

# Circumventing Information and Incentive Problems in Pollution Control

HELGE BERGLANN\*

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**ABSTRACT.** To mitigate pollution this note advocates a two-component system. The polluter pays for the damage he causes *and* for the opportunity to do so. The main motivation is to achieve Pareto efficiency while avoiding problems caused by asymmetric information and strategic moves. The proposed regime induces each polluter to solve the same optimization problem as an altruistic planner. Since the monetary burden of the scheme matches a linear Pigouvian tax, it does not encourage firms to split or merge.

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## 1. INTRODUCTION

Economic theory - and common sense - recommends that polluters better compensate for damage inflicted on other agents. If emitting and receiving parties are easily identified, together with corresponding amounts, one can, in principle, rectify choices to become optimal. However, strategic moves, asymmetric information and measurement problems all make the task of a regulating body hard indeed.

Yet, this note, while mute on measurement issues, advocates a system that does well without access to private knowledge - and also relieves firms from playing the ground. It has two chief components: The first is a proportional tax, calibrated by individual shares and by total damage. It is collected on the basis of the firm's realized and observed emission, so total damage is calculated with the firm itself as a representative polluter of the industry. The second component is a separate payment for the *right* to pollute, recorded as a share of the total emission.

With these two components in place, the resulting outcome is Pareto efficient. Each firm can prudently act as if in a competitive situation, void of problems with information and strategic choice. While taxation leads firms to internalize environmental costs, the market mechanism, being part of the scheme, ensures optimal distribution of damage payments. Broadly, the components induce each and every polluter to solve the same optimization problem as a benevolent, well-informed planner.

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\*Department of Economics, University of Bergen; e-mail: helge.berglann@econ.uib.no. I deeply thank Sjur D. Flåm for his comments and contributions to this paper. I also thank Asbjørn Torvanger for comments on earlier drafts. Financial support from the Research Council of Norway is gratefully acknowledged.

The scheme is spelled out in Section 2. It differs in form from a full-information Pigouvian tax, but complies in outcome because the resulting monetary burdens are equivalent. This feature is at variance with the Kim and Chang (1993) proposal, yielding a total tax burden below the Pigouvian level (McKittrick, 1999). Their proposal is also Pareto efficient, but it may encourage firms to consolidate. We prove this feature in Section 3, and demonstrate that our scheme has no such effect.

## 2. THE INDIVIDUAL TRANSFERABLE FEE SCHEME

Fixed henceforth is a finite set  $I$  of firms. Agent  $i \in I$  produces output  $x_i \geq 0$  of one and the same commodity as the others. He sells this quantity in a common, perfectly competitive market at fixed unit price  $p > 0$ . Production entails the emission of a homogeneous pollutant dispersed into (local or global) commons. Firm  $i$  is accountable for the emission level  $e_i \geq 0$ . Write  $e := \sum e_i$  as shorthand for the aggregate emission level. Let economic damage  $d$  caused by emissions, as measured in monetary terms, be given by

$$d = D(e) \quad (1)$$

where, quite naturally, we posit that  $D(0) = 0$ ,  $D' > 0$  and  $D'' > 0$ .

By assumption each and every firm ignores the social costs of its emissions. It merely cares about its own production cost

$$c^i = C^i(x_i, e_i) \quad (2)$$

where  $C^i$  is strictly convex and twice differentiable. A full-information welfare optimum solves the problem

$$\max_{x_i \geq 0, e_i \geq 0, \forall i} \sum [px_i - C^i(x_i, e_i)] - D\left(\sum e_i\right). \quad (3)$$

Supposing interior solutions, the necessary optimality conditions are

$$p = \frac{\partial C^i}{\partial x_i} \quad (4)$$

$$-\frac{\partial C^i}{\partial e_i} = D'(e) \quad (5)$$

for all  $i$ . Since the objective in problem (3) is strictly concave, conditions (4) and (5) are also sufficient, and the optimum they determine is unique.

Clearly, firm  $i$ 's ignorance of social costs implies that (5) be replaced by  $\partial C^i / \partial e_i = 0$ . Plainly, the resulting  $e_i$  is too large. Thus, there is a need, and ample justification, for a regulator to intervene. We presume that such a central agent is bestowed with the authority to implement an efficient payment system of his own prudent design. In that endeavour he must contend with knowing merely the damage function  $D(\cdot)$ . Private cost functions  $C^i$  are nonobservable, and so is maybe output  $x_i$  as well. We

posit though that he can observe firm  $i$ 's emissions  $e_i$ . Reasonably, we shall require that every firm be a profit maximizing body, well informed about the data that pertains directly to itself.

Interaction between the regulator and the firm takes two forms. First, and right away, firm  $i$  is informed that it will be charged an individual tax

$$t^i = T(e_i, s_i) := s_i D\left(\frac{e_i}{s_i}\right). \quad (6)$$

Here, as said,  $e_i$  is the amount of pollutant emitted by the firm  $i$ . We interpret the entity  $s_i$  as firm  $i$ 's *share* of total emission  $e$ . The reason for this name is that in the final round,  $s_i$  does indeed turn to be of that nature. Note that  $\partial t^i / \partial e_i = D'(e_i/s_i)$ . Since marginal damage  $D'(\cdot)$  increases with its argument  $e_i/s_i$ , a higher  $s_i$  value for the same  $e_i$  means a lower marginal tax. So, a high  $s_i$  appears attractive to firm  $i$ .

The second form of interaction, advocated here, relates to the fact that the equality  $\sum s_i = 1$  must be enforced somehow. Otherwise, talking or thinking about shares makes no sense. To set things right we take an exchange market as appropriate vehicle. Thus there should be a market-clearing price  $\mu$  per unit of  $s_i$ , satisfying the complementarity condition

$$\sum s_i - 1 \leq 0, \mu \geq 0, \mu \left( \sum s_i - 1 \right) = 0. \quad (7)$$

Firm  $i$  seeks a profit maximum. It faces therefore the decision problem

$$\max_{x_i \geq 0, e_i \geq 0, s_i \geq 0} \left\{ px_i - C^i(x_i, e_i) - s_i D\left(\frac{e_i}{s_i}\right) - \mu s_i \right\}. \quad (8)$$

Assuming interior solutions to (8) the necessary condition with respect to  $x_i$  conforms with (4). Two other necessary conditions are

$$-\frac{\partial C^i}{\partial e_i} = D'\left(\frac{e_i}{s_i}\right) \quad (9)$$

$$\mu = \frac{e_i}{s_i} D'\left(\frac{e_i}{s_i}\right) - D\left(\frac{e_i}{s_i}\right). \quad (10)$$

**Proposition 1.** *Conditions (4), (9) and (10) are sufficient for an interior solution of problem (8).*

**Proof.** It suffices to show that  $(e_i, s_i) \mapsto s_i D(e_i/s_i)$  is convex. For this purpose recall that such convexity follows iff the Hessian of  $s_i D(e_i/s_i)$  is positive semidefinite (see Sydsaeter and Hammond, 1995, Theorem 17.13b). The said Hessian equals

$$\frac{1}{s_i} D''(e_i/s_i) \begin{bmatrix} 1 & -e_i/s_i \\ -e_i/s_i & e_i^2/s_i^2 \end{bmatrix}.$$

The eigenvalues 0 and  $\frac{e_i^2 + s_i^2}{s_i^3} D''(e_i/s_i)$  are nonnegative. Finally, to complete the proof, invoke the following (Sydsaeter and Hammond, 1995, Theorem 15.2b): A symmetric matrix is positive semidefinite iff all eigenvalues are  $\geq 0$ .  $\square$

**Proposition 2.** *Suppose the constraint  $\sum s_i = 1$  is enforced. Then  $s_i$ ,  $i \in I$ , will be distributed among firms such that consistency obtains. That is,*

$$e = \frac{e_i}{s_i} \text{ for all } i. \quad (11)$$

**Proof.** Equation (10) expresses  $\mu$  by  $e_i/s_i$ . Let the function  $G$  account for this expression. That is, let  $a_i := e_i/s_i$  and posit

$$\mu = G(a_i) := a_i D'(a_i) - D(a_i)$$

Note that  $G'(a_i) = a_i D''(a_i) > 0$ , this telling that  $G$  is a strictly increasing function. Hence,  $G$  is invertible and  $a_i = G^{-1}(\mu)$ . So all  $a_i$  are equal, and we are justified in setting  $a := a_i$  for all  $i \in I$ . Moreover,  $a = e_i/s_i$  implies  $e_i = s_i a$ . Summing the last expression over  $i \in I$  yields

$$e := \sum e_i = a \sum s_i = a,$$

and the desired assertion follows.  $\square$

The implication of (11) is that (5) is equivalent to (9) for all  $i$ . Hence, each firm solves the same problem as a seemingly well-informed socially planner. This implies

**Proposition 3.** *The tax rule (6) and the enforcement of  $\sum s_i = 1$  yield a socially optimal level of output and pollution for all  $i$ . That is, the Nash solution is Pareto efficient.  $\square$*

Note that (11) also implies

$$\mu = e D'(e) - D(e), \quad (12)$$

whence it follows

**Proposition 4.** *For each firm, the fee (6) plus expenses for  $s_i$  equals the tax the firm would face under full-information, Pigouvian unit tax  $\tau := D'(e)$  determined by*

$$T(e_i, s_i) + \mu s_i = D'(e) e_i = \tau e_i. \quad (13)$$

The overall scheme can thus be construed as a Pigouvian regime where firms themselves choose the optimal emission and thereby also the optimal tax level. Since the scheme relieves firms from strategic considerations the Nash equilibrium, which usually is not Pareto optimal, is indeed so here.

We call our construct the "Individual Transferable Fee" system, a name better defended in Section 4. Before that, however, we briefly address the

## 3. TENDENCY TO MERGE OR SPLIT

Given the aggregate emission  $e$ , Proposition 4 points to the linearity of taxes with respect to individual pollution.

**Proposition 5.** *As long as the total emission  $e$  is constant the above tax scheme does not encourage firms to merge or split.  $\square$*

Kim and Chang (1993) proposed the following  $T^K$  tax based on differential damage:

$$T^K(e_i, e_{-i}) := D(e) - D(e_{-i}). \quad (14)$$

Here  $e_i > 0$  is the emission of polluter  $i$ ;  $e > 0$  is total emission; and  $e_{-i} := \sum_{k \neq i} e_k = e - e_i \geq 0$  is the total amount emitted by all other firms. Since  $D$  is strictly convex, we deduce

**Proposition 6.** *Assume constant total emission  $e$ . The “incremental damage scheme” encourages firms to consolidate.*

**Proof.** It suffices to show that any pair of firms pays less tax by consolidating than what they do individually. Consider a pair where the firm  $i$ , which is not the heaviest polluter of those two, emits quantity  $e_i > 0$ . The other firm emits the quantity  $e_i + r$ , where  $r \geq 0$ . With  $e > 0$  constant, the difference  $\Psi(e_i, r)$  of the sum paid by the two firms individually over a consolidated company is

$$\Psi(e_i, r) := T^K(e_i, e - e_i) + T^K(e_i + r, e - e_i - r) - T^K(2e_i + r, e - 2e_i - r)$$

where the function  $\Psi$  has domain  $\{(e_i, r) : 0 < e_i \leq e, 0 \leq r \leq e - 2e_i\}$ . Inserting (14) and simplifying yields

$$\Psi(e_i, r) = D(e) - D(e - e_i) - D(e - e_i - r) + D(e - 2e_i - r).$$

We have

$$\frac{\partial}{\partial r} \Psi = [D'(e - e_i - r) - D'(e - 2e_i - r)] > 0$$

and

$$\frac{\partial}{\partial e_i} \Psi = [D'(e - e_i) - D'(e - 2e_i - r)] + [D'(e - e_i - r) - D'(e - 2e_i - r)] > 0.$$

This follows from  $D'$  being increasing, and the argument of the positive  $D'$  is higher than for the negative  $D'$  in all segments separated by square brackets. The result

$$\lim_{e_i \rightarrow 0} \Psi(e_i, 0) = 0$$

and the fact that  $\Psi$  increases in both its arguments ensures that  $\Psi(e_i, r) > 0$  within its domain. This proves that each firm pays less tax by consolidating with another company. Hence, a tendency to merge is present.  $\square$

## 4. CONCLUDING REMARKS

This paper introduces a two-component payment system for the purpose of pollution control. When individual emissions can be observed and markets for emission rights are fully competitive, the regulator can induce first best optimum by knowing merely the marginal damage function.

We call our construct the Individual Transferable Fee system. It is "individual" because each firm pays according to its own emission. It is "transferable" because shares are traded on a market.

Note that while the proposed scheme is nonlinear, the total monetary burden equals that caused by a linear Pigouvian tax. The latter feature ensures that the scheme does not encourage restructuring of industry in the form of mergers or splits. Providing such restructuring is deemed undesirable, our proposal seems superior to the scheme advocated by Kim and Chang (1993).

Individual Transferable Fees can be interpreted as a system that blends linear taxation with quantity instruments. To envision this analogy, consider the mixed system of tradable quantities and linear taxation proposed by Robert and Spence (1979). For their proposal to work, the damage function is approximated with a one-step function consisting of two piecewise linear segments. In the appendix of their article, they recommend approximating the damage function better by using several such steps. However, as Baumol and Oates (1988, page 77) note: "In the limit, when the number of steps goes to infinity, we would, of course, have the variable fee system we described earlier and which produces the optimal outcome". What they described earlier (page 76), and called for, was a variable fee system that has precisely the characteristics of our Individual Transferable Fee proposal.

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