^2} & \text{if} \quad s_0 < \frac{A}{\theta_H^2} \end{cases}, \tag{45}$$

where $a - m$ is:

$$\begin{aligned}
a &= A\left(\theta_H + \theta_L - \theta_L\left(\ln\theta_H - \ln\frac{A}{s_0\theta_L}\right)\right), \\
b &= \frac{1}{6}s_0\left(2\theta_H^3 + 2\theta_L^3 - 3\theta_H^2\theta_L\right), \\
c &= \frac{1}{2}\frac{A^2}{s_0\theta_L}, \\
d &= -\frac{8}{3}\frac{A^{\frac{3}{2}}}{\sqrt{s_0}}, \\
e &= A\left(\theta_H + \theta_L + \theta_H\left(\ln\frac{A}{s_0\theta_H} - \ln\theta_L\right)\right), \\
f &= \frac{1}{2}\frac{A^2}{s_0\theta_H}, \\
g &= A\left(2\left(\theta_H + \theta_L\right) + 2\theta_L\left(\ln A - \ln s_0\right) - \theta_H\Theta - \theta_L\left(\ln\theta_H + 3\ln\theta_L\right)\right), \\
h &= \frac{1}{6}s_0\theta_H^3 + \frac{1}{6}s_0\theta_L^3 + \frac{1}{2}s_0\theta_H\theta_L^2 - \frac{1}{2}s_0\theta_H^2\theta_L, \\
i &= \frac{A^2}{s_0\theta_L}, \\
j &= -\frac{16}{3}\frac{A^{\frac{3}{2}}}{\sqrt{s_0}}, \\
k &= A\left(2\left(\theta_H + \theta_L\right) + 2\theta_H\ln\frac{A}{s_0} - \theta_L\Theta - \theta_H\left(3\ln\theta_H + \ln\theta_L\right)\right), \\
l &= \frac{1}{6}s_0\theta_H^3 + \frac{1}{6}s_0\theta_L^3 - \frac{1}{2}s_0\theta_H\theta_L^2 + \frac{1}{2}s_0\theta_H^2\theta_L, \\
m &= \frac{A^2}{s_0\theta_H}.
\end{aligned}$$

Note that for the intervals $s_0 > \frac{A}{\theta_L^2}$ and $\frac{A}{\theta_H^2} > s_0$, I have found s_0 explicitly, while for the two intermediate intervals, s_0 is found by numerical simulations by solving the following two equations:[10]

$$\frac{d}{ds_0} = 0 \Leftrightarrow \begin{cases} \begin{pmatrix} 6A^2+12As_0\theta_L^2-4s_0^2\theta_L^4-4s_0^2\theta_H^3\theta_L \\ +6s_0^2\theta_H^2\theta_L^2+3s_0^{\frac{3}{2}}\theta_L^3-6s_0^{\frac{3}{2}}\theta_H\theta_L^2 \\ +3s_0^{\frac{3}{2}}\theta_H^2\theta_L-16A^{\frac{3}{2}}\sqrt{s_0}\theta_L=0 \end{pmatrix} & \text{if } \frac{A}{\theta_L\theta_H} < s_0 < \frac{A}{\theta_L^2} \\ \begin{pmatrix} 6A^2+12As_0\theta_H^2-4s_0^2\theta_H^4-4s_0^2\theta_H\theta_L^3 \\ +6s_0^2\theta_H^2\theta_L^2+3s_0^{\frac{3}{2}}\theta_H^3+3s_0^{\frac{3}{2}}\theta_H\theta_L^2 \\ -6s_0^{\frac{3}{2}}\theta_H^2\theta_L-16A^{\frac{3}{2}}\sqrt{s_0}\theta_H=0 \end{pmatrix} & \text{if } \frac{A}{\theta_H^2} < s_0 < \frac{A}{\theta_L\theta_H} \end{cases}.$$

Solving these two equations numerically and combining with the explicit solutions for the two other intervals yields the expected consumer surplus and expected profit for the optimal price auctions. The results are illustrated in Figures 2-5.

In Figure 2 and 3, I give a short summary of the simulations where I fix $\theta_L = 1$ and first let θ_H vary between $[1, 20]$ and change A exogenously between $\left[0, \frac{1}{4}\right]$. In Figure 4 and 5, I fix $\theta_L = 1$, let A vary between $\left[0, \frac{1}{4}\right]$, and change θ_H exogenously between $[2, 20]$. These simulations give rise to the conjecture 1.

In the top part of Figure 2, $A = 0$, so I am considering the case where the cost function is given by $c_i = \theta_i s$, in the bottom, $A = \frac{1}{16}$. In the top part of Figure 3, $A = \frac{3}{16}$, and the bottom, $A = \frac{1}{4}$. When A is getting closer to $\frac{1}{4}$, the price auction generates higher profit than the naïve scoring auction. Note also that the auctioneer is able to extract all profit from the bidders in the optimal scoring auction (the dotted black line) as A increases. So bidders might prefer the price-only auction when their comparative advantages are of a character so that they will be moving towards "pseudotype" equality in the scoring auctions, while they, in any price auction, are very different. Next, in Figure 4, in the top part, $\theta_H = 2$, and, in the bottom, $\theta_H = 5$. In Figure 5 $\theta_H = 10$ and $\theta_H = 20$ in the top and bottom respectively.

[10]The numerical simulations were done by using Excel spreadsheets and are available from the author upon request.

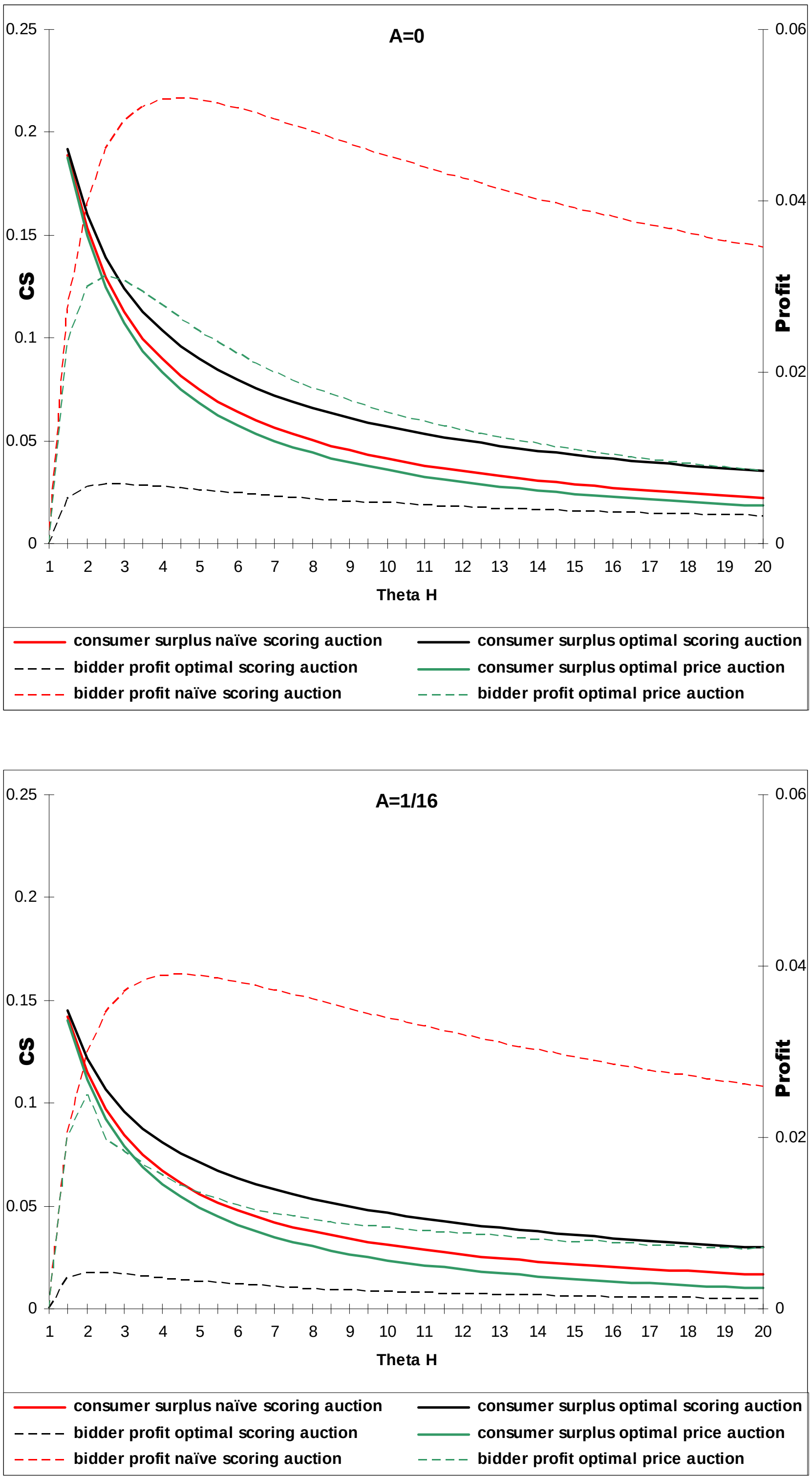
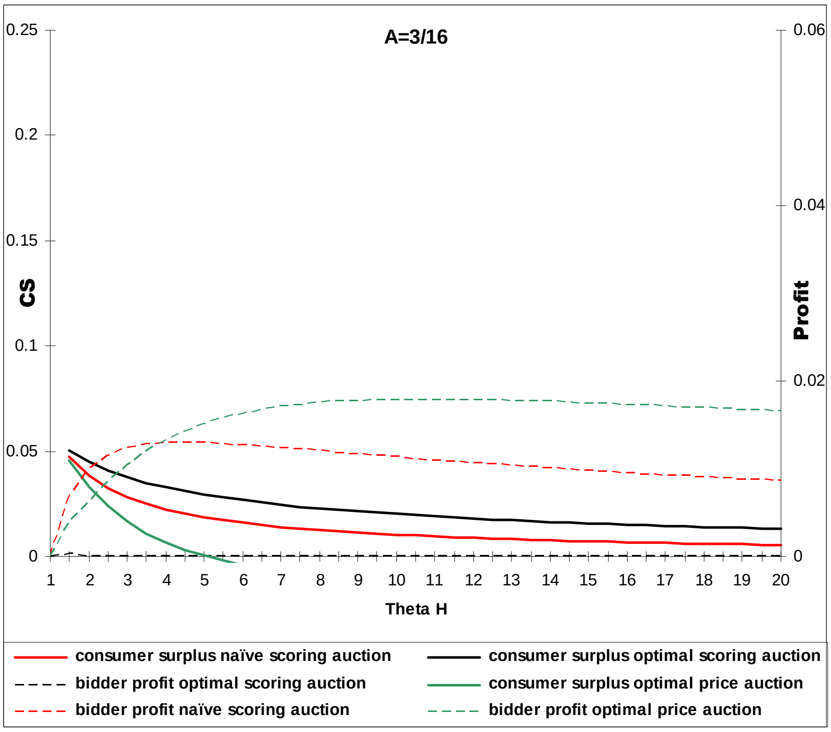


Figure 2: Numerical simulation with $[\theta_L, \theta_H] = [1, 20]$ and $A \in \{0, \frac{1}{16}\}$



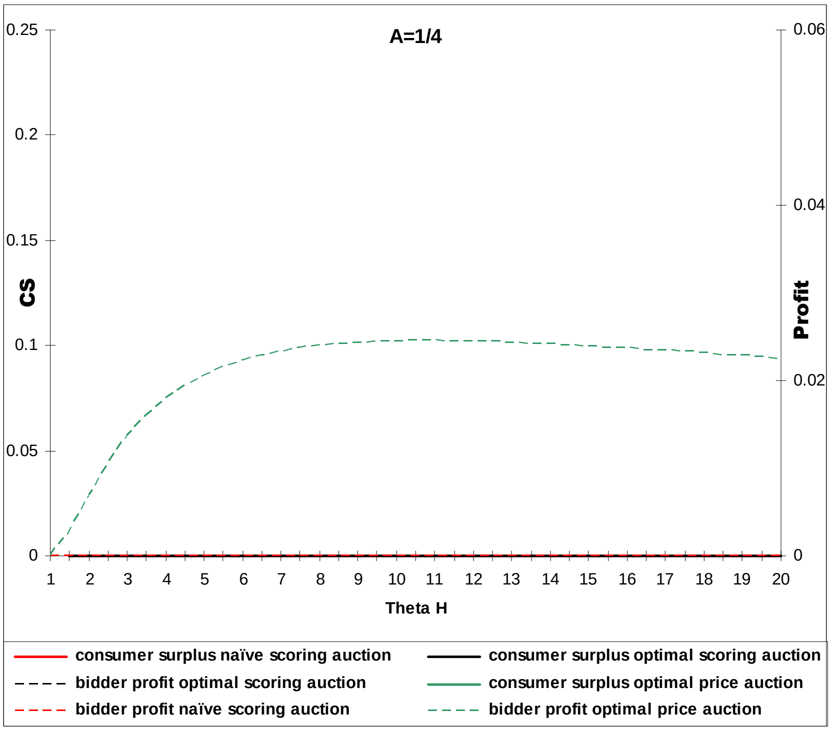
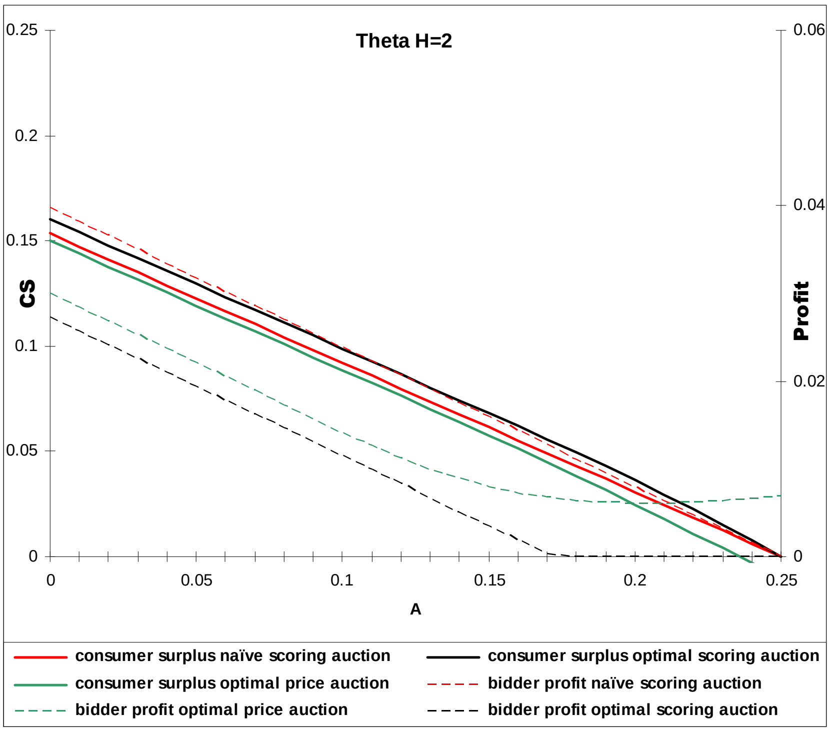


Figure 3: Numerical simulation with $[\theta_L, \theta_H] = [1, 20]$ and $A \in \left\{\frac{3}{16}, \frac{1}{4}\right\}$



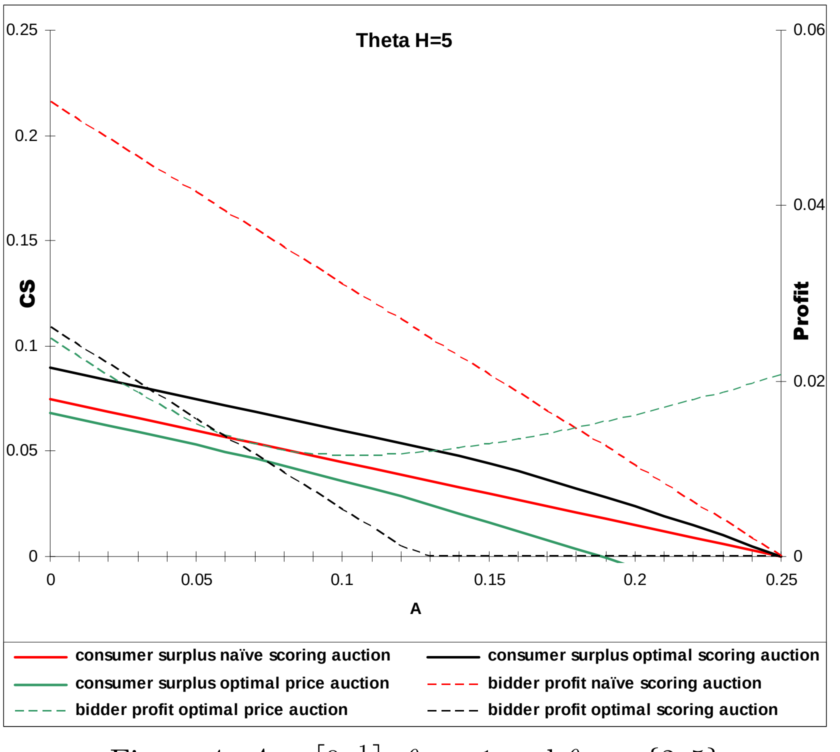
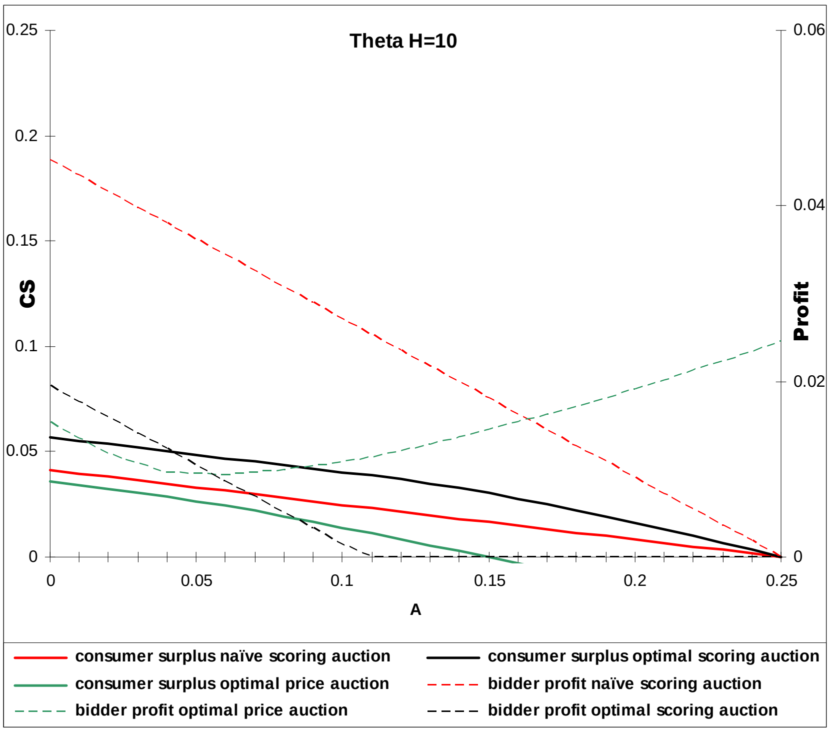


Figure 4: $A = \left[0, \frac{1}{4}\right]$, $\theta_L = 1$ and $\theta_H = \{2, 5\}$



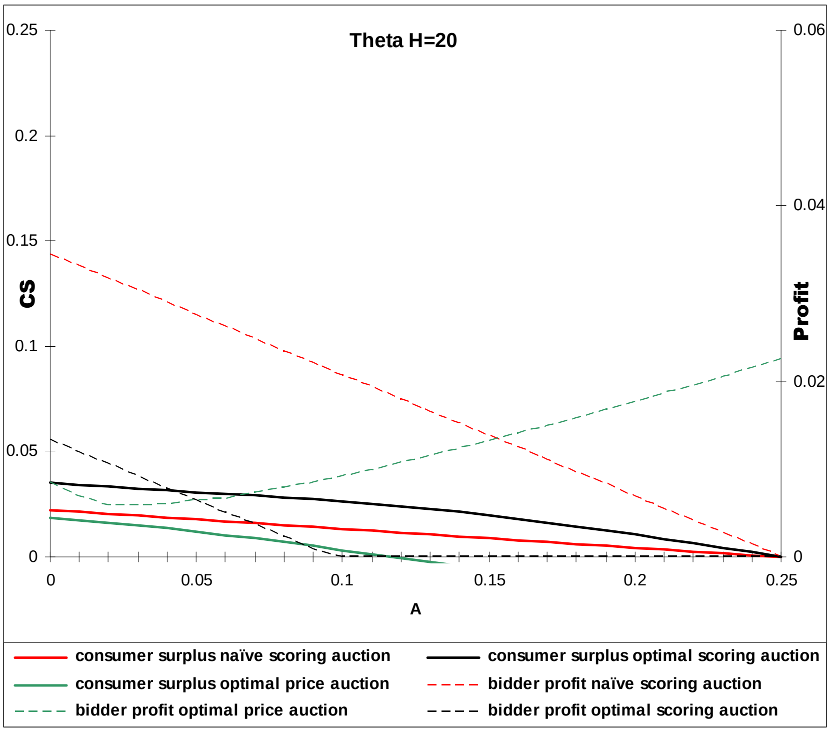


Figure 5: $A = \left[0, \frac{1}{4}\right]$, $\theta_L = 1$ and $\theta_H = \{10, 20\}$

B Proofs

Proof of proposition 1 In the scoring auction, if $A = \frac{1}{4}$, $v^* (\theta_i, s) = v_0$, which is independent of θ_i. For the price auction, inserting $\theta_i = \frac{1}{2\sqrt{s_0}}$ and $A = \frac{1}{4}$ in the surplus function yields v_0.■

Proof of proposition 2 I first prove that the expected consumer surplus in the optimal scoring auction is equal to or larger than the expected consumer surplus in the naïve scoring auction, and that the reverse holds for bidders' expected profits. For the expected consumer surplus in optimal scoring auctions to be larger than in the naïve scoring auction, it must hold that:

$$\begin{aligned} V^* &\geq V^s \\ &\Updownarrow \\ v_0 + \frac{1}{2\Delta^2} \left(\hat{\alpha} \left(\theta_H \Theta - \Delta \right) - 4A \left(\Delta - \theta_L \Theta \right) \right) &\geq v_0 + \frac{(1-4A)}{2\Delta^2} \left(\Delta - \theta_L \Theta \right). \end{aligned}$$

Inserting for $\hat{\alpha} = \frac{(\theta_H \Theta - \Delta)}{((2\theta_H + \theta_L)\Theta - 3\Delta)}$ and rearranging this inequality yields:

$$(\ln \theta_H - \ln \theta_L) \geq \frac{2(\theta_H - \theta_L)}{(\theta_H + \theta_L)}. \tag{46}$$

This expression holds with equality if $\theta_H = \theta_L$, and differentiation of θ_H yields:

$$\begin{aligned} \frac{d}{d\theta_H} (\ln \theta_H - \ln \theta_L) &\geq \frac{d}{d\theta_H} \left(\frac{2(\theta_H - \theta_L)}{(\theta_H + \theta_L)} \right) \\ &\Updownarrow \\ \frac{1}{\theta_H} &\geq \frac{4\theta_L}{(\theta_H + \theta_L)^2}. \end{aligned}$$

Which, after rearranging the terms, gives:

$$(\theta_H - \theta_L)^2 \geq 0.$$

This shows that if $\theta_H > \theta_L$, the left-hand side increases faster than the left-hand side in equation (46); hence, I have proved that the expected consumer surplus in the optimal scoring auction is larger than in the naïve scoring auction. For the expected profit for bidders to be higher in the naïve scoring auction compared with the optimal scoring auction, the following must hold:

$$\begin{aligned} \pi^s &\geq \pi^* \\ &\Updownarrow \\ \frac{(1-4A)}{4\Delta^2}\left((\theta_H+\theta_L)\Theta-2\Delta\right) &\geq \frac{\left(\hat{\alpha}^2-4A\right)}{4\Delta^2}\left((\theta_H+\theta_L)\Theta-2\Delta\right). \end{aligned}$$

Rearranging this yields:

$$(\ln\theta_H-\ln\theta_L)>2\frac{(\theta_H-\theta_L)}{(\theta_H+\theta_L)}.$$

Which is the same condition as in (46) above, and, hence, the expected profit for bidders in the naïve scoring auction is higher than that in the optimal scoring auction.■

Proof of proposition 3 When there are only marginal cost differences ($A=0$), for the expected consumer surplus to be higher in the optimal scoring auction compared with the naïve scoring auction, the following must hold:

$$\begin{aligned} V^* &\geq V^s \\ &\Updownarrow \\ v_0+\frac{\alpha\left(\Delta-\theta_H\Theta\right)}{2\Delta^2} &\geq v_0+\frac{\Delta-\theta_L\Theta}{2\Delta^2}. \end{aligned}$$

Inserting for $\alpha=\frac{(\theta_H\Theta-\Delta)}{((2\theta_H+\theta_L)\Theta-3\Delta)}$, and rearranging this inequality yields:

$$(\ln\theta_H-\ln\theta_L)\geq\frac{2\left(\theta_H-\theta_L\right)}{(\theta_H+\theta_L)}.$$

This is the same condition as (46) in the proof above; hence, it follows that $V^*\geq V^s$. For the expected consumer surplus in the naïve scoring auction to be higher than the expected consumer surplus in the optimal price auction, the following must hold:

$$\begin{aligned} V^s &\geq V^p \\ &\Updownarrow \\ v_0 + \frac{\Delta - \theta_L \Theta}{2\Delta^2} &\geq v_0 + \frac{3}{4\left(2\theta_H + \theta_L\right)}. \end{aligned}$$

Rearranging the inequality yields:

$$\Delta - \theta_L \Theta \geq \frac{3\Delta^2}{2\left(2\theta_H + \theta_L\right)}.$$

This holds with equality if $\theta_H = \theta_L$. Differentiating both sides with respect to θ_H yields:

$$\begin{aligned} \frac{d}{d\theta_H}\left(\Delta - \theta_L \Theta\right) &\geq \frac{d}{d\theta_H}\left(\frac{3\Delta^2}{2\left(2\theta_H + \theta_L\right)}\right) \\ &\Updownarrow \\ \left(\theta_H - \theta_L\right)^2 &\geq 0. \end{aligned}$$

So, if $\theta_H > \theta_L$, the following ranking is true: $V^* > V^s > V^p$. Bidders' expected profits in the naïve scoring auction are higher than in the optimal scoring auction if:

$$\begin{aligned} \pi^s &\geq \pi^* \\ &\Updownarrow \\ \frac{\left(\theta_H + \theta_L\right)\Theta - 2\Delta}{4\Delta^2} &\geq \frac{\alpha^2\left(\left(\theta_H + \theta_L\right)\Theta - 2\Delta\right)}{4\Delta^2}. \end{aligned}$$

This holds with strict equality if $\theta_H = \theta_L$ and $\alpha < 1$, the optimal distortion $\alpha < 1$, if:

$$\begin{aligned} \frac{\left(\ln\theta_H - \ln\theta_L\right)\theta_H - \left(\theta_H - \theta_L\right)}{\left(2\theta_H + \theta_L\right)\left(\ln\theta_H - \ln\theta_L\right) - 3\left(\theta_H - \theta_L\right)} &< 1 \\ \left(\ln\theta_H - \ln\theta_L\right) &> 2\frac{\left(\theta_H - \theta_L\right)}{\left(\theta_H + \theta_L\right)}. \end{aligned}$$

Which, after rearranging, yields:

$$(\ln \theta_H - \ln \theta_L) > 2\frac{(\theta_H - \theta_L)}{(\theta_H + \theta_L)}.$$

This is the same condition as (46) ,and I can conclude that $\alpha < 1 \Longrightarrow \pi^s \geq \pi^*$. For the naïve scoring auction to yield a higher expected profit than the optimal price auction, the following must hold:

$$\begin{aligned} \pi^s &\geq \pi^p \\ &\Updownarrow \\ \frac{(\theta_H + \theta_L)\,\Theta - 2\Delta}{\Delta^2} &\geq \frac{3\Delta}{2\,(2\theta_H + \theta_L)^2}. \end{aligned}$$

Rearranging yields:

$$(\ln \theta_H - \ln \theta_L) \geq \frac{(\theta_H - \theta_L)\left(10\theta_H\theta_L + 19\theta_H^2 + 7\theta_L^2\right)}{2\,(\theta_H + \theta_L)\,(2\theta_H + \theta_L)^2}.$$

If $\theta_H = \theta_L$, this holds with equality. Differentiation of both sides with respect to θ_H yields:

$$\begin{aligned} \frac{d}{d\theta_H}(\ln \theta_H - \ln \theta_L) &\geq \frac{d}{d\theta_H}\left(\frac{(\theta_H - \theta_L)\left(10\theta_H\theta_L + 19\theta_H^2 + 7\theta_L^2\right)}{2\,(\theta_H + \theta_L)\,(2\theta_H + \theta_L)^2}\right) \\ &\Updownarrow \\ \frac{1}{\theta_H} &> \frac{\left(47\theta_H^3 + 16\theta_L^3 + 15\theta_H\theta_L^2 + 30\theta_H^2\theta_L\right)\theta_L}{(2\theta_H + \theta_L)^3\,(\theta_H + \theta_L)^2} \\ \Delta^3\left(8\theta_H^2 + 5\theta_H\theta_L - \theta_L^2\right) &> 0. \end{aligned}$$

This is the same as:

$$(\theta_H - \theta_L)^3\left(8\theta_H^2 + 5\theta_H\theta_L - \theta_L^2\right) > 0.$$

This is true if $\theta_H > \theta_L > 0$■

Department of Economics
University of Bergen
Fosswinckels gate 6
N-5007 Bergen, Norway
Phone: +47 55 58 92 00
Telefax: +47 55 58 92 10
http://www.svf.uib.no/econ