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FRODE MELAND

OVERTIME PAY PREMIUMS IN A UNIONIZED OLIGOPOLY

Department of Economics

UNIVERSITY OF BERGEN

Overtime pay premiums in a unionized oligopoly[*]

Frode Meland[†]

University of Bergen

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Abstract

This paper studies how a high overtime wage rate and a low labor stock may be used as commitment devices by price-setting firms. We show that high overtime pay premiums may both decrease and increase equilibrium employment. If an employment-oriented union or the firm itself sets the overtime wage, then the overtime wage premium will be high enough to ensure that no overtime is used in equilibrium. If the overtime wage is set by a sufficiently wage-oriented union, however, overtime will be used in equilibrium, and employment is substantially lower. Thus the authorities may be able to increase employment if it can make a union act in a less wage-oriented manner. We show that this can be done by setting a minimum overtime pay premium. Minimum wage regulation could have the opposite effect.

Keywords: Overtime, Bertrand competition, unionization, regulation.

JEL classification: J21, J51, J88, L13.

1   Introduction

This paper studies the impact of unionization, minimum wages and regulated overtime pay premiums on labor market outcomes under price competition in the output market. We find that both a high overtime wage and a low

production capacity may be used by price setting firms as a commitment device to reduce competition. The firms themselves would prefer a high overtime wage - which effectively commits the firm not to use any overtime. An employment-oriented union would also want to induce the firm to choose this strategy. A wage-oriented union, however, could prefer a situation where the firm utilizes full overtime in equilibrium, thereby using the constrained production capacity to push up prices. We show that minimum wage legislation may induce unions to act in a more wage-oriented manner, which could in some cases have detrimental effects on employment. A regulated overtime pay premium, however, would pull in the opposite direction, as it induces unions to behave similarly to a more employment-oriented union. A cap on overtime hours is shown to increase labor demand only if wages are set by a wage-oriented union.

In many countries and industries there are restrictions on the regular wages and the overtime pay that can be accepted. These kinds of restrictions often take the form of minimum wages and regulated overtime pay premiums.[1] In the U.S. for instance, The Fair Standards Act of 1938 introduced a minimum wage, and set the overtime wage to at least 150% of the normal (base) wage. However, institutions other than the authorities may also want such regulations. A likely candidate is labor unions. However, with imperfect competition in the output market and firms competing in prices, the firms themselves may also want high overtime wage premiums, as this could in effect serve as a commitment to less fierce competition. In a Bertrand set-up, such a commitment could lead to a favorable response by the competitors.[2] We show that this may reduce the authorities' incentives to provide legislation affecting overtime pay premiums in the non-unionized case, but may make the same kind of legislation even more important in the unionized case.

The effects of overtime pay premiums on labor market outcomes are theoretically indeterminate. A higher premium may make firms hire more labor to avoid overtime pay, thus increasing employment.[3] However this substitution effect is countered by at least two other effects: First there is an 'income'- or scale-effect: Higher overtime pay premiums will raise the (marginal) costs of firms utilizing overtime, and thus employment may go down.[4] Also, there is a capital substitution effect, as the overtime premium may alter the relative

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[1] For a survey on the effects of minimum wages, see for instance Brown *et al* (1982).

[2] Throughout this paper, we assume that prices set by the firms are strategic complements, ensuring that this effect prevails.

[3] The earliest papers focusing on this effect are Rosen (1968) and Ehrenberg (1970, 1971). They assume that the base wage stays unaffected by a higher overtime pay premium.

[4] See Nussbaum and Wise (1977).

costs of capital to labor. Thus the net effect of overtime premiums is highly uncertain, and dependent on the specifics of the markets studied.[5]

Lewis (1969) argues that, despite the above indeterminacies, overtime pay premiums may have no effect on economic behavior: Put simply, he argues that an employee who works overtime does not care what the base and overtime wages are *per se*; what is important is the total wage income. Thus the employee may be willing to work just as much as before with a lower base wage and a higher overtime pay premium. Following this argument, an increased overtime wage may have no real effects, except in the case where the base wage is restricted by a minimum wage. This kind of model is characterized by Trejo (1991) as a 'fixed job' model as it involves the same total work hours at the same total wage. The important aspect of the model is that it incorporates the base wage being affected by an increase in the overtime pay premium. In this paper we also study such a possibility, but through wage setting by unions. Our model thus endogenizes wage decisions while at the same time incorporating both the substitution effect and scale effect. However, we disregard capital substitution.

The 'fixed job' model suggests that overtime may be used independently of the overtime pay legislation as long as the minimum wage does not bind. Accordingly, these models cannot explain the fact that the distribution of hours of work across employees shows a spike at the number of hours corresponding to the normal working day (see Trejo (1991)). The present model offers an explanation for such a spike even when the base wage is endogenized. Also, Trejo (1991) suggests that the scale effect may reduce the role of regulation as a job creation device. In our model, however, we show that regulating overtime pay premiums may have a dramatic (positive) effect on employment in cases where unions are wage-oriented.

Our model is one of imperfect competition. Overtime pay premiums under imperfect competition have also been studied in an interesting paper by King (1997). His set-up is, however, somewhat different from ours: First, where we use price competition in differentiated products, King adopts the Cournot, homogenous product set-up. Second, we endogenize the employment decision (right to manage) whereas King assumes the labor stock to be exogenously determined. For there to be any real focus on overtime, the regular labor stock has to be somehow set in advance. Endogenizing the

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[5]While empirical evidence suggests that overtime premiums increase the ratio of employment to hours per worker (for a recent study from California, see Hamermesh and Trejo (2000)), this does not imply that employment necessarily increases. Bauer and Zimmermann (1999) provide evidence from Germany suggesting that employment might instead fall.

employment decision thus enriches the set-up.[6]

King finds that an overtime pay penalty, initiated either by the authorities or by industry wide unions, may increase firm profits. This is due to the fact that such a move could increase marginal costs (decreasing industry output) without increasing average costs to a large degree.[7] This result is also found in the present paper. However, if the firms were able to choose employment in King's set-up, the underlying specifics of the Cournot competition could typically eradicate the positive profit effect of a high overtime pay penalty: If a firm increases its labor stock (enough to be able to produce without utilizing overtime) in a situation where the competitors use overtime, the firm obtains a cost advantage relative to the other firms. This leads to a strategic reduction in the output of the competitors. Thus, with endogenous employment decisions and Cournot competition, the positive profit effect of a high overtime wage premium could be reversed. In our model we assume price competition. In this case an increase in the overtime pay premium may indeed increase the profits of firms even when they can set the labor stock prior to competition. With price competition, there is a *positive* strategic effect from incurring higher marginal costs, and thus the firms may even choose a high overtime wage themselves.

In our Bertrand set-up, firms could obtain a strategic advantage by *either* facing a high overtime wage *or* choosing a low labor stock. High overtime wages could credibly commit firms not to use overtime, while a low employment level effectively restricts output through the limited amount of overtime that may be utilized.[8] Say that a single worker can produce one unit during regular hours and another additional unit if working full overtime. Then, with n workers, a firm would have a capacity limit of $2n$ units. However, given a very high overtime wage, the *effective* capacity constraint of the firm is the normal production capacity, n. In our model we endogenize the employment demand decision, and study under what circumstances the firm prefers to commit to non-aggressive pricing by restricting the *normal* or the *full* production capacity. The relative profitability of these two options will, of course, depend upon the level of the overtime pay premium. This has important effects on union wage setting, and we show that a slightly more

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[6]Also, we assume that regular wage costs are in effect sunk at the time of price competition. This is not unreasonable given the fact that the labor stock is set in advance. Thus firms effectively compete under zero marginal labor costs up to the point where every worker is producing all they can during regular hours.

[7]Of course, this would only be valid if the employment of every firm is sufficiently large, so that the increase in marginal costs has a small effect on average costs.

[8]In reference to Fudenberg and Tirole (1984), these two strategies would both be characterized as 'puppy-dog ploys'.

wage-oriented union may choose a strategy involving dramatically lower employment than a less wage-oriented union.[9] This could lead to overtime pay premium legislation having a large impact on unemployment.

The next section describes the model, section 3 discusses price setting behavior, and labor demand is addressed in section 4. Section 5 discusses wage setting; both if the firm is allowed to set the overtime premium and if wages are set by unions. In section 6 we discuss what happens if the authorities set a minimum wage, an overtime pay premium or a cap on overtime hours. Section 7 concludes.

2 The model

We model a differentiated product oligopoly, focusing on a single firm (firm 1) facing competitors producing an imperfectly substitutable good. The timing of the model is as follows: First, wages are decided. We discuss what happens both if firm 1 has the power to decide overtime pay premiums, and if a union sets wages. Following wage setting, the firm unilaterally sets the employment level, before finally choosing the product price, competing in the output market. This set-up then follows the right-to-manage literature both by allowing the firm to unilaterally decide employment, and by assuming wage setting to be a more long-term commitment than employment decisions. As already explained, there has to be some fixed number of workers at stage 3 when prices are set in order to make overtime an option. If not, the firm has no credible commitment, and could simply hire as many workers as is needed without using overtime.[10]

We assume that the inverse demand functions for the two differentiated products are given by:[11]

$$p = 1 - x - by \tag{1}$$
$$q = 1 - y - bx \tag{2}$$

[9]Trejo (1993) shows that U.S. data supports a notion that unionization will in and by itself lead to overtime premiums, reducing the overtime hours used and possibly expanding employment.

[10]In our simple one-shot set-up, a union would have an incentive to push up wages (if possible) after the labor stock is set. In a repeated game, however, the union may refrain from opportunistic behavior of this sort, and we therefore assume that wages are in fact fixed when the labor stock and prices are set.

[11]These kind of demand functions can be obtained from a representative consumer maximizing a quadratic utility function.

b is the differentiation parameter, $0 < b < 1$. x is the production by firm 1, p being the corresponding price. y and q are the total production level and (the uniform) price of the competing firms. Solving for quantities yields:

$$x(p,q) = \frac{1 - b - p + bq}{1 - b^2} \tag{3}$$

$$y(p,q) = \frac{1 - b - q + bp}{1 - b^2} \tag{4}$$

We assume that labor is the only factor of production in firm 1, and that a single worker will produce one unit of output during regular hours.[12] In addition, by using overtime, the worker can produce at most γ more units. Total possible sales from firm 1 are thus $n(1 + \gamma)$. The worker receives a compensation w_1 for the regular hours and w_2 for any additional hours. For simplicity we assume $w_2 > w_1$.[13] However, having set the total number of workers, n, the fixed wage cost $w_1 n$ is also determined, and production up to the level n involves a perceived zero marginal cost (at the price-setting stage). Overtime can be used in varying amounts, though, and production levels between n and $n(1 + \gamma)$ involve a marginal production cost of w_2.

With the current set-up, firm 1 has effectively two instruments through which it can push up prices. A high overtime wage commits the firm to a high marginal cost (and thus a high price) if it is to produce using overtime. However, the firm may also use the capacity of its plant(s) to credibly commit to non-aggressive pricing.

We solve by backwards induction. Given wages and employment, prices are determined last.

3 Price setting

When setting prices, firm 1 (the firm, henceforth) is assumed to choose the price p that maximizes profits given the price level of the competitors:[14]
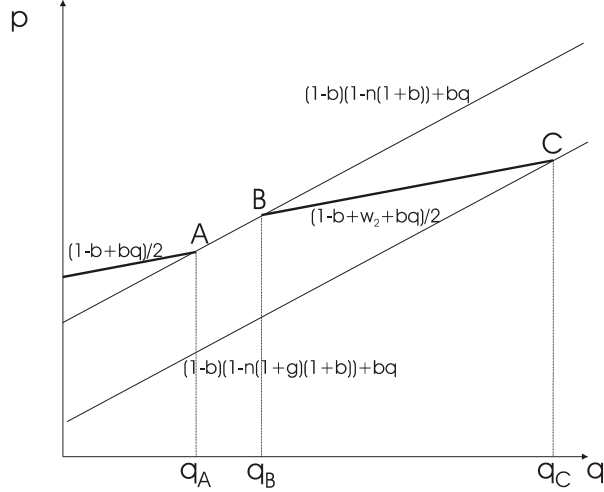
$$\Pi = \begin{cases} px(p,q) - w_1 n & if & x(p,q) < n \\ (p - w_1)n + [p - w_2][x(p,q) - n] & if & n \leq x(p,q) \leq n(1 + \gamma) \\ (p - w_1)n + (p - w_2)n\gamma & if & n(1 + \gamma) < x(p,q) \end{cases} \tag{5}$$

---

[12]Thus we abstract from the before-mentioned capital substitution effect.

[13]This type of restriction is found in most kinds of overtime pay regulations. However, in the present Bertrand set-up, firms and unions could all want a high overtime wage. Nonetheless, to avoid lengthy discussions of less interesting cases, we restrict attention to situations where it is *assumed* that the overtime wage exceeds the regular wage.

[14]Note that the regular wage costs are fixed at this stage since the labor stock has already been determined.

This gives us different best reply functions for the different regimes as expressed in (5). Noting that $x(p,q) < n \Leftrightarrow [1-b][1-n(1+b)] + bq < p$ and $n(1+\gamma) < x(p,q) \Leftrightarrow p < [1-b][1-n(1+\gamma)(1+b)] + bq$, it is easily shown that the following picture describes the situation:

The figure shows three different points, A, B and C, constraining the different possible equilibria. For $q > q_C$, where

$$q_C = \frac{w_2 - 1 + 2n(1-b^2)(1+\gamma) + b}{b}, \tag{6}$$

the best reply to a price q, is to choose the highest possible price that enables the firm to sell everything it produces when utilizing full overtime. In this case, $p = [1-b][1-n(1+\gamma)(1+b)] + bq$.

For $q \in (q_B, q_C]$, where

$$q_B = \frac{w_2 - 1 + 2n(1-b^2) + b}{b}, \tag{7}$$

it turns out to be optimal for the firm only to use part overtime. The best reply to a price q by the competitor is $p = \frac{1}{2}(w_2 + 1 - b + bq)$. In this segment, the price of the competing good is not high enough to warrant setting a price that leads to full overtime production. Neither does the firm want to set a price that makes it produce without using overtime.

For $q \in (q_A, q_B]$, where

$$q_A = \frac{-1 + 2n(1-b^2) + b}{b}, \tag{8}$$

it is optimal to price according to normal capacity. That is to set a price just high enough to avoid any use of overtime, which implies choosing $p = [1 - b][1 - n(1 + b)] + bq$.

For $q \leq q_A$, the best reply involves a price which induces a level of sales below the normal production capacity n (even though workers effectively produce at zero marginal cost). Formally, the best reply function in this segment is given by $p = \frac{1}{2}(1 - b + bq)$.

Thus the best reply function has four segments:

$$
p = \begin{cases}
\frac{1}{2}(1 - b + bq) & if & q < q_A \\
[1 - b][1 - n(1 + b)] + bq & if & q_A \leq q < q_B \\
\frac{1}{2}(w_2 + 1 - b + bq) & if & q_B \leq q < q_C \\
[1 - b][1 - n(1 + \gamma)(1 + b)] + bq & if & q \geq q_C
\end{cases}
\tag{9}
$$

To obtain a situation where committing to non-aggressive pricing yields a positive strategic feedback, we need to specify a market structure where the competitors respond to a higher p by increasing q. To obtain closed form solutions and to make the analysis as tractable as possible, we specify the market pricing response to be given by:

$$
q = \frac{1}{2}(1 - b + bp)
\tag{10}
$$

This would be the optimum price set by a single price setting firm producing under a non-binding capacity constraint and zero marginal costs. However, it should rather be thought of as a simple way to quantify that committing to a high price can induce a positive strategic response in the market.

The Nash-Bertrand equilibrium is then given by:

$$
p, q = \begin{cases}
\frac{1-b}{2-b}, \frac{1-b}{2-b} & if & n \geq n_A \\
\frac{(1-b)(2+b)-2n(1-b^2)}{2-b^2}, \frac{(1-b^2)(1-nb)}{2-b^2} & if & n_B \leq n < n_A \\
\frac{2w_2+(1-b)(2+b)}{4-b^2}, \frac{(1-b)(2+b)+bw_2}{4-b^2} & if & n_C \leq n < n_B \\
(1 - b)\frac{2+b-2n(1+\gamma)(b+1)}{2-b^2}, (1 - b^2)\frac{1-bn(1+\gamma)}{2-b^2} & if & n < n_C
\end{cases}
\tag{11}
$$

where
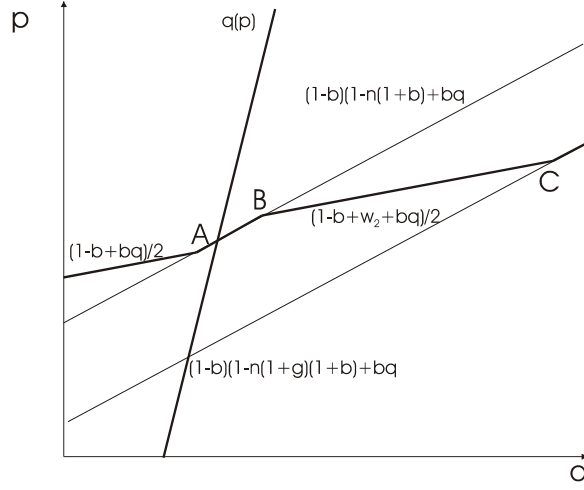
$$
n_A = \frac{1}{(1 + b)(2 - b)}
\tag{12}
$$

$$
n_B = \frac{(b + 2)(1 - b) - (2 - b^2)w_2}{(1 - b^2)(4 - b^2)}
\tag{13}
$$

$$
n_C = \frac{(b + 2)(1 - b) - (2 - b^2)w_2}{(1 - b^2)(4 - b^2)(1 + \gamma)}
\tag{14}
$$

For later reference, the following figure illustrates an equilibrium corresponding to the second regime in (11). Here, overtime is not used in equilibrium. However, the prices are pushed up relative to the situation where the firm has excess capacity: Since it cannot produce more without incurring high marginal costs (w_2), the firm has committed to non-aggressive pricing, yielding a positive strategic feedback.

4   Employment

We use backwards induction, and assume that the firm can estimate the equilibrium prices in the price setting game, choosing employment based on how employment affects profits in this equilibrium. When determining n, the firm then maximizes:

$$
\Pi = \begin{cases}
\frac{1-b}{(1+b)(2-b)^2} - w_1 n & if & n \geq n_A \\
n\big[\frac{(1-b)(2+b)-2n(1-b^2)}{2-b^2} - w_1\big] & if & n_B \leq n < n_A \\
\big[\frac{(1-b)(2+b)+2w_2}{4-b^2} - w_1\big]n + & & \\
\big[\frac{(1-b)(2+b)+2w_2}{4-b^2} - w_2\big]\big[\frac{(1-b)(2+b)-(2-b^2)w_2}{(4-b^2)(1-b^2)} - n\big] & if & n_C \leq n < n_B \\
\frac{(1-b)(2+b)-2n(1+\gamma)(1-b^2)}{2-b^2}n(1+\gamma) - (w_1 + w_2\gamma)n & if & n < n_C
\end{cases}
$$

(15)

In the first case we are in a situation where the two best reply functions cross to the left of A (figure 1). The firm is then utilizing only parts of the regular work force. A lower labor stock does not increase prices, but it does decrease labor costs (some workers are presently idle). And contrary to what is the case in the pricing game, the firm now takes this cost into

consideration. Thus the lowest possible level of n that ensures the desired production is the optimum. That is, $n = n_A$.

However, the firm can possibly do even better. By further decreasing the employment level, it can push prices up. As long as $w_2 > 0$ ($w_2 > w_1$ is not necessary for this result), the firm can credibly commit to non-aggressive pricing, at least if the number of workers is sufficiently large. Restricting the work force beyond the point A reduces equilibrium sales, but pushes up prices by so much that it may pay for the firm to do so. Formally, the unconstrained optimum is

$$n_1 \triangleq \frac{1}{4} \frac{(1-b)(2+b) - w_1(2-b^2)}{1-b^2} \tag{16}$$

for the second case (figure 2).[15] However, for this employment level to be a valid representation of the optimum, it has to satisfy $n_B \leq n_1 < n_A$, or

$$w_1 \leq \frac{-b^2(b+2)(1-b) + 4w_2(2-b^2)}{(4-b^2)(2-b^2)} \triangleq w_1^* \tag{17}$$

$$w_1 > -b^2 \frac{1-b}{(2-b)(2-b^2)} \tag{18}$$

The second inequality always holds. This means that the firm will never choose an employment level corresponding to n_A, as a further reduction in the labor stock induces a relatively high increase in equilibrium prices compared to the reduction in equilibrium sales.

For $w_1 \leq w_1^*$, a candidate for an optimum is thus given by n_1. However, for $w_1 > w_1^*$, the overtime wage is not sufficiently high to make the above optimum level of employment a credible restriction not to use overtime, and thus n is restricted to the point B. That is:

$$n_2 \triangleq \frac{(1-b)(2+b) - w_2(2-b^2)}{(1-b^2)(4-b^2)} \tag{19}$$

Either way, we have shown the following:

Remark 1 *Using the commitment provided by the overtime wage is always more profitable than not doing so. In addition, any non-zero overtime wage will provide some degree of commitment when the labor stock is already set, as it raises marginal costs above zero.*

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[15]It is worth noting that if this is to be positive, we need $w_1 < \frac{2-b-b^2}{2-b^2}$. This means that as products become perfect substitutes ($b = 1$), wages have to drop to zero to make it profitable to produce at all.

This result does not depend critically upon the linear demand system that we use in the formal analysis: Restricting production by increasing prices always involves a positive pricing response from the competitors when prices are strategic complements. As long as the slope of the reaction function between q_A and q_B (figure 1) is larger than for $q < q_A$, there is an additional incentive to decrease the labor stock in order to induce an even higher increase in the competitors' price. This would hold in every situation where the reaction function has an upward kink, which clearly does not apply only to linear demand systems.

In the third regime, the effects on profits from an increase in the employment level turns out to be particularly simple:

$$\frac{d}{dn}\Pi = w_2 - w_1 > 0 \tag{20}$$

Thus in this case the highest possible labor stock, n, would be the equilibrium. This again gives us n_2 as the candidate for an optimum. The intuition behind this result is as follows: When the firm utilizes overtime only in part, an increase in the labor stock has one effect; the firm is able to substitute overtime with regular working hours. This is a direct cost advantage. There is no pricing response, as this substitution of costs alters only the average costs, *not* the marginal costs (still w_2). Thus the firm would like to increase the labor stock to the point where no overtime is used (but the marginal cost of producing *more* is still w_2).

Producing while utilizing overtime (i.e. choosing a low employment level) may nonetheless be a good idea, but only if the firm is able to push up prices beyond point C (figure 1). In this situation average costs are relatively high. However, since the firm is not able to produce more than its full capacity, it credibly commits to choose a high price, inducing a favorable response by the competitors. Formally, the internal optimum in this case is given by:

$$n_3 \triangleq \frac{(b+2)(1-b)(1+\gamma) - (2-b^2)(w_1 + w_2\gamma)}{4(1+\gamma)^2(1-b^2)} \tag{21}$$

Now, if this employment level corresponds to a point to the left of point C in figure 1, we know that the labor stock associated with point C yields higher profits. In addition, n_2 is then better than n_3. Thus a necessary (but not sufficient) restriction for the above employment level to be a maximum is that the resulting pricing game equilibrium lies to the right of point C :

$$\frac{(b+2)(1-b)(1+\gamma) - (2-b^2)(w_1 + w_2\gamma)}{4(1+\gamma)^2(1-b^2)} \tag{22}$$

$$\leq \frac{(b+2)(1-b) - w_2(2-b^2)}{(1-b^2)(4-b^2)(1+\gamma)} \Leftrightarrow \tag{23}$$

$$w_1 \geq \frac{(2-b^2)(4+\gamma b^2)w_2 - b^2(b+2)(1-b)(1+\gamma)}{(2-b^2)(4-b^2)} \triangleq w_1^{**} \qquad (24)$$

This discussion is summarized in the below Proposition:

Proposition 2 *It is never beneficial for the firm to choose the labor stock in such a way that only parts of it is utilized for production, even when using overtime. The firm will always restrict employment such that either the overtime pay or the capacity of the plant (full overtime production) induces a positive pricing response by the competing firm(s).*

The intuition behind this result for the case of overtime follows the same lines as for Remark 1: The best reply function again has an upward kink at point C, and therefore restricting employment more is beneficial. Again, the result does not depend critically upon linear demand.

From the above discussion, we have the following *candidates* for extrema:

$$n = \begin{cases} n_1 & if & w_1 \leq w_1^* \\ n_2 & if & w_1 > w_1^* \\ n_3 & if & w_1 \geq w_1^{**} \end{cases} \qquad (25)$$

If we compare the profits from n_3 and n_2, we can find a switch-off point where n_3 is the better option. Following the above discussion, this would imply that w_1 is *strictly greater* than w_1^{**}. However, we cannot be sure that this is the relevant switch-off point for the firm as n_1 may still dominate n_3. It is possible, nonetheless, to show that n_2 dominates n_3 for $w_1 < w_1^{***}$, where $w_1^{***} > w_1^*$. This is proven in Appendix A. Thus, having determined this, we can conclude that the optimum employment level is:

$$n = \begin{cases} \frac{1}{4}\frac{(1-b)(2+b)-w_1(2-b^2)}{1-b^2} & if & w_1 \leq w_1^* \\ \frac{(1-b)(2+b)-w_2(2-b^2)}{(1-b^2)(4-b^2)} & if & w_1^{***} > w_1 > w_1^* \\ \frac{(2+b)(1-b)(1+\gamma)-(2-b^2)(w_1+w_2\gamma)}{4(1+\gamma)^2(1-b^2)} & if & w_1 \geq w_1^{***} \end{cases} \qquad (26)$$

w_1^{***} is given in Appendix A.

To recapitulate the intuition behind the above equilibrium, it is important to note that the firm faces two possibilities when it comes to inducing higher prices:

1. Choose employment high enough so that the pricing game equilibrium involves no use of overtime, but low enough to induce a positive pricing response (producing at normal capacity n). This is the case if the firm chooses either n_1 or n_2.

2. Choose an even lower employment level, leaving production at the full capacity level, and profiting from using the capacity constraint to increase equilibrium prices.

As is easily shown from the expressions of n_1, n_2 and n_3, a higher overtime wage will never lead to higher employment within any of the three regimes ($\frac{d}{dw_2}n_i \leq 0, \forall i$). We can however not conclude from this that an increase in the overtime wage will reduce employment. It is fairly easy to show that both $\frac{d}{dw_2}w_1^* > 0$ and $\frac{d}{dw_2}w_1^{***} > 0$. This means that as w_2 increases, n_1 becomes a more likely optimum (the interval where n_1 is optimal increases) and n_3 becomes a less likely optimum. Since n_3 is the option related to the lowest employment and n_1 is similarly involves higher employment than either n_2 or n_3, we have proven the following:

Proposition 3 *An increase in the overtime wage might increase equilibrium employment.*

Again, this will be valid for a wider range of demand systems than the family of linear demand systems analyzed in this paper: Suppose that the overtime pay premium is infinite ($w_2 \to \infty$). Then overtime will never be used, and the employment level n effectively becomes the maximum production level. The firm can thus restrict n in order to push prices up. Assume that the optimum employment level in this situation is n^H. On the other hand, for very low w_2, the overtime wage does not constitute a credible output restricting device. In this case, the firm would have to restrict full overtime production, $n(1 + \gamma)$, to increase prices. Assume that the optimum employment level is n^L in this case. Now suppose that the firm aims at producing *the same amount* in both cases.[16] Then $n^H = n^L(1 + \gamma)$ implying $n^H > n^L$. For a level of the overtime wage somewhere between the two extremes discussed above, the firm will be indifferent between the two options. Thus an increase in the overtime wage may lead to a shift in strategies from overtime production to regular production, increasing employment.

In such situations, strict labor regulation, or even unionization, may be favorable to employment. We discuss this in the next sections.

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[16]The *pricing responses* from committing to a lower production through a high overtime wage combined with low n, or only through low full-production capacity, $n(1+\gamma)$, are the same. Thus there is no reason for the firm, on the grounds of pushing up prices, to choose $n^H < n^L(1+\gamma)$. This could apply equally to other demand systems. Furthermore, production using overtime would imply higher costs (marginal as well as average), which rather would lead to $n^H > n^L(1 + \gamma)$ than the other way around. Consequently, these kind of considerations can only strengthen our conclusions.

5 Wage setting

In this section, we discuss different wage setting procedures. In order to form a benchmark case, we start out investigating what the firm itself would do if it could set the overtime wage. We assume that there is a minimum wage equal to $\overline{w}_1$ for regular hours. Also we assume that $w_2 > \overline{w}_1$ by regulation (see footnote 13). We then go on to see what would happen if a union organizing the labor force could determine wages (possibly *both* the base and overtime wages).

5.1 The firm deciding the overtime wage

In Appendix B we show that the optimal overtime wage in this case is given by:

$$w_2 \geq (b+2) \frac{\overline{w}_1 (2-b^2)(2-b) + b^2(1-b)}{2-b^2} \qquad (27)$$

The above level of the overtime wage ensures that overtime will not be used in equilibrium ($n_1 = n_2$). The firm will never want to induce the n_3 option, as this in all cases implies utilizing costly overtime, when it could instead push up prices with the use of a high overtime wage. Note also that an infinite overtime wage is not necessary, as there is a limit to how high the firm wants to push prices due to the direct negative effect on demand. We thus have:

Proposition 4 *If the firm can decide the overtime wage, it will set it sufficiently high to provide a credible commitment not to use overtime.*

Again, this result would hold more generally than for linear demand functions, as should be clear from the above intuitive argument.

Next we study what would happen if a union could set wages:

5.2 Union wage setting

In this section we assume that a union sets either the overtime wage or both the base wage and the overtime wage. There is no obvious way in which we can study negotiations between the firm and the union over wages in this set-up, as negotiations has to be concerned not only with wages, but also with what kind of regime (n_1, n_2, n_3) that will be induced in equilibrium.[17] We therefore stick to a monopoly union framework.

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[17]Technically, the elasticity of employment to labor differs in the three regimes. Simple bargaining models like, for instance, the Nash bargaining solution cannot, then, be applied directly.

While the firm maximizes expected profits, the union will typically maximize some composite function positively related to both wages and employment. For simplicity, we start out with the very simple case where the base wage is exogenous (possibly set by the authorities or a national labor union). If the union can only affect the overtime wage, the following result holds:

Remark 5 *If a union sets the overtime wage only and is sufficiently employment-oriented, it will choose a wage above the same threshold as the firm would have done.*

The intuition behind this result is not difficult to grasp: If the union chooses a lower overtime wage (the only relevant option) than what the firm itself would have wanted, then the firm has two options:

1. Choose a lower employment level to keep prices high (n_2 option). This reduces employment, but does not affect equilibrium pay (w_1 unaffected). Thus it cannot be better for a union caring positively about employment.

2. Choose to significantly reduce the work force, utilizing full overtime and pushing up prices through the restricted capacity of the plant. The employed members of the union *may* benefit from the increased income (working full overtime), but this benefit comes only to a few as the firm will cut employment massively to push up prices. Thus this option will typically not be appealing if the union is employment-oriented.

Following the above Remark, one might wonder if unionization may have any impact in our model. However, if the firm promises a high overtime wage premium, it is not certain that this credibly restricts the firm. If an employee wants to work at a lower premium, there is possibly no one that can prevent this from happening. On the other hand, if a union sets the overtime wage, the commitment could typically carry more weight. Thus, we can conclude:

Remark 6 *If the union can credibly commit to a high overtime wage, while the firm cannot, unionization may be profitable for the firm.*

Above, we discussed the case where the union could only set the overtime wage. However, the union could possibly be involved in the determination of both the base wage and the overtime wage. To this end, we assume that the union can set both these wages. In order to obtain analytical solutions, however, we need to make specific assumptions about union preferences. In

the rest of the analysis, union utility is assumed to be of the Stone-Geary type:

$$U = \begin{cases} (w_1)^a(n_1)^{1-a} & if & w_1 \leq w_1^* \\ (w_1)^a(n_2)^{1-a} & if & w_1^{***} > w_1 > w_1^* \\ (w_1 + \gamma w_2)^a(n_3)^{1-a} & if & w_1 \geq w_1^{***} \end{cases} \quad (28)$$

$a \in (0, 1)$ is a measure of union wage-orientation.[18]

In Appendix C we show that the optimum base and overtime wage levels are given by:

$$\begin{array}{ll} w_1 = a\frac{(1-b)(2+b)}{2-b^2}, \ w_2 \geq w_2^* & if \quad a \leq \frac{1}{2} \\ w_1 + \gamma w_2 = a\frac{(b+2)(1-b)(1+\gamma)}{(2-b^2)}, \ w_2 \leq w_2^{**} & if \quad a > \frac{1}{2} \end{array} \quad (29)$$

w_2^* and w_2^{**} are also provided in Appendix C.

An employment-oriented union ($a \leq \frac{1}{2}$) chooses a high overtime wage, which induces the firm to produce at normal capacity (without using overtime). By contrast, a wage-oriented union ($a > \frac{1}{2}$) induces the firm to operate at full capacity, which involves a higher total wage at the expense of lower employment. These two options results in the same level of utility for a rent-maximizing union ($a = \frac{1}{2}$).

We have thus showed:

Proposition 7 *An employment (wage) oriented union will choose a relatively high (low) overtime wage, inducing the firm to choose a high (low) level of employment and producing with no (full) use of overtime.*

Usually, we would think that workers have a greater disutility from working an hour overtime than from a regular hour. If the union takes this into consideration, it would tilt the union toward choosing a high overtime wage (inducing high employment).

The result, that there will be high employment for $a \leq \frac{1}{2}$ and low employment for $a > \frac{1}{2}$, *is* dependent upon the linear demand system used in this paper. For $a = \frac{1}{2}$, the linearity makes the wage and employment effects (discussed above) cancel out. However, the result in Proposition 7 will hold for a much wider range of demand functions: Say that the union is highly

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[18]It is worth noting that in the overtime case, it is still employment (n) and not full production ($n(1-\gamma)$) that enters the union maximand. This is assumed for two reasons: First, it is by no means clear that a union sees it as beneficial that its members work overtime *per se*. Rather, union members would possibly associate some disutility with working overtime (not included in the analysis). Also, in the case of $a = \frac{1}{2}$, the unions are rent maximizers both under the overtime and the non-overtime regimes, which is a useful property.

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employment-oriented. The union would then set a high overtime wage to induce the firm to choose a large labor stock. In the opposite case, a very wage-oriented union would want the highest possible total wage, not caring much about employment. This means inducing the firm to produce using overtime (at a high total wage $w_1 + \gamma w_2$). Due to continuity, there exists some intermediate level of union wage-orientation for which the union is indifferent between the two options.

6 Regulation

The wage structure may also be influenced by public regulation. As noted earlier, setting a high overtime wage could induce higher employment in the relevant firm (Proposition 3). In addition, it is clear from the last section that if the authorities are successful in passing legislation that effectively causes unions to act in a more employment-oriented manner, unemployment may be reduced in industries where these kinds of considerations are important.

In this section we briefly study three possible ways that the authorities may affect market outcomes: First the impact of minimum wage legislation is discussed. We then turn to assessing the consequences of initiating a minimum overtime pay premium. Third, we study the impact of restricting the number of hours of overtime that any individual may work.

6.1 Minimum wages

In this section, we assume that the authorities set a base wage level, $\overline{w}_1$, which cannot be undercut. If the firm unilaterally chooses the overtime wage, we have showed that this would imply inducing the n_1-option. From (25) it is clear that if the base wage requirement is increased, equilibrium employment falls. However, an increase in the base wage does not make the firm want to pursue another strategy (n_2, n_3), thus an increase in the minimum wage never leads to a *jump* in unemployment. This could, however, happen in the unionized case. For a highly employment-oriented union (a small), it may be that the equilibrium base wage, given by (40), is low enough to violate the minimum wage requirement. Thus the first best option is not possible for the union, which may then choose one of two strategies:

1. Set the base wage equal to the minimum wage and still choose a high overtime wage, which induces a fairly high level of employment

2. Choose a high wage strategy where the firm employs fewer and utilizes full overtime.

The second option is not necessarily affected by the minimum wage legislation, as what is important for the union in this case is the *total* wage (see (28)). The union could then possibly increase w_1 and reduce w_2 accordingly without violating any constraints. Thus a wage-oriented union would typically still induce the firm to produce using full overtime. However, an employment-oriented union may change its behavior: It could choose to switch to a low employment/high wage strategy because the low base wage is no longer possible:

Remark 8 *A marginal increase in the minimum wage will likely have no effect if wages are set by a wage-oriented union, and it will only have a marginal negative effect on employment in the non-unionized case. If the union is employment-oriented, however, stricter minimum wage legislation may cause a discontinuous fall in employment.*

It could be argued that minimum wage regulations may prove to be non-restrictive in industries where unions have an important role in wage setting. However, minimum wages could still restrict the options of an employment-oriented union when, say, demand has dropped and (local) unions would like to temporarily adjust their wage claims in order to prevent massive layoffs. Nonetheless, it would not be unreasonable to argue that minimum overtime mark-ups and caps on overtime hours may possibly turn out to be more restrictive than minimum wages in a unionized set-up. We will discuss these cases next.

6.2 Minimum overtime mark-up

We now assume that the authorities set a mark-up β, so that $w_2 \geq (1+\beta)w_1$ is required by law.

If the firm can itself choose the overtime wage, this kind of regulation would never change the optimal employment level. As we saw from the last section, the firm will always be willing to choose a very high (even infinite is possible) overtime wage. This also holds for the case where an employment-oriented union sets the wage.

The same is no longer necessarily true when a wage-oriented union chooses the wages. In this case we found that there was an *upper* limit on the overtime wage (which ensured that the firm would choose to produce utilizing overtime). The lowest possible w_2 is such that $w_2 = (1+\beta)w_1$. The internal

optimum would then involve:

$$\frac{w_2}{(1+\beta)} + \gamma w_2 = a\frac{(b+2)\,(1-b)\,(1+\gamma)}{(2-b^2)} \Leftrightarrow \tag{30}$$

$$w_2 = a(1+\beta)\frac{(b+2)\,(1-b)\,(1+\gamma)}{(2-b^2)(1+\gamma(1+\beta))} \tag{31}$$

It is easy to show that this overtime wage is increasing in β, which means that the authorities, by increasing the mark-up requirement, can make the first-best option for a wage-oriented union impossible. Thus:

Remark 9 *A minimum overtime mark-up will affect employment only if wage setting is unionized and unions are wage-oriented. In this case, such regelation may induce a wage-oriented union to change its behavior, opting for a strategy similar to what an employment-oriented union would choose. Employment may then be significantly increased.*

Of course, a highly wage-oriented union strictly prefers the low employment/ high wage option. This would still be the case if the authorities sets a low β. Thus, the regulated overtime pay premium would have to be quite substantial for a highly wage-oriented union to choose a strategy involving high employment and low total wages.

6.3   A cap on overtime hours

In this section we discuss the possibility that the authorities may regulate the number of hours of overtime any single employee is allowed to work. This would amount to restricting γ.

As is clear from the previous discussion, a cap on overtime hours will affect the outcome neither when the firm sets the overtime wage nor when wages are set by an employment-oriented union. This is, of course, due to the fact that overtime is not utilized in equilibrium in either of these cases. However, if wage setting is unionized and the union is wage-oriented, regulation may have an effect. To see this, note that from (29), the total wage obtained by the union is an increasing function of γ. Thus a cap on overtime hours reduces the total wage that is obtainable. There are two effects that determine the impact of such a regulatory scheme on employment: First, a cap on overtime hours is the same as lowering plant capacity. Thus, if the firm wants to operate at a given capacity, it would have to increase its labor stock. Second, wages are cut as a result of regulation. This also induces the firm to increase labor demand. Both these effects are readily observed from (21). Thus we have:

Remark 10 *A cap on overtime hours will affect employment only if wage setting is unionized and unions are wage-oriented. In this case, the cap increases equilibrium employment.*

7 Conclusions and further remarks

In this paper, we utilized an imperfect competition model to study the impacts of unionization, overtime pay premiums and regulation on labor demand. We found that the authorities cannot obtain increased employment by imposing stricter overtime pay premium legislation or caps on overtime hours when labor is non-unionized, or if unions are employment-oriented. This corresponds to the results from 'fixed job'-models. However, if unions are wage-oriented, this result no longer applies, and stricter legislation of this sort may induce increases in employment.

We also show that minimum wages may have the opposite effect: Such legal requirements will have no effect in the case where a wage-oriented union sets wages, will have only a limited employment-reducing effect in the non-union case, but may possibly have a negative impact on employment if unions are employment-oriented.

Our model also predicts a spike in the distribution of hours per worker at maximum regular hours (n). We found that in both the non-unionized and the employment-oriented union cases, every employee works full normal hours. In the wage-oriented union case, however, full overtime will be utilized in equilibrium.

A serious shortcoming of the present model is that it does not include hiring and firing costs, which are potentially very important for the labor demand and overtime decisions of firms. Also, the model neglects randomness. Overtime may to a high degree be used to deal with temporary demand shocks, and this is why we never specified that the overtime wage should be infinite in the cases where an employment-oriented union or the firm itself sets the overtime wage. Instead we determined the *minimum* overtime pay premium that a union or firm would choose. Thus the model does not preclude the possibility that overtime may be used in specific periods. Rather, it provides a *sufficient* overtime wage level, ensuring that overtime would not be used under *normal* circumstances.

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A   Profits

From the text we find that profits, given that employment is given by n_2 and n_3, are:

$$\Pi(n_2) = \left((2+b)(1-b) - (2-b^2)w_2\right) \frac{(2+b)(1-b) + 2w_2 - (4-b^2)w_1}{(1-b^2)(2-b)^2(2+b)^2}$$
(32)

$$\Pi(n_3) = \frac{1}{8} \frac{\left((2+b)(1-b)(1+\gamma) - (2-b^2)(w_1+\gamma w_2)\right)^2}{(1+\gamma)^2(1-b^2)(2-b^2)}$$
(33)

We leave it to the interested reader to verify that $\Pi(n_2) > \Pi(n_3)$ for $w_1^- < w_1 < w_1^+$, where

$$w_1^+ = \frac{w_2(2-b^2)(\gamma(4+b^2)+4(\gamma^2+1)) - (1+\gamma)(2+b)(1-b)(b^2+4\gamma)}{(2-b^2)(4-b^2)}$$
(34)

$$+ \frac{2(1+\gamma)((2+b)(1-b)-w_2(2-b^2))\sqrt{2\gamma(2\gamma+b^2)}}{(2-b^2)(4-b^2)}$$

$$w_1^- = \frac{w_2(2-b^2)(\gamma(4+b^2)+4(\gamma^2+1)) - (1+\gamma)(b+2)(1-b)(b^2+4\gamma)}{(2-b^2)(4-b^2)}$$
(35)

$$- \frac{2(1+\gamma)((2+b)(1-b)-w_2(2-b^2))\sqrt{2\gamma(2\gamma+b^2)}}{(2-b^2)(4-b^2)}$$

We can then show that w_1^+ does not satisfy the necessary restriction from the main analysis:

$$w_1^+ < w_1^{**} \Leftrightarrow$$
(36)

$$w_2 > \frac{(2+b)(1-b)}{2-b^2}$$
(37)

Since we have implicitly assumed that $w_2 < \frac{(2+b)(1-b)}{2-b^2}$ ($n_2 > 0$), the above restriction does not hold. Thus $w_1^{***} = w_1^-$. Following the previous discussion, we need to show that $w_1^{***} > w_1^*$, but

$$w_1^{***} > w_1^* \Leftrightarrow$$
(38)

$$w_2 < \frac{(2+b)(1-b)}{2-b^2}$$
(39)

As noted above, this is the case by assumption.

B   Overtime wages set by the firm

The profits in the three relevant regimes are easily found from the results in the main text. When setting the overtime wage, the firm then chooses

$$
w_2 = \arg\max \begin{cases} \frac{1}{8}\frac{[(1-b)(2+b)-w_1(2-b^2)]^2}{(1-b^2)(2-b^2)} & if & \overline{w}_1 \leq w_1^* \\ \frac{[(2+b)(1-b)-(2-b^2)w_2][(2+b)(1-b)+2w_2-(4-b^2)\overline{w}_1]}{(1-b^2)(2-b)^2(2+b)^2} & if & w_1^{***} > \overline{w}_1 > w_1^* \\ \frac{1}{8}\frac{[(b+2)(1-b)(1+\gamma)-(2-b^2)(\overline{w}_1+w_2\gamma)]^2}{(1+\gamma)^2(1-b^2)(2-b^2)} & if & \overline{w}_1 \geq w_1^{***} \end{cases}
$$

(40)

In the first case, w_2 does not affect the profits directly, but it does however affect the constraint $\overline{w}_1 \leq w_1^*$. As w_2 increases, w_1^* increases. Thus if the fixed wage, $\overline{w}_1$, is such that the restriction $\overline{w}_1 \leq w_1^*$ is initially violated, an increase in the overtime wage might make the n_1 option possible again. We know also that this option is always better than choosing n_2. Thus since it does not bring about any costs for the firm, an increase in the overtime wage so that $\overline{w}_1 = w_1^*$ will be profitable as long as n_3 is not even more profitable.

In the n_3 case, it is not the overtime wage, but rather the low labor stock that induces a positive strategic feedback. A high overtime wage only reduces profits through more expensive overtime (which is fully in use). Thus setting the lowest possible overtime wage would be profitable in this situation (it is easily shown that $\frac{d^2}{dw_2^2}[\frac{1}{8}\frac{\left((b+2)(1-b)(1+\gamma)-(2-b^2)(\overline{w}_1+w_2\gamma)\right)^2}{(1+\gamma)^2(1-b^2)(2-b^2)}] > 0$, implying a corner solution). Since w_1^{***} is increasing in w_2, the lowest possible w_2 coincides with the solution to $\overline{w}_1 = w_1^{***}$. In this case, however, the n_2 and n_3 options are equally profitable, and as we have noted before, the n_1 option can never be worse than n_2 when n_1 is possible. Thus the equilibrium entails the firm setting w_2 such that $\overline{w}_1 \leq w_1^*$, that is:

$$
w_2 \geq (b+2)\frac{w_1(2-b^2)(2-b)+b^2(1-b)}{2-b^2}
$$

(41)

This level of the overtime wage provides the firm with a sufficient commitment not to produce more than what can be achieved during regular hours. Instead using the commitment provided by full overtime production involves paying costly overtime. This is never favored by the firm.

C   Union wage setting

From the main text we have union utility given by:

$$U = \begin{cases} (w_1)^a(n_1)^{1-a} & if & w_1 \le w_1^* \\ (w_1)^a(n_2)^{1-a} & if & w_1^{***} > w_1 > w_1^* \\ (w_1 + \gamma w_2)^a(n_3)^{1-a} & if & w_1 \ge w_1^{***} \end{cases} \tag{42}$$

The union does not strictly care about the overtime wage in the n_1 case as long as it can choose w_1 freely (overtime is not used). Instead of inducing the n_2 state, it could do better (higher employment) if it instead chose w_2 sufficiently high as to make the n_1 option possible. Thus, competing with a possible n_3 alternative is again an option where $w_1 \le w_1^*$. In addition, the optimal (base) wage claim under n_1 is easily shown to be $w_1 = a\,(b+2)\,\frac{1-b}{2-b^2}$. Thus the relevant condition for being able to induce the n_1 employment level is

$$a\,(1-b)\,\frac{b+2}{2-b^2} \le \frac{4w_2(2-b^2) - b^2\,(b+2)\,(1-b)}{(2-b)\,(b+2)\,(2-b^2)} \Leftrightarrow \tag{43}$$

$$w_2 \ge \frac{1}{4}\,(b+2)\,(1-b)\,\frac{a(4-b^2)+b^2}{2-b^2} \triangleq w_2^* \tag{44}$$

Thus by setting $w_2 \ge w_2^*$, the union could also choose $w_1 = a\,(1-b)\,\frac{b+2}{2-b^2}$ (which maximizes union utility assuming that no-overtime should be induced) without fear that the firm may choose to set a low labor stock and utilize overtime.

By choosing w_1 and w_2 such that $w_1 \ge w_1^{***}$, the union could also induce the n_3 option. The optimum would then solve (the second order condition is satisfied):

$$\frac{d}{d(w_1 + w_2\gamma)}[(w_1 + \gamma w_2)^a n_3^{1-a}] = 0 \Leftrightarrow \tag{45}$$

$$w_1 + \gamma w_2 = a\,(b+2)\,(1-b)\,\frac{1+\gamma}{2-b^2} \tag{46}$$

Combining this with the condition $w_1 \ge w_1^{***}$ yields:

$$a\,(b+2)\,(1-b)\,\frac{1+\gamma}{2-b^2} - \gamma w_2 \ge w_1^{***} \Leftrightarrow \tag{47}$$

$$w_2 \le \frac{1}{2}\,\frac{(b+2)\,(1-b)}{2-b^2}\,\frac{b^2(1-a)+4(a+\gamma)+2\sqrt{2\gamma\,(2\gamma+b^2)}}{2\,(1+\gamma)+\sqrt{2\gamma\,(2\gamma+b^2)}} \triangleq w_2^{**} \tag{48}$$

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Thus the union would choose between the following two options:

$$U = \begin{cases} U_1 & \text{for} & w_1 = a\frac{(1-b)(2+b)}{2-b^2}, \ w_2 \geq w_2^* \\ U_3 & \text{for} & w_1 + \gamma w_2 = a\frac{(2+b)(1-b)(1+\gamma)}{(2-b^2)}, \ w_2 \leq w_2^{**} \end{cases} \quad (49)$$

where

$$U_1 = (2+b)(a\frac{(1-b)}{2-b^2})^a(\frac{1}{4}\frac{(1-a)}{1+b})^{1-a} \quad (50)$$

$$U_3 = (2+b)(a\frac{(1-b)(1+\gamma)}{2-b^2})^a(\frac{(1-a)}{4(1+\gamma)(1+b)})^{1-a} \quad (51)$$

The union will, of course, induce the regime that produces the higher utility. By comparing the two expressions, it is easily shown that the equilibrium is given by (29).